Selected Exercises for the Feynman Lectures on Physics by Richard Feynman, Et Al. Chapter 4 Kinematics – Detailed Work by James Pate Williams, Jr. BA, BS, MSwE, PhD

4.1

(a)

$$\ddot{x}(t) = a$$

$$\int_{v_{x_0}}^{v_x} dv = v_x(t) - v_{x_0} = a\int_0^t d\tau = at$$

$$v_x(t) = v_{x_0} + at$$

$$\int_{x_0}^{x} d\xi = x - x_0 = v_{x_0}\int_0^t d\tau + a_x\int_0^t d\tau = v_{x_0}t + \frac{1}{2}at^2$$

$$x(t) = x_0 + v_{x_0}t + \frac{1}{2}at^2$$

(b)

$$t = \frac{v_x - v_{x_0}}{a}$$

$$x(t) = x_0 + v_{x_0}\left(\frac{v_x - v_{x_0}}{a}\right) + \frac{1}{2}a\left(\frac{v_x - v_{x_0}}{a}\right)^2 = x_0 + \frac{v_x v_{x_0} - {v_{x_0}}^2}{a} + \frac{1}{2}a\left(\frac{{v_x}^2 - 2v_x v_{x_0} + {v_{x_0}}^2}{a^2}\right)$$

$$a(x - x_0) = \frac{1}{2}{v_x}^2 - \frac{1}{2}{v_{x_0}}^2$$

$$2a(x - x_0) = {v_x}^2 - {v_{x_0}}^2$$

$${v_x}^2 = {v_{x_0}}^2 + 2a(x - x_0)$$

4.2

(a)

$$\ddot{x}(t) = a_x$$

$$\ddot{y}(t) = a_y$$

$$\ddot{z}(t) = a_z$$

$$\int_{v_{x_0}}^{v_x} dv = v_x(t) - v_{x_0} = a_x \int_0^t d\tau = a_x t$$

$$\int_{v_{y_0}}^{v_y} dv = v_x(t) - v_{y_0} = a_y \int_0^t d\tau = a_y t$$

$$\int_{v_{z_0}}^{v_z} dv = v_z(t) - v_{z_0} = a_z \int_0^t d\tau = a_z t$$

$$v_x(t) = v_{x_0} + a_x t$$

$$v_y(t) = v_{y_0} + a_y t$$

$$v_z(t) = v_{z_0} + a_z t$$

$$\int_{x_0}^{x} d\xi = x - x_0 = v_{x_0} \int_0^t d\tau + a_x \int_0^t d\tau = v_{x_0} t + \frac{1}{2} a_x t^2$$

$$\int_{y_0}^{y} d\eta = y - y_0 = v_{y_0} \int_0^t d\tau + a_y \int_0^t d\tau = v_{y_0} t + \frac{1}{2} a_y t^2$$

$$\int_{z_0}^{z} d\zeta = z - z_0 = v_{z_0} \int_0^t d\tau + a_z \int_0^t d\tau = v_{z_0} t + \frac{1}{2} a_z t^2$$

$$x(t) = x_0 + v_{x_0} t + \frac{1}{2} a_x t^2$$

$$y(t) = y_0 + v_{y_0} t + \frac{1}{2} a_y t^2$$

$$z(t) = z_0 + v_{z_0} t + \frac{1}{2} a_z t^2$$

(b)

$$x(t) = x_0 + v_{x_0} \left(\frac{v_x - v_{x_0}}{a_x}\right) + \frac{1}{2} a_x \left(\frac{v_x - v_{x_0}}{a_x}\right)^2 = x_0 + \frac{v_x v_{x_0} - {v_{x_0}}^2}{a_x} + \frac{1}{2} a_x \left(\frac{{v_x}^2 - 2 v_x v_{x_0} + {v_{x_0}}^2}{{a_x}^2}\right)$$

$$a_x(x - x_0) = \frac{1}{2}{v_x}^2 - \frac{1}{2}{v_{x_0}}^2$$

$$2a_x(x - x_0) = {v_x}^2 - {v_{x_0}}^2$$

$${v_x}^2 = {v_{x_0}}^2 + 2a_x(x - x_0)$$

$$y(t) = y_0 + v_{y_0}\left(\frac{v_y - v_{y_0}}{a_y}\right) + \frac{1}{2}a_y\left(\frac{v_y - v_{y_0}}{a_y}\right)^2 = y_0 + \frac{v_y v_{y_0} - {v_{y_0}}^2}{a_y} + \frac{1}{2}a_y\left(\frac{{v_y}^2 - 2v_y v_{y_0} + {v_{y_0}}^2}{{a_y}^2}\right)$$

$$a_y(y - y_0) = \frac{1}{2}{v_y}^2 - \frac{1}{2}{v_{y_0}}^2$$

$$2a_y(y - y_0) = {v_y}^2 - {v_{y_0}}^2$$

$${v_y}^2 = {v_{y_0}}^2 + 2a_y(y - y_0)$$

$$z(t) = z_0 + v_{z_0}\left(\frac{v_z - v_{z_0}}{a_z}\right) + \frac{1}{2}a_z\left(\frac{v_z - v_{z_0}}{a_z}\right)^2 = z_0 + \frac{v_z v_{z_0} - {v_{z_0}}^2}{a_z} + \frac{1}{2}a_z\left(\frac{{v_z}^2 - 2v_z v_{z_0} + {v_{z_0}}^2}{{a_z}^2}\right)$$

$$a_z(z - z_0) = \frac{1}{2}{v_z}^2 - \frac{1}{2}{v_{z_0}}^2$$

$$2a_z(z - z_0) = {v_z}^2 - {v_{z_0}}^2$$

$${v_z}^2 = {v_{z_0}}^2 + 2a_z(z - z_0)$$

4.3

(a)

$$\theta = \frac{s}{R} \approx \frac{R\sin\theta}{R} = \sin\theta \;\forall\theta \ll 1$$

$$e^{\pm i\theta} = \cos\theta \pm i\sin\theta$$

$$e^{i\theta}e^{-i\theta} = e^0 = 1 = (\cos\theta + i\sin\theta)(\cos\theta - i\sin\theta) = \cos^2\theta - i^2\sin^2\theta = \cos^2\theta + \sin^2\theta$$

$$\begin{aligned} e^{i(\theta\pm\phi)} = \cos(\theta \pm \phi) + i\sin(\theta \pm \phi) = e^{i\theta}e^{i\phi} &= (\cos\theta + i\sin\theta)(\cos\phi \pm i\sin\phi) \\ &= \cos\theta\cos\phi \pm i^2\sin\theta\sin\phi + i(\sin\theta\cos\phi \pm \cos\theta\sin\phi) \end{aligned}$$

$$\cos(\theta \pm \phi) = \cos\theta\cos\phi \mp \sin\theta\sin\phi$$

$$\sin(\theta \pm \phi) = \sin\theta\cos\phi \pm \cos\theta\sin\phi$$

$$\sin\theta \approx \theta$$

$$\cos\theta = 1 - \sin^2\theta \approx 1 - \theta^2 \approx 1$$

$$\frac{dy}{dx} = \lim_{\Delta x\to\infty}\frac{y(x + \Delta x) - y(x)}{\Delta x}$$

$$y(x) = \sin(x)$$

$$\frac{dy}{dx} = \lim_{\Delta x\to 0}\frac{\sin(x+\Delta x)-\sin(x)}{\Delta x} = \lim_{\Delta x\to 0}\frac{\sin(x)\cos(\Delta x)+\cos(x)\sin(\Delta x)-\sin(x)}{\Delta x}$$
$$= \lim_{\Delta x\to 0}\frac{\sin(x)\cos(\Delta x)-\sin(x)}{\Delta x} + \lim_{\Delta x\to 0}\frac{\cos(x)\sin(x)}{\Delta x}$$
$$= \lim_{\Delta x\to 0}\frac{\sin(x)-\sin(x)}{\Delta x} + \lim_{\Delta x\to 0}\frac{\cos(x)\Delta x}{\Delta x} = 0+\cos(x) = \cos(x)$$

$$y(x) = \cos(x)$$

$$\frac{dy}{dx} = \lim_{\Delta x\to 0}\frac{\cos(x+\Delta x)-\cos(x)}{\Delta x} = \lim_{\Delta x\to 0}\frac{\cos(x)\cos(\Delta x)-\sin(x)\sin(\Delta x)-\cos(x)}{\Delta x}$$
$$= \lim_{\Delta x\to 0}\frac{\cos(x)-\sin(x)\Delta x-\cos(x)}{\Delta x} = \lim_{\Delta x\to 0}\frac{\cos(x)-\cos(x)}{\Delta x} + \lim_{\Delta x\to 0}\frac{-\sin(x)\Delta x}{\Delta x}$$
$$= -\sin(x)$$

4.4

(a)

$$\theta = \omega t = \frac{Vt}{R}$$

$$x = R\cos\theta = R\cos\omega t$$

$$y = R\sin\theta = R\sin\omega t$$

$$v_x = \frac{dx}{dt} = -R\omega\sin\omega t = -V\sin\omega t$$

$$v_y = \frac{dy}{dt} = R\omega\cos\omega t = V\cos\omega t$$

$$a_x = \ddot{x} = \frac{dv_x}{dt} = -R\omega^2\cos\omega t = -\frac{V^2}{R}\cos\omega t$$

$$a_y = \ddot{y} = \frac{dv_y}{dt} = -R\omega^2\sin\omega t = -\frac{V^2}{R}\cos\omega t$$

$$a^2 = \sqrt{a_x^2+a_y^2} = \sqrt{\left(\frac{V^2}{R}\right)^2} = \frac{V^2}{R}$$

Since

$$\cos^2\theta + \sin^2\theta = 1$$

(b)

$$a_x + \omega^2 x = \ddot{x} + \omega^2 x = -R\omega^2\cos\omega t + R\omega^2\cos\omega t = 0$$

$$a_y + \omega^2 y = \ddot{y} + \omega^2 y = -R\omega^2\sin\omega t + R\omega^2\sin\omega t = 0$$

4.5

$$v_{y_0} = 1000\ ft\ min^{-1}$$

$$a_y = \ddot{y} = \frac{dv_y}{dt} = -g = -32.174\ ft\ sec^{-2} = -32.174\ \cdot 3600\ ft\ min^{-2}$$

$$a_y = -115830\ ft\ min^{-2}$$

$$v_y = -gt + v_{y_0}$$

$$y = -\frac{1}{2}gt^2 - v_{y_0}t + y_0$$

$$y_0 = 30000 ft$$

$$y = 0$$

$$t = \frac{-v_{y_0} \pm \sqrt{{v_{y_0}}^2 + 2gy_0}}{-g} = \frac{-1000 \pm \sqrt{10^6 + 2 \cdot 1115830 \cdot 30000}}{-115830} = \frac{-1000 \pm 1711.85}{-115830}$$

$$t \approx\ 0.720\,\mathrm{min} =\ 43.2\ sec$$

The time the balloon is in the air:

$$T = 1800\,\mathrm{sec} + 43.2\,\mathrm{sec} = 1843.2\ sec$$

$$v_y = -32.174 \cdot 43.2\ ft\ sec^{-1} + 0\ ft\ sec^{-1} = -1390\ ft\ sec^{-1}$$

4.6

$$x = x_0 + v_{x_0}t_x + \frac{1}{2}a_x t_x^2 = \frac{1}{2}a_x t_x^2$$

$$y = y_0 + v_{y_0}t_y + \frac{1}{2}a_y t_y^2$$

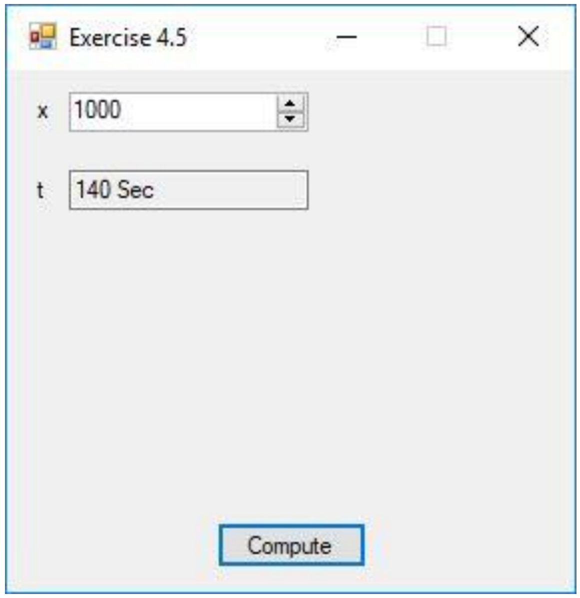
$$y_0 = x = \frac{1}{2}a_x t_x^2$$

$$v_{y_0} = a_x t_x$$

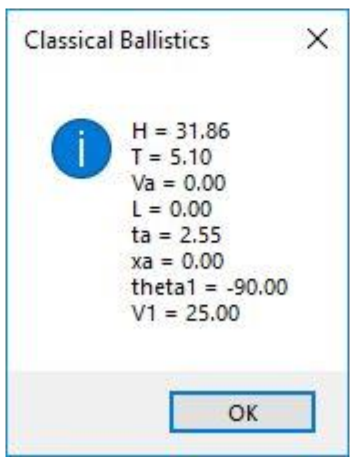
$$a_x = 20\ cm\ sec^{-2} = 0.20\ m\ sec^{-2}$$

$$a_y = -100\ cm\ sec^{-2} = -1.0\ m\ sec^{-2}$$

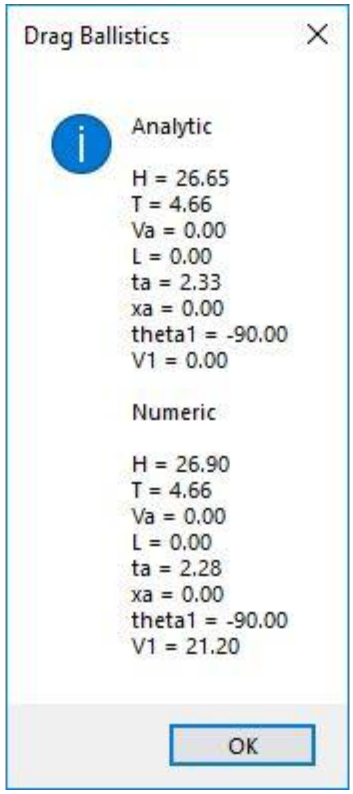
$$x + y = 2\ km = 2000\ m$$



4.7



The velocity of 25 meters per second is approximately 56 miles per hour. We use a velocity squared retardation function:



The analytic solution is not valid for theta0 = 90 degrees. The numeric shows a time to apogee of 2.28 seconds and time of flight 4.66 seconds. The difference is 4.66 – 2.28 seconds = 2.38 seconds so the time to return from apogee is greater than the time to reach apogee.