Exercises for the Feynman Lectures on Physics by Richard Feynman, Et Al. Chapter 18 Algebra – Detailed Work by James Pate Williams, Jr. BA, BS, MSwE, PhD

18.1

(a)

$$b = a\tan\alpha$$

$$d = c\tan\beta$$

$$u + iv = (a+ib)(c+id) = ac - bd + i(ad+bc)$$

$$\begin{aligned}(u+iv)(u-iv) = u^2+v^2 &= [(ac-bd)+i(ad+bc)][(ac-bd)-i(ad+bc)]\\ &= a^2c^2 - 2abcd + b^2d^2 + a^2d^2 + 2abcd + b^2c^2 = (a^2+b^2)c^2 + (a^2+b^2)d^2\\ &= (a^2+b^2)(c^2+d^2)\end{aligned}$$

$$\sqrt{u^2+v^2} = \sqrt{a^2+b^2}\sqrt{c^2+d^2}$$

(b)

$$\frac{v}{u} = \frac{ad+bc}{ac-bd} = \frac{ac\tan\beta + ac\tan\alpha}{ac - ac\tan\alpha\tan\beta} = \frac{ac}{ac}\cdot\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta} = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

18.2

(a)

$$b = a\tan\alpha$$

$$d = c\tan\beta$$

$$b\cos\alpha = a\sin\alpha$$

$$d\cos\beta = c\sin\beta$$

$$u+iv = (a+ib)(c+id) = (a+ia\tan\alpha)(c+ic\tan\beta) = ac(1+i\tan\alpha)(1+i\tan\beta)$$

$$u-iv = (a-ib)(c-id) = (a-ia\tan\alpha)(c-ic\tan\beta) = ac(1-i\tan\alpha)(1-i\tan\beta)$$

$$\begin{aligned}(u+iv)(u-iv) = u^2+v^2 = a^2c^2[(1+\tan^2\alpha)(1+\tan^2\beta)] &= a^2c^2\left(1+\frac{b^2}{a^2}\right)\left(1+\frac{d^2}{c^2}\right)\\ &= (a^2+b^2)(c^2+d^2)\end{aligned}$$

(b)

$$e^{i(\alpha+\beta)} = \cos(\alpha+\beta) + i\sin(\alpha+\beta) = e^{i\alpha}e^{i\beta} = (\cos\alpha + i\sin\alpha)(\cos\beta + i\sin\beta)$$
$$= \cos\alpha\cos\beta - \sin\alpha\sin\beta + i(\sin\alpha\cos\beta + \cos\alpha\sin\beta)$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\tan(\alpha+\beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\cos\alpha\cos\beta - \sin\alpha\sin\beta} = \frac{1}{\cos\alpha\cos\beta}\cdot\frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\left(1 - \frac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}\right)}$$
$$= \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = \frac{v}{u}$$

18.3

$$e^{+i\theta} = \cos\theta + i\sin\theta$$

$$e^{-i\theta} = \cos\theta - i\sin\theta$$

$$2\cos\theta = e^{i\theta} + e^{-i\theta}$$

$$\cos\theta = \frac{1}{2}\left(e^{i\theta} + e^{-i\theta}\right)$$

$$2i\sin\theta = e^{i\theta} - e^{-i\theta}$$

$$\sin\theta = \frac{1}{2i}\left(e^{i\theta} - e^{-i\theta}\right) = -\frac{i}{2}\left(e^{i\theta} - e^{-i\theta}\right)$$

18.4

$$\frac{a+ib}{c+id}\cdot\frac{c-id}{c-id} = \frac{ac+bd+i(bc-ad)}{c^2+d^2}$$

18.5

$$\cos i\theta = \frac{1}{2}\left(e^{-\theta} + e^{\theta}\right) = \cosh\theta$$

$$\sin i\theta = \frac{1}{2i}\left(e^{-\theta} - e^{\theta}\right) = -\frac{i}{2}\left(e^{-\theta} - e^{\theta}\right) = \frac{i}{2}\left(e^{\theta} - e^{-\theta}\right) = i\sinh\theta$$

$$\cos^2 i\theta + \sin^2 i\theta = 1 = \cosh^2\theta + i^2\sinh^2\theta = \cosh^2\theta - \sinh^2\theta$$

18.6

$$\frac{df}{dx} = \lim_{\Delta x\to\infty}\frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$\frac{de^{\alpha x}}{dx} = \lim_{\Delta x\to\infty}\frac{e^{\alpha(x+\Delta x)} - e^{\alpha x}}{\Delta x}$$

$$\cosh\alpha\theta + \sinh\alpha\theta = e^{\alpha x}$$

$$\cosh[\alpha(x+\Delta x)] + \sinh[\alpha(x+\Delta x)] = e^{\alpha(x+\Delta x)} = e^{\alpha x}e^{\alpha\Delta x}$$
$$= (\cosh\alpha x + \sinh\alpha x)(\cosh\alpha\Delta x + \sinh\alpha\Delta\mathrm{x})$$

$$e^{\alpha x}e^{\alpha\Delta x}-e^{\alpha x}=(\cosh\alpha x+\sinh\alpha x)(\cosh\alpha\Delta x+\sinh\alpha\Delta \mathrm{x}-1)$$

$$\lim_{\Delta x\to\infty}\frac{(\cosh\alpha x+\sinh\alpha x)(\cosh\alpha\Delta x+\sinh\alpha\Delta \mathrm{x}-1)}{\Delta x}=e^{\alpha x}\lim_{\Delta x\to\infty}\frac{(\cosh\alpha\Delta x+\sinh\alpha\Delta \mathrm{x}-1)}{\Delta x}$$
$$=e^{\alpha x}\lim_{\Delta x\to\infty}\frac{(1+\sinh\alpha\Delta \mathrm{x}-1)}{\Delta x}=e^{\alpha x}\lim_{\Delta x\to\infty}\frac{\sinh\alpha\Delta \mathrm{x}}{\Delta x}=e^{\alpha x}\lim_{\Delta x\to\infty}\frac{\alpha\Delta \mathrm{x}}{\Delta x}=\alpha e^{\alpha x}$$

18.7

$$f(x)=\sum_{n=0}^{\infty}\frac{f^{(n)}(0)}{n!}x^n$$

(a)

$$f(x)=e^x, f(0)=1$$

$$f'^{(x)}=e^x, f'^{(0)}=1$$

$$f''(x)=e^x, f''(0)=1$$

$$f(x)=e^x=\sum_{n=0}^{\infty}\frac{1}{n!}x^n=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots$$

(b)

$$f(x)=\cos x\,, f(0)=1$$

$$f'^{(x)}=-\sin x\,, f'^{(0)}=0$$

$$f''^{(x)}=-\cos x\,, f''^{(0)}=-1$$

$$f'''^{(x)}=\sin x\,, f'''^{(0)}=0$$

$$f'^{v(x)}=\cos x\,, f^{iv(0)}=1$$

$$f(x)=\cos x=\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}x^{2n}=1-\frac{x^2}{2!}+\frac{x^2}{4!}-\frac{x^3}{6!}+\cdots$$

$$f(x)=\sin x\,, f(0)=0$$

$$f'^{(x)}=\cos x\,, f'^{(0)}=1$$

$$f''^{(x)}=-\sin x\,, f''^{(0)}=0$$

$$f'''^{(x)}=-\cos x\,, f'''^{(0)}=-1$$

$$f'^{v(x)}=\sin x\,, f^{iv(0)}=0$$

$$f(x)=\sin x=\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}x^{2n+1}=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\cdots$$

18.8

$$y = \sqrt[n]{1}$$

$$y = x^{1/n}$$

$$\ln y = \frac{1}{n} \ln x$$

$$y = e^{\ln y} = e^{\ln x/n}$$

$$f(z) = e^{2\pi kzi/n} = \cos\left(\frac{2\pi kz}{n}\right) + i \sin\left(\frac{2\pi kz}{n}\right)$$

$$f(1) = e^{2\pi ki/n} = \cos\left(\frac{2\pi k}{n}\right) + i \sin\left(\frac{2\pi k}{n}\right)$$

18.9

$$e^{in\theta} = \left(e^{i\theta}\right)^n = \cos n\theta + i \sin n\theta = (\cos\theta + i \sin\theta)^n = \sum_{k=0}^{n} \binom{n}{k} i^{n-k} \cos^n \theta \sin^{n-k} \theta$$

$$\cos n\theta = \cos^n \theta + \binom{n}{2} \cos^{n-2} \theta \sin^2 \theta + \cdots$$

18.10

(a)

$$e^{i(\theta+\phi)} = \cos(\theta + \phi) + i \sin(\theta + \phi) = e^{i\theta} e^{i\phi} = (\cos\theta + i \sin\phi)(\cos\phi + i \sin\phi)$$
$$= \cos\theta \cos\phi - \sin\theta \sin\phi + i(\sin\theta \cos\phi + \cos\theta \sin\phi)$$

$$\cos(\theta + \phi) = \cos\theta \cos\phi - \sin\theta \sin\phi$$

$$\sin(\theta + \phi) = \sin\theta \cos\phi + \cos\theta \sin\phi$$

(b)

$$u = Ae^{i\theta}$$

$$v = Be^{i\phi}$$

$$w = uv = ABe^{i(\theta+\phi)} = AB[\cos(\theta + \phi) + i \sin(\theta + \phi)]$$

18.11

$$\log_a x = \log_a b \cdot \log_b x$$

$$y = x^{1/n}$$

$$\log_a y = \log_a b \cdot \log_b x/n$$

$$\log_a y = \log_a(x)/n$$

Root r	1/r	11^(1/r)	Log(r,11)
1	1.00000	11.0000	0.000
2	0.50000	3.3166	0.289
4	0.25000	1.8212	0.578
8	0.12500	1.3495	0.867
16	0.06250	1.1617	1.156
32	0.03125	1.0778	1.445
64	0.01563	1.0382	1.734
128	0.00781	1.0189	2.023

Log(7,11)	0.81151

$$\log_{11} 2 = 0.289$$

$$\log_{11} 7 = 0.811$$