A Modern Reincarnation of Ordnance Pamphlet 770 October 1941 by James Pate Williams, Jr. (jamespate@mac.com)

Abstract

This paper is dedicated to the effort of reproducing the first seven columns of United States Naval Ordnance Pamphlet 770 October 1941 using the exterior ballistics of the time, but on a modern digital computer. It should be well known that digital computers did not exist in 1941 and only electromechanical analog computers were available in that time frame. The pamphlet references "Exterior Ballistics 1935" by Lieutenant Commander Ernest Edward Herrmann of the United States Naval Academy so we rely heavily on this textbook. Exterior ballistics deals with the trajectory of a projectile after it leaves the barrel of the rifle. Interior ballistics is concerned with the dynamics of a projectile in the barrel of the gun.

Introduction

Ordnance Pamphlet 770 October 1941 [1] gives a range table for the 16-Inch/50 caliber naval artillery rifle that was used on the Iowa class of fast battleships. The projectile had a weight of 2700 pounds, an initial velocity in a new gun of 2500 feet per second, length of 4.5 calibers, and a radius of ogive of 9.0 calibers. It is to be noted that 4.5 / 9 = 0.5 and this number is the coefficient of form that we use in most of our ballistic calculations. According to Explanatory Note 5 of the document, the weather conditions to be assumed for the range table calculations are a density of 1.2034 kg/m^3 corresponding to a temperature of 59 degrees Fahrenheit, a barometric pressure of 29.53 inches Mercury (Hg), and a humidity of 78%.

As stated in Explanatory Note 14 of the pamphlet the firings upon which the range table is based are given in a table consisting of the number of rounds, angle of elevation, initial velocity, range, mean error, and pattern. All the range related data are in yards. We make extensive use of this data.

The range table of the pamphlet consists of nineteen columns, but we are just interested in the primary first seven columns. The columns are the range in yards, the angle of elevation in degrees and minutes, the angle of fall (obliquity) in degrees and minutes, the time of flight in seconds, the striking velocity in feet per second, the drift in yards, and the maximum y-ordinate also called the height at apogee or summit. It is surmised that the main cause of drift is the gyroscopic action of the projectile.

As was stated in the abstract no digital computers existed at the time of the writing of the pamphlet. The Harvard Mark I electromechanical analog computer was not running until 1944. Grace Hopper worked on the Harvard Mark I for the U.S. Navy Bureau of Ships [2] during the war years. The U.S. Navy did have the Ford Mark I Fire Control Computer (FCC) in the 1930s and 1940s [3]. The FCC was a very sophisticated electromechanical analog computer (calculator) that worked via cams, gears, differentials, motors, etc. A manual for the Ford Mark I can be found on the website [4]. The Ford Mark I cost astounding $75,000 U.S. dollars in 1945 which is $1,017,269.66 in 2017 U.S. dollars.

This paper consists of four more sections. The first section explains the theoretical foundations of the experimental computations, the second section describes our computer application, the third section gives our experimental results, and finally, a conclusions section wraps up the paper.

Theoretical Foundations

All the theory in our actual calculations comes from the United States Naval Academy undergraduate textbook "Exterior Ballistics 1935" [5]. The system of ordinary non-linear differential equations to be solved are given in Equations (601) to (606) in [5] which for clarity are reproduced in this paper below.

$$E = \frac{G_v \times H_y}{C}$$

$$x'' = -E\cos\theta$$

$$y'' = -E\sin\theta - g$$

$$x' = v\cos\theta$$

$$y' = v\sin\theta$$

$$v = \sqrt{(x')^2 + (y')^2}$$

The velocity dependent G function above is the Mayevski extensions of the Gavre functions. The H function is a height dependent ballistic density function. Both functions are found in Equations (403) and (405) of [6], and are restated here.

$$R_a = \frac{A_1}{C}v^{1.55}, \log A_1 = 7.60905 - 10\,, 2600 < v \leq 3600$$

$$R_a = \frac{A_2}{C}v^{1.7}, \log A_2 = 7.09620 - 10\,, 1800 < v \leq 2600$$

$$R_a = \frac{A_3}{C}v^{2}, \log A_3 = 6.11926 - 10, 1370 < v \leq 1800$$

$$R_a = \frac{A_4}{C}v^{3}, \log A_4 = 2.98090 - 10\,, 1230 < v \leq 1370$$

$$R_a = \frac{A_5}{C}v^{5}, \log A_5 = 6.80187 - 10\,, 970 < v \leq 1230$$

$$R_a = \frac{A_6}{C}v^{3}, \log A_6 = 2.77344 - 10\,, 790 < v \leq 970$$

$$R_a = \frac{A_7}{C}v^{2}, \log A_7 = 5.66989 - 10{,}0 < v \leq 790$$

$$G_v = CR_a$$

$$H_y = 10^{-0.00001372y}$$

The C in the E equation is the ballistic coefficient defined by Equation (406) in [6] and is repeated for the benefit of the reader here:

$$C = \frac{w}{\delta i d^2}$$

Where w is the weight in pounds which is 2700 pounds in our case, delta is the ratio of the density of the humid air to the standard density of 1.2034 kg/m^3, i is the coefficient of form which we use 0.5 in most cases, and d is the diameter of the projectile in our case 16 inches.

We solve the ordinary differential equations using the Runge-Kutta fifth-order method code found in the treatise [7]. We translated the excellent C code from C to C#. Incidentally, the C code was translated from an Algol scientific computing library. Also for angles of elevation less than 10 degrees we use Siacci's Method as described in [8].

The Computer Application

The application was written in C# on a personal computer using Windows Forms. The Microsoft Visual Studio 2015 integrated development environment was utilized. C# is a modern strongly typed object - oriented computer language. It is based on the venerable class of C computer languages which includes C, C#, C++, and Java.

The application consists at the time of this writing of six forms and thirteen non-form classes. The non-form classes are composed of around 3,000 lines of code and comments. The basic form is menu driven. The following two pictures illustrate the opening form and a range table generating form.

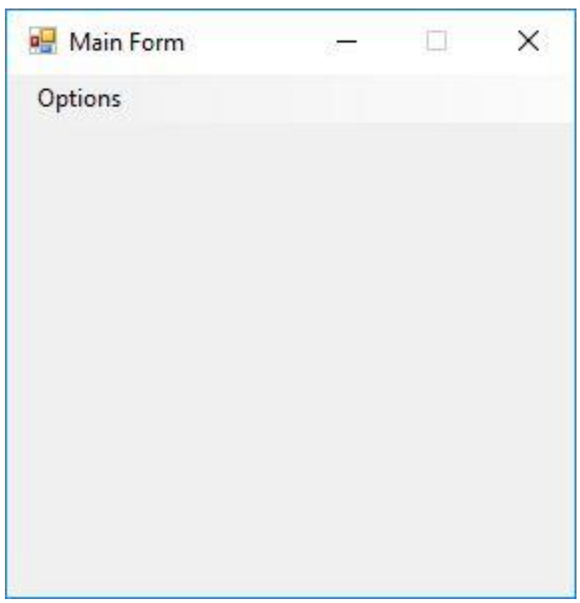

Figure 1 Main Application Form

The main application's Options Menu currently has three options: the determination of the optimal coefficients of form to be used in one of the range table generator functions, range table generation suing the Runge-Kutta firth-order numerical integration method, and range table generator using Siacci's method. The coefficients of for are found using an evolutionary hill-climber designed and implemented by the current author.

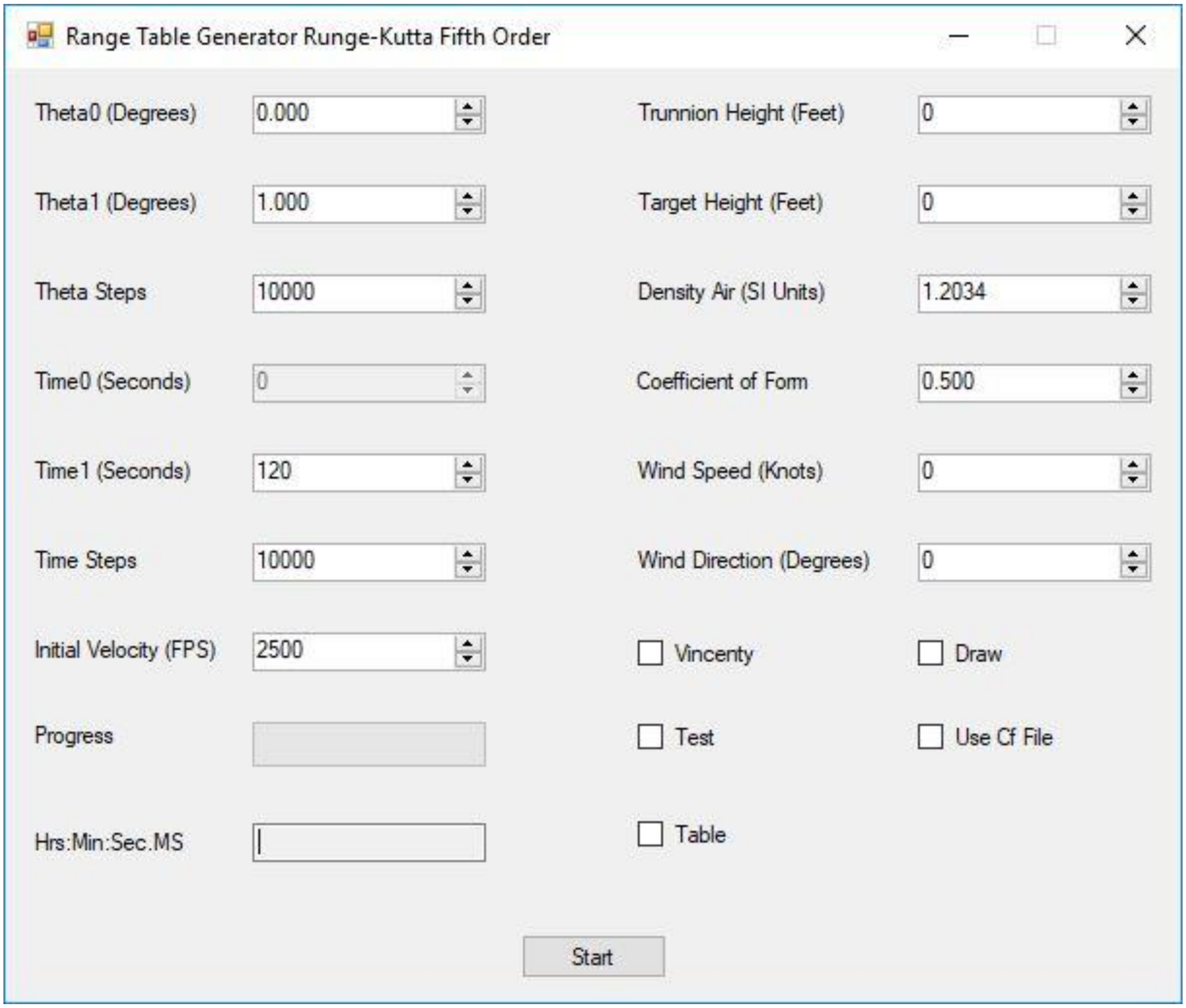

Figure 2 Range Table Generator Form

The theta angles in the preceding user interface are the beginning and ending angles of elevation for the table generation. If Theta 0 = Theta 1 then the trajectory (path of motion) of the projectile can be graphed. Also, if Theta 0 = Theta 1 = 0 and the Trunnion Height is greater than zero then the rifle can be fired horizontally and the drop of the projectile can be calculated and/or graphed. We illustrate such a case in the next section. If the Target Height is greater than zero there exists two points where the target intersects the trajectory. When Theta 0 = Theta 1 and Target Height greater than zero we compute both the ascending target and descending target data. Ascending targets could be aircraft in an anti-aircraft fire scenario. When Theta 0 not equal Theta 1 a table of range values in 100-yard increments beginning with 1000 yards is created. The number of range values to interpolated by angles of elevation is equal to the number of Theta steps. The Vincenty check box option is to include the curvature of the Earth in the calculations [9]. The density of the humid air is defaulted to the value of the pamphlet mentioned in the introduction. We display the meaning of the Test, Use Cf File, and Table check box options in the Experimental Results section. The coefficient of form can be experimented with to see its impact on the experimental results.

Experimental Results

The first experiment was to compare our results with the pamphlet's range table at very low angles of elevation namely 0 to 1 degrees. Our range table is shown in the following figure. The coefficient of form was 0.5 and the ballistic coefficient 21.09375. We used 20,000 angle of elevation steps which yielded a table of 20,000 entries to generate the resulting 12 entry range table. We chose Time 1 to be 5 seconds and used 10,000-time steps.

	A	B	C	D	E	F	G	H	I
1	Range	Start	Angle	Final	Angle	Time	V1	Drift	Y-Max
2	Yards	Degrees	Minutes	Degrees	Minutes	Sec	FPS	Yards	Feet
3	1000	0	26.9	0	27	1.21	2454	0.2	6
4	1100	0	29.6	0	30	1.33	2449	0.2	7
5	1200	0	32.3	0	33	1.46	2444	0.2	9
6	1300	0	35.1	0	36	1.58	2440	0.3	10
7	1400	0	37.8	0	38	1.7	2435	0.3	12
8	1500	0	40.6	0	41	1.83	2431	0.4	13
9	1600	0	43.3	0	44	1.95	2426	0.4	15
10	1700	0	46.1	0	47	2.07	2422	0.5	17
11	1800	0	48.9	0	50	2.2	2417	0.5	19
12	1900	0	51.7	0	53	2.32	2413	0.6	22
13	2000	0	54.5	0	56	2.45	2408	0.7	24
14	2100	0	57.2	0	59	2.57	2404	0.7	27

Figure 3 Range Table 1000 to 2100 Yards

Next, we display our detailed deviations from the pamphlet's range table at 1,000 to 2,100 yards.

	A	B	C	D	E	F	G	H	I
1	Detailed Deviations from Ordnance Pamphlet 770								
2	Range	Start	Angle	Final	Angle	Time	V1	Drift	Y-Max
3	Yards	Degrees	Minutes	Degrees	Minutes	Sec	FPS	Yards	Feet
4	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	_
6	0	0	0	0	0	-0.01	1	0	_
7	0	0	-0.1	0	0	0	0	0	_
8	0	0	-0.1	0	0	0	1	0	_
9	0	0	-0.1	0	0	-0.01	0	0	0
10	0	0	0	0	0	0	1	0	_
11	0	0	0	0	0	0	1	0	_
12	0	0	0	0	-1	0	1	0.1	_
13	0	0	0	0	-1	0	1	0	_
14	0	0	0	0	-1	0	1	0	0
15	0	0	0.1	0	-1	0	1	0.1	_

Figure 4 Detailed Deviations from the Pamphlet

We now show an error summary for the 12-entry range table. As you can easily detect we are having the most trouble accurately reproducing the drift numbers. We proceed to discuss our drift methodology.

	A	B	C	D
1	Total	30		
2	Out of	87		
3	% Error	34.48		
4				
5		Err Cnt	Avg Abs Dev	
6	Range	0	0	
7	Degrees	0	0	
8	Minutes	4	0.1	
9	Degrees	0	0	
10	Minutes	4	1	
11	Time	2	0.01	
12	V1	8	1	
13	Drift	12	0.028	

Figure 5 Error Summary for 12-Entry Range Table

Column 8 of our reproduction (Column 6 of the pamphlet) was created using Hamilton's drift formula which is Equation (901) on page 113 of Herrmann's book and given below:

$$D = X(1 - D')\frac{d^3}{\mu w}(\phi^0 + \omega^0)\sec\phi$$

Where X is the range, the primed D is the drift coefficient, d is the diameter of the projectile, w is the weight, mu is the final twist of the rifling, and the superscripted angles of elevation and fall must be expressed in radians. The pamphlet contains ten test firings of the guns in question using different angles of elevation and initial velocities. Unfortunately, the angle of fall and the drift are not tabulated. We compute the drift coefficient by transposing the preceding equation:

$$D' = 1 - \frac{D\mu w}{Xd^3(\phi^0 + \omega^0)}\cos\phi$$

The ranges and angles of elevation are known from the test firings. We approximate that the angle of elevation and the angle of fall are the same as in the vacuum case. We use 3 times the mean test firing errors as the drifts. The bore length of the rifle is 800 inches. Mu according to the pamphlet is 800 calibers / 32 = 25 calibers. The pamphlet states that for a gun with mu = 800 calibers / 25 = 32 calibers to multiply column 6 by 0.78 which is a rounded version of 25 / 32. We average the drift coefficients found using the above equation and get a value of 0.82700994 for the drift coefficient.

Now we perform the same experiments for between 4 and 5 degrees elevation. This time we use Time 1 of 20 seconds and the same Theta Steps (20,000) and Time Steps 10,000. The three tables generated are illustrated on the next page.

	A	B	C	D	E	F	G	H	I
1	Range	Start	Angle	Final	Angle	Time	V1	Drift	Y-Max
2	Yards	Degrees	Minutes	Degrees	Minutes	Sec	FPS	Yards	Feet
3	8200	4	2.4	4	28	10.66	2148	12.8	457
4	8300	4	5.7	4	32	10.8	2144	13.2	469
5	8400	4	9	4	36	10.94	2141	13.5	481
6	8500	4	12.3	4	40	11.08	2137	13.9	494
7	8600	4	15.6	4	44	11.22	2133	14.2	507
8	8700	4	18.9	4	48	11.36	2129	14.6	519
9	8800	4	22.3	4	52	11.5	2125	14.9	533
10	8900	4	25.6	4	56	11.65	2122	15.3	546
11	9000	4	29	5	0	11.79	2118	15.7	559
12	9100	4	32.3	5	4	11.93	2114	16.1	573
13	9200	4	35.7	5	9	12.08	2110	16.5	587
14	9300	4	39.1	5	13	12.22	2107	16.9	601
15	9400	4	42.5	5	17	12.36	2103	17.3	615
16	9500	4	45.9	5	21	12.51	2099	17.7	630
17	9600	4	49.3	5	25	12.65	2095	18.1	644
18	9700	4	52.7	5	29	12.8	2092	18.5	659
19	9800	4	56.2	5	34	12.94	2088	18.9	674
20	9900	4	59.6	5	38	13.09	2084	19.3	689

Figure 6 Range Table for 4 to 5 Degrees of Elevation

This time we generate an 18-entry range table. We are still in the second Mayevski zone of 1800 to 2600 feet per second velocities. That means our drag velocity exponent is less than quadratic at 1.7 instead of 2. Unfortunately, there are no short-range test firings. The number of errors between our results and the pamphlet range table is almost at 50%.

	A	B	C	D	E	F	G	H	I
1	Detailed Deviations from Ordnance Pamphlet 770								
2	Range	Start	Angle	Final	Angle	Time	V1	Drift	Y-Max
3	Yards	Degrees	Minutes	Degrees	Minutes	Sec	FPS	Yards	Feet
4	0	0	-0.2	0	0	-0.01	-2	1.2	_
5	0	0	-0.2	0	0	-0.01	-2	0.8	_
6	0	0	-0.2	0	0	-0.01	-3	0.5	_
7	0	0	-0.2	0	0	-0.01	-3	0.1	-3
8	0	0	-0.2	0	0	-0.01	-3	0.8	_
9	0	0	-0.2	0	0	0	-3	0.4	_
10	0	0	-0.3	0	0	0	-3	0.1	_
11	0	0	-0.2	0	0	-0.01	-4	0.7	_
12	0	0	-0.2	0	0	0	-4	0.3	-2
13	0	0	-0.1	0	0	0	-4	0.9	_
14	0	0	-0.1	0	0	0	-4	0.5	_
15	0	0	-0.1	0	0	0	-5	0.1	_
16	0	0	-0.1	0	0	0.01	-5	0.7	_
17	0	0	-0.1	0	0	0	-5	0.3	-2
18	0	0	-0.1	0	0	0.01	-5	0.9	_
19	0	0	-0.1	0	0	0	-6	0.5	_
20	0	0	-0.2	0	0	0.01	-6	0.1	_
21	0	0	-0.2	0	0	0	-6	0.7	_

Figure 7 Detailed Deviations from Pamphlet for 4 to 5 Degrees Elevation

	A	B	C	D
1	Total	63		
2	Out of	129		
3	% Error	48.84		
4				
5		Err Cnt	Avg Abs Dev	
6	Range	0	0	
7	Degrees	0	0	
8	Minutes	18	0.167	
9	Degrees	0	0	
10	Minutes	0	0	
11	Time	9	0.01	
12	V1	18	4.056	
13	Drift	18	0.543	

Figure 8 Detailed Deviations from Pamphlet for 4 to 5 Degrees Elevation

We try to duplicate the data in the test firings table of the pamphlet. First, we use a constant coefficient of form of 0.5. The results are displayed in the following figure.

	A	B	C	D	E	F	G	H	I	J	K
1	A	B	C	D	E	F	G	H	I	J	K
2	Init	V0	Range	Corr1	Range	Corr2	Range	Range	% Error	% Error	% Error
3	Angle	FPS	Yards	Yards	Yards	Yards	Yards	Real	C & H	E & H	G & H
4	10	2502	17734	-104	17630	-104	17630	17402	1.9092	1.312	1.3105
5	15	2502	24100	-118	23982	-118	23981	23642	1.9352	1.4361	1.4344
6	25	2500	33655	-124	33531	-124	33530	32971	2.0744	1.6987	1.6969
7	45	2500	43201	-101	43099	-102	43099	42643	1.3077	1.0702	1.0687
8	15	2478	23733	-115	23618	-115	23618	23182	2.377	1.8824	1.8807
9	20	2476	28852	-119	28733	-120	28732	28110	2.6392	2.2157	2.2139
10	25	2481	33249	-121	33128	-122	33127	32544	2.1665	1.7941	1.7923
11	30	2475	36688	-120	36569	-120	36568	36539	0.4081	0.0809	0.0791
12	45	2475	42462	-98	42364	-99	42364	43188	1.6804	1.9072	1.9086
13	15	2439	23138	-109	23029	-110	23028	23028	0.4777	0.0029	0.0014

Figure 9 Test Firings Comparison Constant Coefficients of Form

Columns A and H are taken from the test firings table in the pamphlet. Column D is the curvature of the Earth correction using the formula given in Herrmann. The column F is Vincenty formulas correction for the curvature or Earth or the great circle distance correction. Remember on a sphere the shortest distance between two points is a geodesic not a straight line. Columns I-J are percentage errors between our values and the pamphlet's test firing table. The percentage error is defined as 100 * |experimental value – true value| / | true value|. We take the pamphlet's values as the true values. We now repeat the same experiment using five different values of the coefficient of form, one for each initial velocity zone.

	A	B	C	D	E	F	G	H	I	J	K
1	A	B	C	D	E	F	G	H	I	J	K
2	Init	V0	Range	Corr1	Range	Corr2	Range	Range	% Error	% Error	% Error
3	Angle	FPS	Yards	Yards	Yards	Yards	Yards	Real	C & H	E & H	G & H
4	10	2502	17480	-100	17380	-100	17380	17402	0.4461	0.1269	0.1283
5	15	2502	23638	-111	23527	-112	23527	23642	0.0161	0.4867	0.4883
6	25	2500	33080	-118	32962	-119	32961	32971	0.3299	0.0286	0.0303
7	45	2500	42401	-96	42305	-97	42304	42643	0.5665	0.7926	0.794
8	15	2478	23177	-107	23071	-107	23070	23182	0.0195	0.4799	0.4814
9	20	2476	28066	-110	27956	-111	27956	28110	0.1552	0.5472	0.5487
10	25	2481	32280	-112	32168	-112	32168	32544	0.8114	1.1552	1.1568
11	30	2475	36794	-120	36673	-121	36673	36539	0.6968	0.3671	0.3654
12	45	2475	42586	-99	42488	-99	42487	43188	1.3928	1.6214	1.6228
13	15	2439	23011	-108	22904	-108	22903	23028	0.0736	0.5406	0.5422

Figure 10 Test Firing Comparison Variable Coefficients of Form

Next, we display the coefficients of form utilized and the corresponding ballistic coefficients.

	A	B	C	D	E
1	Coefficients of Form for Each Initial Velocity Zone				
2					
3	V0	2502	0.538038		
4	V0	2500	0.526946		
5	V0	2476	0.547196		
6	V0	2475	0.49574		
7	V0	2439	0.510899		
8					
9	Ballistic Coefficients for Each Initial Velocity Zone				
10					
11	V0	2502	19.60249		
12	V0	2500	20.01511		
13	V0	2476	19.2744		
14	V0	2475	21.27503		
15	V0	2439	20.64378		

Figure 11 Coefficients of Form Etc.

A word about the generation of the variable coefficients of form. We utilized an evolutionary hill climber to determine the coefficients. We used the sum of the percentage differences between the uncorrected range and pamphlet's range for each initial velocity zone as the fitness to be minimized. The hill climber used tournament selection with a tournament size of two. After each mutation of one of the five potential coefficients of form, we would replace the worst individual in the population. Therefore, the hill climber is elitist in nature.

As a final pamphlet related test, we look at angles of elevation form 5 degrees to the maximum elevation of 45 degrees in 5-degree steps. The results illustrated next.

Range Table Elevations 5 to 45 Degrees Runge-Kutta Fifth Order Method

Range Yards	Start Degrees	Angle Minutes	Final Degrees	Angle Minutes	Time Sec	V1 FPS	Drift Yards	Y-Max Feet
9904	5	0.0	5	38	13.10	2084	19	691
17711	10	0.0	12	14	25.39	1837	73	2598
24066	15	0.0	19	26	37.07	1659	157	5523
29297	20	0.0	26	41	48.19	1582	267	9320
33655	25	0.0	33	15	58.70	1567	396	13866
37284	30	0.0	39	0	68.71	1584	544	19034
40179	35	0.0	44	2	78.26	1618	710	24704
42250	40	0.0	48	31	87.36	1660	894	30760
43201	45	0.0	52	53	95.99	1699	1096	37065

Ordnance Pamphlet 770 Linear Interpolation Results

Range Yards	Start Degrees	Angle Minutes	Final Degrees	Angle Minutes	Time Sec	V1 FPS	Drift Yards	Y-Max Feet
9917	5	0.0	5	39	13.12	2077	20	–
17645	10	0.0	12	22	25.41	1807	75	–
23868	15	0.0	19	38	37.00	1653	159	–
28983	20	0.0	26	43	47.95	1576	267	–
33249	25	0.0	33	21	58.42	1554	397	–
36728	30	0.0	39	17	68.35	1564	543	–
39492	35	0.0	44	27	77.89	1598	705	–
41424	40	0.0	48	52	86.86	1643	877	–
42300	45	0.0	52	48	94.44	1682	1037	–

Deviations from Ordnance Pamphlet Results

Range Yards	Start Degrees	Angle Minutes	Final Degrees	Angle Minutes	Time Sec	V1 FPS	Drift Yards
13	0	0.0	0	1	0.02	-7	1.0
-66	0	0.0	0	8	0.02	-30	2.5
-198	0	0.0	0	12	-0.07	-6	2.4
-314	0	0.0	0	2	-0.24	-6	0.5
-406	0	0.0	0	6	-0.28	-13	1.0
-556	0	0.0	0	17	-0.36	-20	-0.6
-687	0	0.0	0	25	-0.37	-20	-4.6
-826	0	0.0	0	21	-0.50	-17	-17.1
-901	0	0.0	0	-5	-1.55	-17	-59.0

```
Ballistic coefficient   21.09375
Coefficient of form     0.5
Density 1.2034
Diameter        16
Weight  2700
V0      2500
```

Figure 12 Range Table Comparison 5 to 45 Degrees

Just for the fun of it consider the following scenario. Suppose you had a 16"/50 caliber naval artillery gun on a 32-foot trunnion overlooking the English Channel on Dover Cliffs (300 feet above sea level). Now we simulate the firing of the rifle. The projectile drops 332 feet in 3,699.23 yards or 2.1018 miles. The time of flight would be 4.595 seconds. See the accompanying figures. The angle of elevation was, of course, 0 degrees. Now suppose France is 20 miles away. We crank the elevation to 27 degrees and the AP projectile carries 35,468.95 yards or 20.1528 miles reaching an apogee (summit) of 16,230.65 feet or 3.0740 miles and the time of flight is 63.192 seconds. Raise the elevation to 45 degrees and the projectile goes 43,444.67 yards or 24.6850 miles reaching an apogee of 37,488.62 feet or 7.1001 miles and the time of flight is 96.348 seconds.

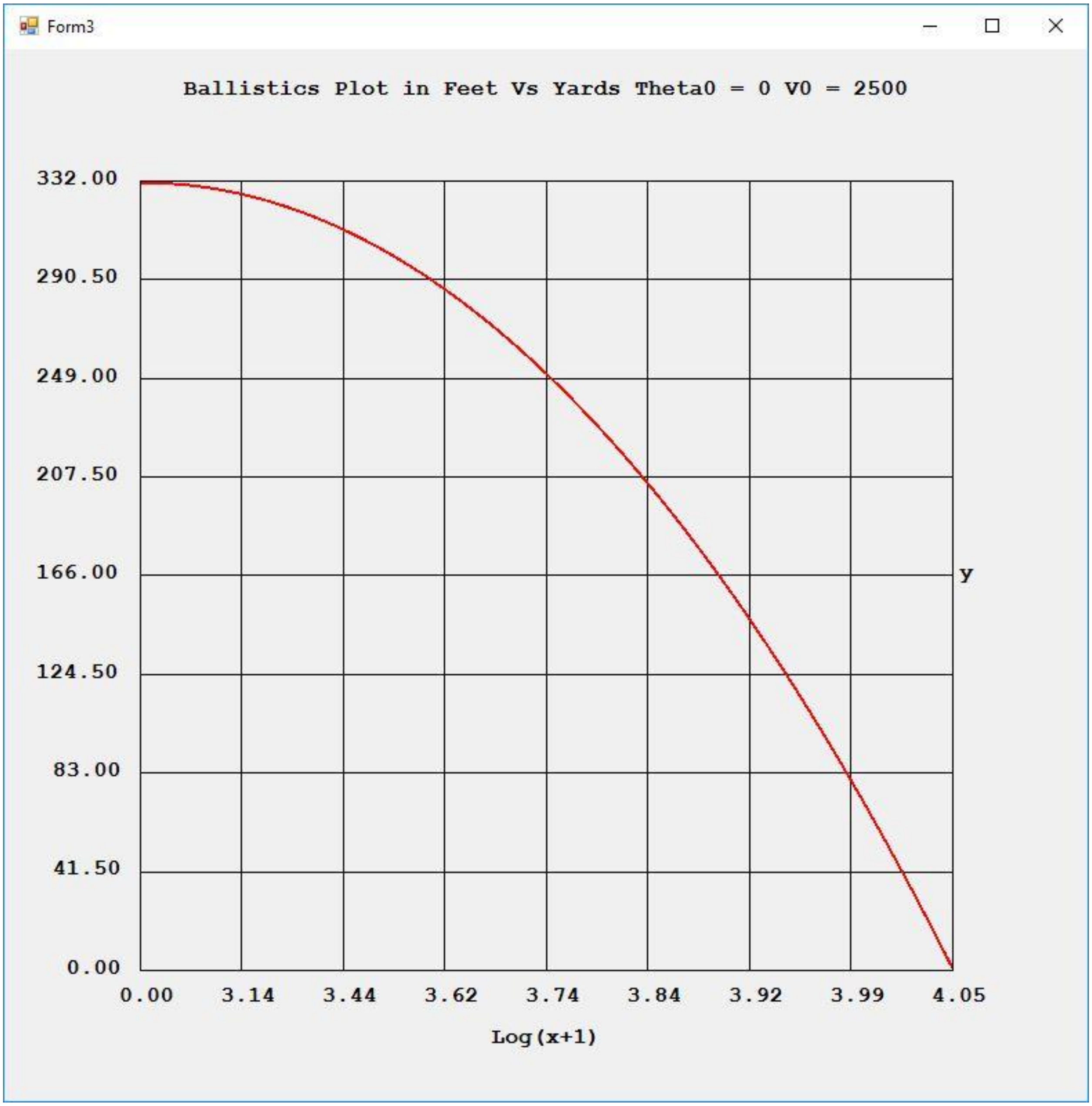

Figure 13 Firing from the Cliffs of Dover into the Straits

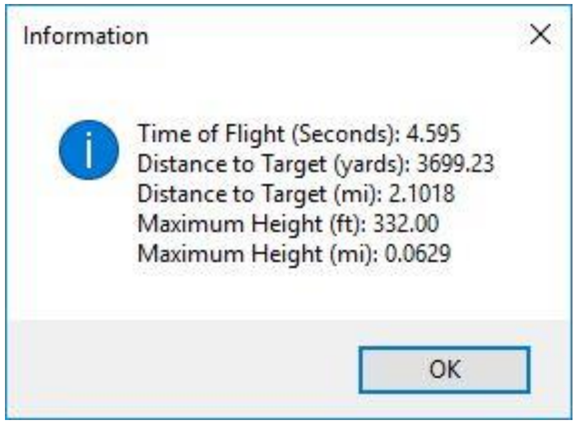

Figure 14 Message Box from Draw Option of Our Application

Conclusions

Although our efforts at exactly duplicating the results of the pamphlet did not come to fruition, we did come relatively close to the 1941 results. We had the distinct advantage of using a modern digital computer with all computations in double precision. Our double precision means 15 to 17 digits of significance. Back in the day, I am sure this would be considered an exciting amount of precision. The electromechanical analog computers in the early 1940s epoch must have had not many digits of precision. Manufacturing tolerances were just not that tight in that bygone but certainly not forgotten era. I suspect that the actual numerical integration of the 1940s computation was done using the Adams-Bashforth-Moulton predictor-corrector algorithm and not the Runge-Kutta or Siacci methods. The relative agreement between our numbers and the pamphlet's data at low angles of elevation is certainly satisfying from our perspective.

In the future, we will look deeper into the discrepancies between our results and those of the pamphlet. Also, we will publish our Siacci method experiments at low elevations of 0 to 10 degrees elevation.

References

[1] E. L. Slover, "Gene Slovers US Navy Pages," [Online]. Available: http://www.eugeneleeslover.com/USN-GUNS-AND-RANGE-TABLES/OP-770-1.html. [Accessed 1 7 2017].

[2] "Harvard Mark I," Wikipedia, [Online]. Available: https://en.wikipedia.org/wiki/Harvard_Mark_I. [Accessed 14 September 2017].

[3] G. Slover, "The Mark I Fire Control Computer," www.navweaps.com, [Online]. Available: http://www.navweaps.com/index_tech/tech-056.htm. [Accessed 14 Spetember 2017].

[4] "Computer Mark I," San Francisco Maritime National Park Association, [Online]. Available: https://maritime.org/doc/computermk1/index.htm. [Accessed 14 September 2017].

[5] E. E. Herrmann, "Chapter Six," in *Exterior Ballistics 1935*, Annapolis, U.S. Naval Institute, 1935, pp. 54-76.

[6] E. E. Herrmann, "Chapter Four," in *Exterior Ballistics 1935*, Annapolis, U.S. Naval Institute, 1935, p. 27 and 35.

[7] H. Lau, "Chapter 5 Analytic Problems," in *A Numerical Library in C for Scientists and Engineers*, Boca Raton, CRC Press, 1995, pp. 459-462.

[8] E. E. Herrmann, "Chapter Five and Appendix A," in *Exterior Ballistics*, Annapolis, U.S. Naval Institute, 1935, pp. 43-53, 247-267.

[9] "Great Circle Distance," Wikipedia, [Online]. Available: https://en.wikipedia.org/wiki/Great-circle_distance. [Accessed 14 September 2017].