The following exercise is from ***Stellar Atmospheres*** (Second Edition) by Dimitri Mihalas page 7.

Exercise 1-3: Derive Stefan's Law from the following equation:

$$E_R^* = \frac{8\pi h}{c^3}\int_0^\infty \frac{\nu^3 d\nu}{e^{h\nu/kT}-1} = a_R T^4 (Stefan'sLaw)$$

$$a_R = \frac{8\pi^5 k^4}{15c^3h^3}$$

$$F(\nu) = \left(e^{h\nu/kT}-1\right)^{-1}$$

$$G(x) = (e^x - 1)^{-1}$$

$$x = \frac{h\nu}{kT}$$

$$G(x) = e^{-x}(1-e^{-x})^{-1}$$

$$H(y) = y(1-y)^{-1}$$

$$y = e^{-x}$$

$$H'(y) = (1-y)^{-1} + y(1-y)^{-2}$$

$$H''(y) = 2(1-y)^{-2} + 2y(1-y)^{-3}$$

$$H'''(y) = 6(1-y)^{-3} + 6y(1-y)^{-4}$$

$$H^{iv}(y) = 24(1-y)^{-4} + 24y(1-y)^{-5}$$

$$H^{v}(y) = 120(1-y)^{-5} + 120y(1-y)^{-6}$$

$$H(0) = 0$$

$$H'(0) = 1$$

$$H''(0) = 2$$

$$H'''(0) = 6$$

$$H^{iv}(0) = 24$$

$$H^{v}(0) = 120$$

$$H^{n}(0) = n!$$

$$H(y) = \sum_{n=1}^{\infty} y^n$$

$$G(x) = \sum_{n=1}^{\infty} e^{-nx}$$

$$F(\nu) = \sum_{n=1}^{\infty} e^{-nh\nu/kT}$$

$$E_R^* = \frac{8\pi h}{c^3} \sum_{n=1}^{\infty} \int_0^{\infty} \nu^3 \, e^{-nh\nu/kT} d\nu$$

$$\int_0^{\infty} \nu^3 \, e^{-nh\nu/kT} d\nu$$
$$= -\frac{\nu^3 kT}{nh} e^{-nh\nu/kT} \Bigg]_{\nu=0}^{\nu\to\infty} - \frac{3\nu^2 k^2 T^2}{n^2 h^2} e^{-nh\nu/kT} \Bigg]_{\nu=0}^{\nu\to\infty} - \frac{6\nu k^3 T^3}{n^3 h^3} e^{-nh\nu/kT} \Bigg]_{\nu=0}^{\nu\to\infty} - \frac{6k^4 T^4}{n^4 h^4} e^{-nh\nu/kT} \Bigg]_{\nu=0}^{\nu\to\infty}$$
$$= \frac{6k^4 T^4}{n^4 h^4}$$

$$E_R^* = \frac{48\pi k^4 T^4}{c^3 h^3} \sum_{n=0}^{\infty} \frac{1}{n^4} = \frac{48\pi k^4 T^4}{c^3 h^3} \zeta(4) = \frac{48\pi k^4 T^4}{c^3 h^3} \left(\frac{\pi^4}{90} \right) = \frac{8\pi^5 k^4}{15 c^3 h^3} T^4 = a_R T^4$$