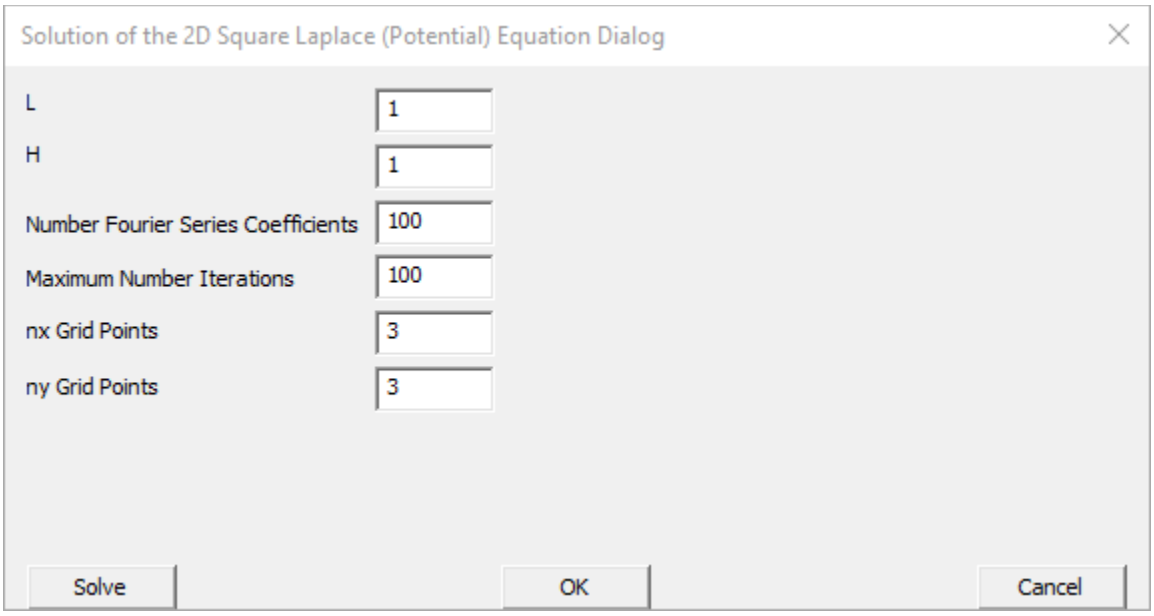
Solution of the 2D Square Laplace (Potential) Equation by James Pate Williams, Jr. © April 6-8, 2024

I reproduced the following MATLAB via translation to vanilla C:

https://people.uncw.edu/hermanr/pde1/PDE1notes/Numerical.pdf




```
Jacobi MSE       = 0.116887
Gauss-Seidel MSE = 7.24976e-25
Gradient MSE     = 1.06374e-24
Gaussian MSE     = 0.00732156

U Matrix
+0.000000 +0.000000 +0.000000 +0.000000 +0.000000
+0.707107 +0.000000 +0.000000 +0.000000 +0.000000
+1.000000 +0.000000 +0.000000 +0.000000 +0.000000
+0.707107 +0.000000 +0.000000 +0.000000 +0.000000
+0.000000 +1.000000 +1.000000 +1.000000 +1.000000

A Matrix
-4.000000 +1.000000 +0.000000 +1.000000 +0.000000 +0.000000 +0.000000 +0.000000 +0.000000
+1.000000 -4.000000 +1.000000 +0.000000 +1.000000 +0.000000 +0.000000 +0.000000 +0.000000
+0.000000 +1.000000 -4.000000 +0.000000 +0.000000 +1.000000 +0.000000 +0.000000 +0.000000
+1.000000 +0.000000 +0.000000 -4.000000 +1.000000 +0.000000 +1.000000 +0.000000 +0.000000
+0.000000 +1.000000 +0.000000 +1.000000 -4.000000 +1.000000 +0.000000 +1.000000 +0.000000
+0.000000 +0.000000 +1.000000 +0.000000 +1.000000 -4.000000 +0.000000 +0.000000 +1.000000
+0.000000 +0.000000 +0.000000 +1.000000 +0.000000 +0.000000 -4.000000 +1.000000 +0.000000
+0.000000 +0.000000 +0.000000 +0.000000 +1.000000 +0.000000 +1.000000 -4.000000 +1.000000
+0.000000 +0.000000 +0.000000 +0.000000 +0.000000 +1.000000 +0.000000 +1.000000 -4.000000

Exact Solution
+0.388070 +0.236318 +0.121159
+0.634716 +0.449268 +0.257246
+0.752127 +0.681433 +0.485215

Jacobi Solution
+0.176777 +0.044194 +0.011049
+0.294194 +0.084597 +0.023911
+0.500325 +0.396231 +0.355035
```

```
Jacobi Absolute Error
+0.211293 +0.192124 +0.110110
+0.340522 +0.364671 +0.233335
+0.251802 +0.285203 +0.130180

Gauss-Seidel Solution
+0.403241 +0.249103 +0.129782
+0.656753 +0.463388 +0.270024
+0.760383 +0.677674 +0.486924

Gauss-Seidel Absolute Error
+0.015170 +0.012784 +0.008623
+0.022037 +0.014120 +0.012777
+0.008257 +0.003759 +0.001709

Gradient Method Solution
+0.403241 +0.249103 +0.129782
+0.656753 +0.463388 +0.270024
+0.760383 +0.677674 +0.486924

Gradient Method Absolute Error
+0.015170 +0.012784 +0.008623
+0.022037 +0.014120 +0.012777
+0.008257 +0.003759 +0.001709

Gaussian Elimination Solution
+0.398470 +0.240309 +0.120687
+0.646464 +0.442081 +0.242437
+0.745306 +0.639113 +0.406982

Gaussian Elimination Error
+0.010400 +0.003991 +0.000472
+0.011748 +0.007187 +0.014809
+0.006821 +0.042320 +0.078234

Jacobi MSE       = 0.0885001
Gauss-Seidel MSE = 2.87893e-25
Gradient MSE     = 4.74286e-23
Gaussian MSE     = 0.0043631

U Matrix
+0.000000 +0.000000 +0.000000 +0.000000 +0.000000 +0.000000
+0.587785 +0.000000 +0.000000 +0.000000 +0.000000 +0.000000
+0.951057 +0.000000 +0.000000 +0.000000 +0.000000 +0.000000
+0.951057 +0.000000 +0.000000 +0.000000 +0.000000 +0.000000
+0.587785 +0.000000 +0.000000 +0.000000 +0.000000 +0.000000
+0.000000 +1.000000 +1.000000 +1.000000 +1.000000 +1.000000

Exact Solution
+0.355761 +0.233911 +0.152344 +0.077784
+0.611027 +0.433346 +0.301367 +0.161251
+0.722748 +0.596522 +0.464544 +0.272972
+0.768443 +0.769774 +0.688206 +0.490466

Jacobi Solution
+0.146946 +0.036737 +0.009184 +0.002296
+0.274501 +0.077809 +0.021748 +0.006011
+0.306389 +0.096050 +0.029450 +0.008865
+0.473544 +0.392398 +0.355462 +0.341082

Jacobi Absolute Error
+0.208814 +0.197175 +0.143160 +0.075488
+0.336526 +0.355537 +0.279619 +0.155240
+0.416358 +0.500473 +0.435094 +0.264107
+0.294899 +0.377375 +0.332744 +0.149384
```

Gauss-Seidel Solution
+0.363543 +0.241860 +0.158553 +0.081804
+0.624526 +0.445342 +0.310550 +0.168664
+0.738162 +0.604433 +0.469641 +0.282300
+0.772634 +0.764587 +0.681281 +0.490895

Gauss-Seidel Absolute Error
+0.007782 +0.007948 +0.006209 +0.004020
+0.013499 +0.011996 +0.009183 +0.007413
+0.015414 +0.007911 +0.005097 +0.009328
+0.004191 +0.005187 +0.006926 +0.000429

Gradient Method Solution
+0.363543 +0.241860 +0.158553 +0.081804
+0.624526 +0.445342 +0.310550 +0.168664
+0.738162 +0.604433 +0.469641 +0.282300
+0.772634 +0.764587 +0.681281 +0.490895

Gradient Method Absolute Error
+0.007782 +0.007948 +0.006209 +0.004020
+0.013499 +0.011996 +0.009183 +0.007413
+0.015414 +0.007911 +0.005097 +0.009328
+0.004191 +0.005187 +0.006926 +0.000429

Gaussian Elimination Solution
+0.361327 +0.237639 +0.153193 +0.077508
+0.619885 +0.436035 +0.297624 +0.156840
+0.731121 +0.588992 +0.444429 +0.252227
+0.764552 +0.744382 +0.638873 +0.407638

Gaussian Elimination Error
+0.005567 +0.003727 +0.000849 +0.000276
+0.008858 +0.002689 +0.003743 +0.004411
+0.008373 +0.007531 +0.020115 +0.020745
+0.003891 +0.025392 +0.049334 +0.082828