Text and Exercise from “Boundary Value Problems Second Edition” by David L. Powers

We want to solve the equation for the pressure of the lubricant in a plate bearing. This problem is found in Chapter 5 Section C of the textbook [1].

$$J_1(\lambda_n a)Y_1(\lambda_n b) - J_1(\lambda_n b)Y_1(\lambda_n a) = 0 (in\ text\ unnumbered)$$

$$X_n(x) = \frac{1}{x}[Y_1(\lambda_n a)J_1(\lambda_n x) - J_1(\lambda_n a)Y_1(\lambda_n x)] (Equation\ 34)$$

$$u(x,y) = \sum_{n=1}^{\infty} a_n X_n(x) \frac{\cosh(\lambda_n y)}{\cosh(\lambda_n c)} (Equation\ 35)$$

Now we answer Exercise 13 on page 234 by finding the coefficients in Equation 35.

$$\int_a^b u(x,y)\, X_m(x) x^3 dx = \sum_{n=1}^{\infty} a_n \frac{\cosh(\lambda_n y)}{\cosh(\lambda_n c)} \int_a^b X_m(x) X_n(x)\, x^3 dx$$

$$\int_a^b X_m(x) X_n(x)\, x^3 dx = 0\ m \neq n (in\ text\ unnumbered)$$

$$\int_a^b u(x,y)\, X_m(x) x^3 dx = a_m \frac{\cosh(\lambda_m y)}{\cosh(\lambda_m c)}$$

$$f = J_1(ea)Y_1(eb) - J_1(eb)Y_1(ea) = 0 (in\ text\ unnumbered)$$

We solve for e using a home-grown stochastic hill-climber. Suppose that a = 1 and b = 2.5 so that b / a = 2.5. We carryout eigenvalue computations as follows:

```
e =  2.156163
e * a =  2.156163       e * b =  5.390408
abs(f(e * a, e * b)) =  0.000092
abs(f(e * b, e * a)) =  0.000092
e =  6.331492
e * a =  6.331492       e * b = 15.828730
abs(f(e * a, e * b)) =  0.002397
abs(f(e * b, e * a)) =  0.002397
e =  6.308084
e * a =  6.308084       e * b = 15.770211
abs(f(e * a, e * b)) =  0.000145
abs(f(e * b, e * a)) =  0.000145
e = 12.566668
e * a = 12.566668       e * b = 31.416669
abs(f(e * a, e * b)) =  0.000558
abs(f(e * b, e * a)) =  0.000558
```

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e = 14.667806
e * a = 14.667806       e * b = 36.669515
abs(f(e * a, e * b)) =  0.000130
abs(f(e * b, e * a)) =  0.000130
e = 16.760887
e * a = 16.760887       e * b = 41.902219
abs(f(e * a, e * b)) =  0.000115
abs(f(e * b, e * a)) =  0.000115
e = 25.134587
e * a = 25.134587       e * b = 62.836467
abs(f(e * a, e * b)) =  0.000099
abs(f(e * b, e * a)) =  0.000099
e = 25.133335
e * a = 25.133335       e * b = 62.833338
abs(f(e * a, e * b)) =  0.000129
abs(f(e * b, e * a)) =  0.000129
e = 25.141331
e * a = 25.141331       e * b = 62.853328
abs(f(e * a, e * b)) =  0.000063
abs(f(e * b, e * a)) =  0.000063
e = 29.330119
e * a = 29.330119       e * b = 73.325297
abs(f(e * a, e * b)) =  0.000072
abs(f(e * b, e * a)) =  0.000072
e = 18.857479
e * a = 18.857479       e * b = 47.143696
abs(f(e * a, e * b)) =  0.000001
abs(f(e * b, e * a)) =  0.000001
e = 37.700003
e * a = 37.700003       e * b = 94.250008
abs(f(e * a, e * b)) =  0.000049
abs(f(e * b, e * a)) =  0.000049
e = 41.889065
e * a = 41.889065       e * b = 104.722663
abs(f(e * a, e * b)) =  0.000035
abs(f(e * b, e * a)) =  0.000035
e = 27.230506
e * a = 27.230506       e * b = 68.076266
abs(f(e * a, e * b)) =  0.000047
abs(f(e * b, e * a)) =  0.000047
e = 54.457228
e * a = 54.457228       e * b = 136.143071
abs(f(e * a, e * b)) =  0.000002
abs(f(e * b, e * a)) =  0.000002
e = 50.266671
e * a = 50.266671       e * b = 125.666677
abs(f(e * a, e * b)) =  0.000022
```

```
abs(f(e * b, e * a)) =  0.000022
e = 52.370190
e * a = 52.370190       e * b = 130.925474
abs(f(e * a, e * b)) =  0.000086
abs(f(e * b, e * a)) =  0.000086
e = 56.550005
e * a = 56.550005       e * b = 141.375011
abs(f(e * a, e * b)) =  0.000014
abs(f(e * b, e * a)) =  0.000014
e = 60.724967
e * a = 60.724967       e * b = 151.812418
abs(f(e * a, e * b)) =  0.000149
abs(f(e * b, e * a)) =  0.000149
e = 62.833338
e * a = 62.833338       e * b = 157.083346
abs(f(e * a, e * b)) =  0.000009
abs(f(e * b, e * a)) =  0.000009
e = 75.384533
e * a = 75.384533       e * b = 188.461333
abs(f(e * a, e * b)) =  0.000126
abs(f(e * b, e * a)) =  0.000126
e = 67.027924
e * a = 67.027924       e * b = 167.569811
abs(f(e * a, e * b)) =  0.000045
abs(f(e * b, e * a)) =  0.000045
e = 83.776940
e * a = 83.776940       e * b = 209.442351
abs(f(e * a, e * b)) =  0.000005
abs(f(e * b, e * a)) =  0.000005
e = 75.400006
e * a = 75.400006       e * b = 188.500015
abs(f(e * a, e * b)) =  0.000002
abs(f(e * b, e * a)) =  0.000002
e = 87.958159
e * a = 87.958159       e * b = 219.895398
abs(f(e * a, e * b)) =  0.000056
abs(f(e * b, e * a)) =  0.000056
e = 83.778130
e * a = 83.778130       e * b = 209.445326
abs(f(e * a, e * b)) =  0.000004
abs(f(e * b, e * a)) =  0.000004
e = 77.488113
e * a = 77.488113       e * b = 193.720283
abs(f(e * a, e * b)) =  0.000050
abs(f(e * b, e * a)) =  0.000050
e = 75.400128
e * a = 75.400128       e * b = 188.500320
```

```
abs(f(e * a, e * b)) =  0.000001
abs(f(e * b, e * a)) =  0.000001
e = 113.095309
e * a = 113.095309      e * b = 282.738273
abs(f(e * a, e * b)) =  0.000018
abs(f(e * b, e * a)) =  0.000018
e = 64.929350
e * a = 64.929350       e * b = 162.323374
abs(f(e * a, e * b)) =  0.000007
abs(f(e * b, e * a)) =  0.000007
e = 31.420057
e * a = 31.420057       e * b = 78.550142
abs(f(e * a, e * b)) =  0.000012
abs(f(e * b, e * a)) =  0.000012
e = 127.759758
e * a = 127.759758      e * b = 319.399396
abs(f(e * a, e * b)) =  0.000002
abs(f(e * b, e * a)) =  0.000002
```

The distinct eigenvalues found above are:

```
2.156163
6.308084
12.566668
14.667806
16.760887
18.857479
25.141331
27.230506
29.330119
31.420057
37.700003
41.889065
50.266671
52.370190
54.457228
62.833338
64.929350
67.027924
75.400128
77.488113
83.776940
87.958159
113.095309
127.759758
```