From 'Numerical Analysis : An Algorithmic Approach Third Edition'
(c) 1980 by S.D. Conte and Carl de Boor
'Chapter 3 The Solution of Nonlinear Equations'
Also see 'A Numerical Library in C for Scientists
and Engineers' (c) 1995 by H. T. Lau, PhD

Algorithm 3.1
The zero is: 1.324717957245   Plus/Minus: 0.000000000001

Example 3.2a Part 1
Finds a zero of f(x) = x^3 - x - 1 = 0

n	Bisection x	Error	Mod Reg x	Error
0	1.500000000000	3.000000000000	1.500000000000	3.000000000000
1	1.312500000000	0.260742187500	1.583333333333	2.789351851852
2	1.320312500000	0.033044815063	1.661654135338	2.503001950577
3	1.325195312500	0.004168387502	1.324925618462	0.006900068696
4	1.324768066406	0.000260316851	1.325629263357	0.003903228795
5	1.324714660645	0.000032536368	1.325630459710	0.003898126800
6	1.324717521667	0.000004067068	1.325630459710	0.003898126800
7	1.324717938900	0.000000254192	1.325630459710	0.003898126800
8	1.324717961252	0.000000031774	1.324717957238	0.000000017332
9	1.324717956595	0.000000003972	1.324717955209	0.000000008681
10	1.324717957235	0.000000000248	1.324717955209	0.000000008681
11	1.324717957243	0.000000000031	1.324717955209	0.000000008681
12	1.324717957245	0.000000000004	1.324717955209	0.000000008681
13	1.324717957245	0.000000000000	1.324717955209	0.000000008681
14	1.324717957245	0.000000000000	1.324717955209	0.000000008681
15	1.324717957245	0.000000000000	1.324717955209	0.000000008681
16	1.324717957245	0.000000000000	1.324717957245	0.000000000000
17	1.324717957245	0.000000000000	1.324717957245	0.000000000000
18	1.324717957245	0.000000000000	1.324717957245	0.000000000000
19	1.324717957245	0.000000000000	1.324717957245	0.000000000000
20	1.324717957245	0.000000000000	1.324717957245	0.000000000000

Example 3.2a Part 2
Finds a zero of f(x) = x^3 - x - 1 = 0

n	Secant x	f(x)
0	1.166666666667	0.578703703704
1	1.166666666667	0.578703703704
2	1.253112033195	0.285363029639
3	1.337206445842	0.053880586676
4	1.323850096388	0.003698115440
5	1.324707936532	0.000042734263
6	1.324717965354	0.000000034582
7	1.324717957245	0.000000000000

Example 3.2a Part 3
Finds a zero of f(x) = x^3 - x - 1 = 0

n	Newton x	f(x)
0	1.500000000000	0.875000000000
1	1.500000000000	0.875000000000
2	1.347826086957	0.100682173091
3	1.325200398951	0.002058361917
4	1.324718173999	0.000000924378
5	1.324717957245	0.000000000000

Example 3.2b
Finds a zero of f(v) = 50I(exp(qv) - 1) + v - 20

xn	fxn
4.370573714636e-01	+7.481659736186e-10
4.370573715746e-01	+8.767604953164e-08
4.370573714627e-01	+1.669775429036e-12
4.370573715884e-01	+9.853834015416e-08
6.475263584576e-01	+8.862579746440e+04
6.475597665962e-01	+8.874433546093e+04

xn	fxn

```
 5.500000000000e-01    -1.645634291424e+01
 8.203338798801e-01    +6.480871366799e+02
 5.246418300000e-01    -1.767259368331e+01
 1.001553949650e+00    +2.500502145089e+04
 5.158064500000e-01    -1.797344032352e+01
 1.091581794982e+00    +1.514519219823e+05
 5.149556200000e-01    -1.799978238554e+01
 1.101148603919e+00    +1.833920480298e+05
 5.149485300000e-01    -1.800000007785e+01
 1.101229024889e+00    +1.836872872044e+05

Example 3.1
Finds a zero of f(x) = x - 0.2 sin(x) - 0.5 = 0

n       Bisection x     Error         Mod Reg x      Error
0       0.750000000000 0.213795455380 0.750000000000 0.213795455380
1       0.562500000000 0.051932826566 0.750000000000 0.213795455380
2       0.617187500000 0.006538279529 0.806061240609 0.167252246506
3       0.616210937500 0.000817173036 0.617013286655 0.001293671712
4       0.615417480469 0.000051067718 0.617013286655 0.001293671712
5       0.615470886230 0.000006383512 0.617013286655 0.001293671712
6       0.615468025208 0.000000797939 0.617013286655 0.001293671712
7       0.615468204021 0.000000049871 0.617013643329 0.001293373283
8       0.615468166769 0.000000006234 0.617013643329 0.001293373283
9       0.615468169563 0.000000000779 0.617013643329 0.001293373283
10      0.615468169504 0.000000000049 0.615468169414 0.000000000064
11      0.615468169497 0.000000000006 0.615468169414 0.000000000064
12      0.615468169491 0.000000000001 0.615468169414 0.000000000064
13      0.615468169490 0.000000000000 0.615468169414 0.000000000064
14      0.615468169490 0.000000000000 0.615468169414 0.000000000064
15      0.615468169490 0.000000000000 0.615468169414 0.000000000064
16      0.615468169490 0.000000000000 0.615468169414 0.000000000064
17      0.615468169490 0.000000000000 0.615468169414 0.000000000064
18      0.615468169490 0.000000000000 0.615468169414 0.000000000064
19      0.615468169490 0.000000000000 0.615468169414 0.000000000064
20      0.615468169490 0.000000000000 0.615468169414 0.000000000064

 n      Secant x       Error          Newton x        Error
0       0.612122481218 0.002798689973 0.616841656450 0.001149304633
1       0.615367725887 0.000084040515 0.615468299804 0.000000109034
2       0.615367725887 0.000084040515 0.615468299804 0.000000109034
3       0.615468192646 0.000000019375 0.615468169490 0.000000000000
4       0.615468169490 0.000000000000 0.615468169490 0.000000000000
5       0.615468169490 0.000000000000 0.615468169490 0.000000000000
6       0.615468169490 0.000000000000 0.615468169490 0.000000000000
7       0.615468169490 0.000000000000 0.615468169490 0.000000000000
8       0.615468169490 0.000000000000 0.615468169490 0.000000000000
9       0.615468169490 0.000000000000 0.615468169490 0.000000000000
10      0.615468169490 0.000000000000 0.615468169490 0.000000000000
11      0.615468169490 0.000000000000 0.615468169490 0.000000000000
12      0.615468169490 0.000000000000 0.615468169490 0.000000000000
13      0.615468169490 0.000000000000 0.615468169490 0.000000000000
14      0.615468169490 0.000000000000 0.615468169490 0.000000000000
15      0.615468169490 0.000000000000 0.615468169490 0.000000000000
16      0.615468169490 0.000000000000 0.615468169490 0.000000000000
17      0.615468169490 0.000000000000 0.615468169490 0.000000000000
18      0.615468169490 0.000000000000 0.615468169490 0.000000000000
19      0.615468169490 0.000000000000 0.615468169490 0.000000000000
20      0.615468169490 0.000000000000 0.615468169490 0.000000000000

Example 3.3
Finds a fixed point of g(x) = x + exp(x) - sin(x)
Solves the equation exp(-x) = sin(x)
Fixed Point Iteration

xn                    f(xn)                  Error
 6.000000000000e-01    -1.583083730101e-02
 5.841691626990e-01    +6.062405756870e-03     2.709974834664e-02
 5.902315684559e-01    -2.354492761297e-03     1.027123264981e-02
 5.878770756946e-01    +9.095832398562e-04     4.005076671029e-03
 5.887866589344e-01    -3.521181775338e-04     1.544843494760e-03
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 5.884345407569e-01    +1.362031442000e-04     5.983982127916e-04
 5.885707439011e-01    -5.270118489875e-05     2.314133782751e-04
 5.885180427162e-01    +2.038926609294e-05     8.954897058979e-05
 5.885384319823e-01    -7.888654636190e-06     3.464389916604e-05
 5.885305433276e-01    +3.052084145305e-06     1.340398510430e-05
 5.885335954118e-01    -1.180845493276e-06     5.185913207163e-06
 5.885324145663e-01    +4.568656264414e-07     2.006423884309e-06
 5.885328714319e-01    -1.767601424119e-07     7.762788598874e-07
 5.885326946718e-01    +6.838801958065e-08     3.003403957200e-07
 5.885327630598e-01    -2.645914332078e-08     1.162008708319e-07
 5.885327366007e-01    +1.023697171476e-08     4.495781062852e-08
 5.885327468376e-01    -3.960657246616e-09     1.739405626920e-08

Example 3.7
Finds a zero of f(x) = exp(x) - 3
Using the secant method

n       Secant x       f(x)
0       1.060315090224 0.112719400152
1       1.060315090224 0.112719400152
2       1.083843744504 0.043980072301
3       1.098897600116 0.000856056460
4       1.098610176766 0.000006335700
5       1.098612288367 0.000000000904
6       1.098612288668 0.000000000000

Example 3.11
Finds the real positive root of the polynomial: P(x) = x^5 - 3.7x^4 + 7.4x^3 - 10.8x^2 + 10.8x -
6.8
Using Newton's Method

i        x              ap             app
1       1.5000000e+00 -1.0625000e+00  3.7125000e+00
2       1.7861953e+00  7.2393309e-01  9.6004867e+00
3       1.7107894e+00  8.0013394e-02  7.5470613e+00
4       1.7001875e+00  1.3665626e-03  7.2905682e+00
5       1.7000000e+00 -1.7763568e-15  7.2861000e+00
6       1.7000000e+00 -1.7763568e-15  7.2861000e+00
7       1.7000000e+00 -1.7763568e-15  7.2861000e+00

Exercise 3.6-1
Finds the real root of the polynomial:
P(x) = x^3 + 2x - 1
Using Newton's Method

n       x              P(x)           P'(x)
1       +5.0000000e-01 -1.0000000e+00 +2.0000000e+00
2       +4.5454545e-01 +1.2500000e-01 +2.7500000e+00
3       +4.5339834e-01 +3.0052592e-03 +2.6198347e+00
4       +4.5339765e-01 +1.7928715e-06 +2.6167102e+00
5       +4.5339765e-01 +6.3860028e-13 +2.6167083e+00

Exercise 3.6-2 (a)
Finds the real roots of the polynomial:
P(x) = x^5 - 3x^3 + x^2 - 1
Using Lau's zerpol function

n       x              P(x)           P'(x)
5       +1.6180340e+00 +2.2963187e-11 -6.1803399e-01
5       -1.8392868e+00 -1.7945090e-11 +1.0954441e+02
5       -6.1803399e-01 -2.2204460e-16 +1.6180340e+00

Exercise 3.6-2 (b)
Finds the real roots of the polynomial:
P(x) = x^3 + 3x - 1
Using Lau's zerpol function

n       x              P(x)           P'(x)
3       +3.2218535e-01 +0.0000000e+00 +1.2779663e+00

Example 3.13
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Finds the first sixteen roots of the J_0(z) Bessel Function
Using Lau's besszeros Function

Zeros[k] Lau's besszeros Function

n	zero[k]	J0(zero[k])
1	2.4048256e+00	-2.4980018e-16
2	5.5200781e+00	-2.7755576e-17
3	8.6537279e+00	-1.3785938e-15
4	1.1791534e+01	1.0250181e-15
5	1.4930918e+01	-7.6726702e-16
6	1.8071064e+01	1.0629067e-15
7	2.1211637e+01	-7.2802621e-16
8	2.4352472e+01	9.0524273e-16
9	2.7493479e+01	-7.4756458e-16
10	3.0634606e+01	5.0664482e-16
11	3.3775820e+01	-2.0719851e-16
12	3.6917098e+01	-4.2382465e-16
13	4.0058426e+01	-4.0462601e-16
14	4.3199792e+01	2.5233467e-16
15	4.6341188e+01	3.8290824e-16
16	4.9482610e+01	6.1882047e-17

Example 3.14
Finds all the roots of P(x) = x^3 + x - 3
Using Mueller's Method

Zeros[i]

Re Part	Im Part
1.2134117e+00	0.0000000e+00
-6.0670583e-01	1.4506122e+00
-6.0670583e-01	-1.4506122e+00

P(zeros[i])

Re Part	Im Part
-4.4408921e-16	0.0000000e+00
-8.8817842e-16	2.2204460e-16
8.8817842e-16	8.8817842e-16

Example 3.15
Finds all the roots of the polynomial:
P(x) = x^5 - 3.7x^4 + 7.4x^3 - 10.8x^2 + 10.8x - 6.8
Using Mueller's Method

Zeros[i]

Re Part	Im Part
1.0000000e+00	-1.0000000e+00
1.0000000e+00	1.0000000e+00
-4.0344338e-17	1.4142136e+00
-5.9557026e-18	-1.4142136e+00
1.7000000e+00	1.1832914e-30

P(zeros[i])

Re Part	Im Part
-8.8817842e-16	1.3322676e-15
0.0000000e+00	8.8817842e-16
0.0000000e+00	-2.3181592e-15
0.0000000e+00	2.4835110e-15
3.5527137e-15	8.6215792e-30

Example 3.16
Finds all the roots of the polynomial:
P(x) = x^7 - 28x^6 + 322x^5 - 1,960x^4 + 6,769x^3 - 13,132x^2 + 13,068x - 5,040
Using Mueller's Method

Zeros[i]

Re Part	Im Part
2.0000000e+00	8.6636639e-25
3.0000000e+00	-2.7268912e-15
1.0000000e+00	1.9334983e-24
6.0000000e+00	-1.2805718e-16
5.0000000e+00	-1.0120991e-17

```
 4.0000000e+00  1.3753001e-26
 7.0000000e+00  4.6391332e-26

P(zeros[i])
Re Part        Im Part
 0.0000000e+00 -1.0396397e-22
-4.5474735e-12 -1.3089078e-13
 0.0000000e+00  1.3921187e-21
 3.0195224e-10  1.5366862e-14
 1.1823431e-10 -4.8580758e-16
-6.1845640e-11 -4.9510804e-25
 2.0372681e-10  3.3401759e-23

Example 3.17
Finds all the roots of the polynomial:
P(x) = x^8 - 170x^6 + 7,392x^4 - 39,712x^2 + 51,200
Using Mueller's Method

Zeros[i]
Re Part        Im Part
-1.4142136e+00  0.0000000e+00
 1.4142136e+00  0.0000000e+00
 2.0000000e+00  0.0000000e+00
-2.0000000e+00  0.0000000e+00
-8.0000000e+00  0.0000000e+00
 8.0000000e+00  0.0000000e+00
 1.0000000e+01  0.0000000e+00
-1.0000000e+01  0.0000000e+00

P(zeros[i])
Re Part        Im Part
 0.0000000e+00  0.0000000e+00
 0.0000000e+00  0.0000000e+00
-1.4551915e-11  0.0000000e+00
 0.0000000e+00  0.0000000e+00
 2.8085196e-09  0.0000000e+00
 0.0000000e+00  0.0000000e+00
-1.4333636e-09  0.0000000e+00
 0.0000000e+00  0.0000000e+00

Exercise 3.7-1(a)
Finds all the roots of the polynomial:
P(x) = x^7 - 1
Using Mueller's Method

Zeros[i]
Re Part        Im Part
 1.0000000e+00  0.0000000e+00
-9.0096887e-01 -4.3388374e-01
-2.2252093e-01  9.7492791e-01
-2.2252093e-01 -9.7492791e-01
-9.0096887e-01  4.3388374e-01
 6.2348980e-01  7.8183148e-01
 6.2348980e-01 -7.8183148e-01

P(zeros[i])
Re Part        Im Part
 0.0000000e+00  0.0000000e+00
 2.2204460e-16 -5.5511151e-17
 2.2204460e-16  1.9428903e-16
 0.0000000e+00  1.1102230e-16
 2.2204460e-16  5.5511151e-17
 4.4408921e-16 -2.7755576e-16
-2.2204460e-16 -5.5511151e-17

Exercise 3.7-1(b)
Finds all the roots of the polynomial:
P(x) = x^4 - 7x^3 + 18x^2 -20x + 8
Using Mueller's Method

Zeros[i]
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Re Part        Im Part
 5.0000139e-01  3.6975781e-06
 5.0000106e-01 -1.7993959e-06
 5.0000003e-01  1.4210537e-08
 1.0000000e+00 -1.2777929e-28

P(zeros[i])
Re Part        Im Part
-4.4408921e-16  1.1656867e-16
-2.2204460e-16  7.6402372e-19
 0.0000000e+00 -1.4558379e-22
 0.0000000e+00 -1.2777929e-28

Exercise 3.7-1(c)
Finds all the roots of the polynomial:
P(x) = x^6 - 2x^5 + x^4 + 3x^3 + 5x^2 + x + 1
Using Mueller's Method

Zeros[i]
Re Part        Im Part
-6.0848516e-01  1.4398620e-01
-6.0848516e-01 -1.4398620e-01
 3.3484731e-01 -6.6416705e-01
 3.3484731e-01  6.6416705e-01
-2.2636215e-01  2.1381690e+00
-2.2636215e-01 -2.1381690e+00

P(zeros[i])
Re Part        Im Part
 0.0000000e+00  0.0000000e+00
 2.2204460e-16  5.5511151e-17
 0.0000000e+00  5.5511151e-17
 0.0000000e+00 -5.5511151e-17
 2.4202862e-14  7.3552275e-16
-1.5543122e-14  8.3266727e-17

Exercise 3.7-2
Finds roots of the function:
f(x) = x - tan(x)
Using Newton's Method

i = 1   root =  7.6332987e-05  f(root) = -1.4825710e-13
i = 2   root =  1.0678547e-04  f(root) = -4.0589647e-13
i = 3   root =  7.7877860e-05  f(root) = -1.5744207e-13

Example 3.13
Finds the first sixteen roots of the J_1(z) Bessel Function
Using Lau's besszeros Function

Zeros[k] Lau's besszeros Function
 n       zero[k]       J1(zero[k])
 1       3.8317060e+00  6.9128283e-16
 2       7.0155867e+00 -2.2392989e-15
 3       1.0173468e+01 -1.0084908e-15
 4       1.3323692e+01  1.3233766e-15
 5       1.6470630e+01 -5.1498213e-16
 6       1.9615859e+01  3.6813910e-16
 7       2.2760084e+01 -8.5354427e-16
 8       2.5903672e+01  3.6822080e-16
 9       2.9046829e+01 -4.6252442e-16
10       3.2189680e+01  6.1940497e-16
11       3.5332308e+01  1.3179555e-16
12       3.8474766e+01  2.7691986e-16
13       4.1617094e+01 -2.8578450e-16
14       4.4759319e+01  6.6410021e-17
15       4.7901461e+01  8.6593209e-17
16       5.1043535e+01  2.0826122e-17
```