

Table 7.1

| h      | exp(h)           | exp(-h)          | Dh               | D2h              |
|--------|------------------|------------------|------------------|------------------|
| 1.0000 | 2.7182818285e+00 | 3.6787944117e-01 | 1.1752011936e+00 | 1.0861612696e+00 |
| 0.1000 | 1.1051709181e+00 | 9.0483741804e-01 | 1.0016675002e+00 | 1.0008336112e+00 |
| 0.0100 | 1.0100501671e+00 | 9.9004983375e-01 | 1.0000166667e+00 | 1.0000083334e+00 |
| 0.0010 | 1.0010005002e+00 | 9.9900049983e-01 | 1.0000001667e+00 | 1.0000000834e+00 |
| 0.0001 | 1.0001000050e+00 | 9.9990000500e-01 | 1.0000000017e+00 | 1.0000000050e+00 |

Exercise 7.1-1

f'(1.4) = cosh(1.4) = 2.150898  
Forward f'(1.4) = 2.250000  
Error = 0.099102

Central f'(1.4) = 2.154500  
Error = 0.003602

Interpo f'(1.4) = 2.143500  
Error = 0.007398

f''(1.4) = sinh(1.4) = 1.904302  
Central f''(1.4) = 1.910000  
Error = 0.005698

Exercise 7.1-2

f'(0.4) = cosh(0.4) = 1.0810723718  
Central1 f'(0.4) = 1.0815000000  
Error = 0.0004276282

f'(0.4) = cosh(0.4) = 1.0810723718  
Central2 f'(0.4) = 1.0810000000  
Error = 0.0000723718

Exercise 7.1-3

E<sub>+</sub> - E<sub>-</sub> = 4.9999985871e-08  
h = 2.4999980435e-08

Exercise 7.1-5

f(x) = ln(x)  
f'(2) = 0.5000000000  
a[16] = 0.5000000000

f(x) = ln(x)  
f'(2) = 0.5000000000  
a[24] = 0.5000000000

f(x) = ln(x)  
f'(2) = 0.5000000000  
a[32] = 0.5000000000

f(x) = ln(x)  
f'(2) = 0.5000000000  
a[48] = 0.5000000000

f(x) = ln(x)  
f'(2) = 0.5000000000  
a[64] = -nan(ind)

Exercise 7.2-2

f(x) = x \* sin(x), a = 0, b = 1  
Rectangle Rule = 0.00000000  
Error = 0.30116868  
Mid-point Rule = 0.23971277

```
Error = 0.06145591
Trapezoidal Rule = 0.42073549
Error = 0.11956681
Simpson's Rule = 0.30005368
Error = 0.00111500
Corrected Rule = 0.30558772
Error = 0.00441904
```

#### Exercise 7.2-3

```
f(x) = x, a = 0, b = 1 / 2
f(x) = 1 - x, a = 1 / 2, b = 1
Trapezoidal Rule = 0.00000000
Trapezoidal Rule = 0.25000000
Simpson's Rule = 0.33333333
Corrected Rule = 0.16666667
```

#### Exercise 7.2-5

```
Simpson's Rule = 0.59967937
```

#### Exercise 7.2-6

```
f(x) = exp(-x * x), a = 0, b = 1
Trapezoidal Rule = 0.68393972
```

#### Example 7.2.1

```
f(x) = exp(-x * x), a = 0, b = 1
Rectangle Rule = 1.00000
Error = 0.25318
Mid-point Rule = 0.77880
Error = 0.03198
Trapezoidal Rule = 0.68394
Error = 0.06288
Simpson's Rule = 0.74718
Error = 0.00036
Corrected Rule = 0.74525
Error = 0.00157
```

#### Example 7.2a

```
f(x) = exp(-x * x), a = 0, b = 1
Abscissa Weight
x[1] = -0.86113631 w[1] = +0.34785485
x[2] = +0.86113631 w[2] = +0.34785485
x[3] = -0.33998104 w[3] = +0.65214515
x[4] = +0.33998104 w[4] = +0.65214515
```

```
Gaussian Quadrature = 0.74682447
```

#### Example 7.3

```
Abscissa Weight
f(x) = (sin(x))^2 / x, a = 1, b = 3
x[1] = -0.86113631 w[1] = +0.34785485
x[2] = +0.86113631 w[2] = +0.34785485
x[3] = -0.33998104 w[3] = +0.65214515
x[4] = +0.33998104 w[4] = +0.65214515
```

```
Gaussian Quadrature = 0.79482835
Error = 0.00000317
```

#### Exercise 7.3-4

```
f(x) = exp(-x * x), a = 0, b = 1
Gaussian Quadrature = 0.85294432
Error = 0.03328261
```

```
Ext Trapezoidal Rule = 0.88622693
Error = 0.00000000
```

Ext Simpson's Rule = 0.88622693  
Error = 0.00000000

#### Exercise 7.3-9

$f(x) = [1 - \text{pow}(-1, n + 1)] / (n + 1)$

$f(x) = x^n, a = -1, b = 1$   
 $n = 0$   
Gaussian Quadrature = +0.00000000  
Error = +2.00000000  
 $f(x) = x^n, a = -1, b = 1$   
 $n = 1$   
Gaussian Quadrature = +0.00000000  
Error = +0.00000000  
 $f(x) = x^n, a = -1, b = 1$   
 $n = 2$   
Gaussian Quadrature = +0.22222222  
Error = +0.44444444  
 $f(x) = x^n, a = -1, b = 1$   
 $n = 3$   
Gaussian Quadrature = -0.00000000  
Error = +0.00000000  
 $f(x) = x^n, a = -1, b = 1$   
 $n = 4$   
Gaussian Quadrature = +0.40000000  
Error = +0.00000000  
 $f(x) = x^n, a = -1, b = 1$   
 $n = 5$   
Gaussian Quadrature = +0.00000000  
Error = +0.00000000  
 $f(x) = x^n, a = -1, b = 1$   
 $n = 6$   
Gaussian Quadrature = +0.28571429  
Error = +0.00000000  
 $f(x) = x^n, a = -1, b = 1$   
 $n = 7$   
Gaussian Quadrature = +0.00000000  
Error = +0.00000000  
 $f(x) = x^n, a = -1, b = 1$   
 $n = 8$   
Gaussian Quadrature = +0.22222222  
Error = +0.00000000

#### Exercise 7.3-9

$f(x) = x * \ln(x) a = 1, b = 2$   
Ext Simpson's Rule = 0.63629477

$f(x) = x * \ln(x) a = 1, b = 2$   
Ext Simpson's Rule = 0.63629439

#### Exercise 7.4-4

|                                         |                    |        |
|-----------------------------------------|--------------------|--------|
| $f(x) = x * \exp(-x) a = 0, b = 1$      |                    |        |
| Ext Simpson's Rule = 0.32975270         | Delta = 0.00000000 | N = 10 |
| Ext Simpson's Rule = 0.32975301         | Delta = 0.00000031 | N = 20 |
| $f(x) = x * \cos(x) a = 0, b = 1$       |                    |        |
| Ext Simpson's Rule = 0.02067687         | Delta = 0.00000000 | N = 10 |
| Ext Simpson's Rule = 0.02067486         | Delta = 0.00000201 | N = 20 |
| Ext Simpson's Rule = 0.02067473         | Delta = 0.00000013 | N = 40 |
| $f(x) = (1 + x * x) ^ 1.5 a = 0, b = 1$ |                    |        |
| Ext Simpson's Rule = 6.24063260         | Delta = 0.00000000 | N = 10 |
| Ext Simpson's Rule = 6.24063229         | Delta = 0.00000031 | N = 20 |

#### Exercise 7.4-5

|                                             |                    |        |
|---------------------------------------------|--------------------|--------|
| $f(x) = x * \ln(x) a = 1, b = 2$            |                    |        |
| Ext Corrected Trapezoidal Rule = 0.63629426 | Delta = 0.00000000 | N = 10 |
| Ext Corrected Trapezoidal Rule = 0.63629435 | Delta = 0.00000010 | N = 20 |

|                                 |                    |        |
|---------------------------------|--------------------|--------|
| Ext Simpson's Rule = 0.63629477 | Delta = 0.00000000 | N = 10 |
| Ext Simpson's Rule = 0.63629439 | Delta = 0.00000039 | N = 20 |

#### Exercise 7.4-6

$f(x) = x * \ln(x)$   $a = 1, b = 2$   
 Gaussian Quadrature = +0.21204983 N = 2  
 Gaussian Quadrature = +0.63629433 N = 4

#### Exercise 7.4-6

$f(x) = \text{erf}(0.5)$   $a = 1, b = 2$   
 Gaussian Quadrature = 0.1734772771 Error = 0.3470225989 N = 2  
 Gaussian Quadrature = 0.5204998768 Error = 0.0000000008 N = 4

#### Exercise 7.4-5

$f(x) = (\sin(x)) ^ 1 / 3$   $a = 0, b = \pi$   
 Ext Simpson's Rule = 2.28843984 N = 5  
 Ext Simpson's Rule = 2.52869542 N = 10  
 Ext Simpson's Rule = 2.51811474 N = 15  
 Ext Simpson's Rule = 2.56391812 N = 20

#### Example 7.8

$f(x) = \text{sqrt}(x)$   $a = 0, b = 1$   
 Adaptive Ext Simpson's Rule 16 = 0.6665081031  
 Adaptive Ext Simpson's Rule 32 = 0.6666106059  
 Adaptive Ext Simpson's Rule 48 = 0.6666361511  
 Adaptive Ext Simpson's Rule 64 = 0.6666468462  
 Adaptive Ext Simpson's Rule 80 = 0.6666524843  
 Adaptive Ext Simpson's Rule 96 = 0.6666558778  
 Extended Simpson's Rule 16 = 0.6653981886  
 Extended Simpson's Rule 32 = 0.6662181827  
 Extended Simpson's Rule 48 = 0.6664225423  
 Extended Simpson's Rule 64 = 0.6665081031  
 Extended Simpson's Rule 80 = 0.6665532078  
 Extended Simpson's Rule 96 = 0.6665803555

#### Exercise 7.5-3

$f(x) = (1 - x * x) ^ 1.5$   $a = 0, b = 1$   
 Adaptive Ext Simpson's Rule 16 = 0.5890498584  
 Adaptive Ext Simpson's Rule 32 = 0.5890488392  
 Adaptive Ext Simpson's Rule 48 = 0.5890487009  
 Adaptive Ext Simpson's Rule 64 = 0.5890486607  
 Adaptive Ext Simpson's Rule 80 = 0.5890486444  
 Extended Simpson's Rule 16 = 0.5890901605  
 Extended Simpson's Rule 32 = 0.5890557315  
 Extended Simpson's Rule 48 = 0.5890511738  
 Extended Simpson's Rule 64 = 0.5890498584  
 Extended Simpson's Rule 80 = 0.5890493276

#### Exercise 7.5-3

$f(x) = (1 - x * x) ^ 1.5$   $a = 0, b = 1$   
 Adaptive Ext Simpson's Rule 16 = 1.7778147064  
 Adaptive Ext Simpson's Rule 32 = 1.8238986445  
 Adaptive Ext Simpson's Rule 48 = 1.8443145686  
 Adaptive Ext Simpson's Rule 64 = 1.8564848780  
 Adaptive Ext Simpson's Rule 80 = 1.8647903026  
 Extended Simpson's Rule 16 = 1.6204721387  
 Extended Simpson's Rule 32 = 1.7126419182  
 Extended Simpson's Rule 48 = 1.7534740156  
 Extended Simpson's Rule 64 = 1.7778147064  
 Extended Simpson's Rule 80 = 1.7944255846

#### Example 7.9

Romberg T-Table  
 0.7313702518  
 0.7429840978 0.7468553798

|              |              |              |              |
|--------------|--------------|--------------|--------------|
| 0.7458656148 | 0.7468261205 | 0.7468241699 |              |
| 0.7465845968 | 0.7468242574 | 0.7468241332 | 0.7468241326 |
| 0.7467642547 | 0.7468241406 | 0.7468241328 | 0.7468241328 |
| 0.7468241328 |              |              |              |
| 0.7468091636 | 0.7468241333 | 0.7468241328 | 0.7468241328 |
| 0.7468241328 | 0.7468241328 |              |              |

Table of ratios

|      |       |       |        |
|------|-------|-------|--------|
| 4.03 |       |       |        |
| 4.01 | 15.70 |       |        |
| 4.00 | 15.95 | 89.20 |        |
| 4.00 | 15.99 | 69.89 | 299.13 |

Exercise 7.7-2 (a)

Romberg T-Table

|              |              |              |              |
|--------------|--------------|--------------|--------------|
| 0.3750000000 |              |              |              |
| 0.3437500000 | 0.3333333333 |              |              |
| 0.3359375000 | 0.3333333333 | 0.3333333333 |              |
| 0.3339843750 | 0.3333333333 | 0.3333333333 | 0.3333333333 |
| 0.3334960938 | 0.3333333333 | 0.3333333333 | 0.3333333333 |
| 0.3333333333 |              |              |              |
| 0.3333740234 | 0.3333333333 | 0.3333333333 | 0.3333333333 |
| 0.3333333333 | 0.3333333333 |              |              |

Table of ratios

|      |      |      |      |
|------|------|------|------|
| 4.00 |      |      |      |
| 4.00 | 0.00 |      |      |
| 4.00 | 0.00 | 0.00 |      |
| 4.00 | 0.00 | 0.00 | 0.00 |

Exercise 7.7-2 (b)

Romberg T-Table

|               |               |               |               |
|---------------|---------------|---------------|---------------|
| 0.0000000000  |               |               |               |
| 0.5000000000  | 0.6666666667  |               |               |
| -0.1035533906 | -0.3047378541 | -0.3694981555 |               |
| 0.0835223297  | 0.1458809032  | 0.1759221537  | 0.1845796189  |
| 0.1169292757  | 0.1280649244  | 0.1268771925  | 0.1260987010  |
| 0.1258693641  |               |               |               |
| -0.0078277175 | -0.0494133819 | -0.0612452690 | -0.0642313398 |
| 0.0649777321  | -0.0651642884 |               |               |

Table of ratios

|       |        |        |      |
|-------|--------|--------|------|
| -0.83 |        |        |      |
| -3.23 | -2.16  |        |      |
| 5.60  | -25.29 | -11.12 |      |
| -0.27 | 0.10   | 0.26   | 0.31 |

Exercise 7.7-2 (c)

Romberg T-Table

|              |              |              |              |
|--------------|--------------|--------------|--------------|
| 1.0000000000 |              |              |              |
| 1.0000000000 | 1.0000000000 |              |              |
| 1.0000000000 | 1.0000000000 | 1.0000000000 |              |
| 1.0000000000 | 1.0000000000 | 1.0000000000 | 1.0000000000 |
| 1.0000000000 | 1.0000000000 | 1.0000000000 | 1.0000000000 |
| 1.0000000000 |              |              |              |
| 1.0000000000 | 1.0000000000 | 1.0000000000 | 1.0000000000 |
| 1.0000000000 | 1.0000000000 |              |              |

Table of ratios

|       |       |       |       |
|-------|-------|-------|-------|
| 0.00  |       |       |       |
| 0.00  | 0.00  |       |       |
| -0.00 | -0.00 | -0.00 |       |
| 0.00  | -3.00 | -3.00 | -3.00 |

Exercise 7.7-2 (d)

Romberg T-Table

|              |              |              |              |
|--------------|--------------|--------------|--------------|
| 0.5000000000 |              |              |              |
| 0.3333333333 | 0.2777777778 |              |              |
| 0.2916666667 | 0.2777777778 | 0.2777777778 |              |
| 0.2812500000 | 0.2777777778 | 0.2777777778 | 0.2777777778 |

|              |              |              |              |
|--------------|--------------|--------------|--------------|
| 0.2786458333 | 0.2777777778 | 0.2777777778 | 0.2777777778 |
| 0.2777777778 |              |              |              |
| 0.2779947917 | 0.2777777778 | 0.2777777778 | 0.2777777778 |
| 0.2777777778 | 0.2777777778 |              |              |

Table of ratios

|      |      |      |      |
|------|------|------|------|
| 4.00 |      |      |      |
| 4.00 | 0.00 |      |      |
| 4.00 | 0.00 | 0.00 |      |
| 4.00 | 0.00 | 0.00 | 0.00 |

#### Exercise 7.7-2 (d)

##### Romberg T-Table

|              |              |              |              |
|--------------|--------------|--------------|--------------|
| 0.2777777778 |              |              |              |
| 0.2777777778 | 0.2777777778 |              |              |
| 0.2777777778 | 0.2777777778 | 0.2777777778 |              |
| 0.2777777778 | 0.2777777778 | 0.2777777778 | 0.2777777778 |
| 0.2777777778 | 0.2777777778 | 0.2777777778 | 0.2777777778 |
| 0.2777777778 |              |              |              |
| 0.2777777778 | 0.2777777778 | 0.2777777778 | 0.2777777778 |
| 0.2777777778 | 0.2777777778 | 0.2777777778 | 0.2777777778 |

##### Table of ratios

|      |      |      |      |
|------|------|------|------|
| 0.00 |      |      |      |
| 0.00 | 0.00 |      |      |
| 0.00 | 0.00 | 0.00 |      |
| 0.00 | 0.00 | 0.00 | 0.00 |

#### Exercise 7.7-2 (e)

##### Romberg T-Table

|              |              |              |              |
|--------------|--------------|--------------|--------------|
| 0.6035533906 |              |              |              |
| 0.6432830462 | 0.6565262648 |              |              |
| 0.6581302216 | 0.6630792801 | 0.6635161478 |              |
| 0.6635811969 | 0.6653981886 | 0.6655527825 | 0.6655851101 |
| 0.6655589363 | 0.6662181827 | 0.6662728490 | 0.6662842786 |
| 0.6662870205 |              |              |              |
| 0.6662708114 | 0.6665081031 | 0.6665274311 | 0.6665314721 |
| 0.6665324415 | 0.6665326814 |              |              |

##### Table of ratios

|      |      |      |      |
|------|------|------|------|
| 2.68 |      |      |      |
| 2.72 | 2.83 |      |      |
| 2.76 | 2.83 | 2.83 |      |
| 2.78 | 2.83 | 2.83 | 2.83 |

#### Exercise 7.7-2 (f)

##### Romberg T-Table

|              |              |              |              |
|--------------|--------------|--------------|--------------|
| 0.9397932848 |              |              |              |
| 0.9445135217 | 0.9460869340 |              |              |
| 0.9456908636 | 0.9460833109 | 0.9460830694 |              |
| 0.9459850299 | 0.9460830854 | 0.9460830704 | 0.9460830704 |
| 0.9460585610 | 0.9460830713 | 0.9460830704 | 0.9460830704 |
| 0.9460830704 |              |              |              |
| 0.9460769431 | 0.9460830704 | 0.9460830704 | 0.9460830704 |
| 0.9460830704 | 0.9460830704 |              |              |

##### Table of ratios

|      |       |       |        |
|------|-------|-------|--------|
| 4.01 |       |       |        |
| 4.00 | 16.07 |       |        |
| 4.00 | 16.02 | 64.31 |        |
| 4.00 | 16.00 | 64.07 | 343.50 |

#### My Example

$f(x) = e^{(-x*x)}$   $a = 0$ ,  $b = 100$   
 Exact Value = 0.886226925453  
 Adaptive Ext Simpson's Rule 16 = 0.702223048723  
 Error 16 = 0.184003876729  
 Adaptive Ext Simpson's Rule 32 = 0.875857083838  
 Error 32 = 0.010369841614  
 Adaptive Ext Simpson's Rule 48 = 0.886160675431

|                                |                  |
|--------------------------------|------------------|
| Error 48                       | = 0.000066250022 |
| Adaptive Ext Simpson's Rule 64 | = 0.886226869380 |
| Error 64                       | = 0.000000056073 |
| Adaptive Ext Simpson's Rule 80 | = 0.886226925446 |
| Error 80                       | = 0.000000000006 |
| Adaptive Ext Simpson's Rule 96 | = 0.886226925446 |
| Error 96                       | = 0.000000000006 |
| Extended Simpson's Rule 16     | = 2.083333333333 |
| Error 16                       | = 1.197106407881 |
| Extended Simpson's Rule 32     | = 1.041905795370 |
| Error 32                       | = 0.155678869917 |
| Extended Simpson's Rule 48     | = 0.730647005206 |
| Error 48                       | = 0.155579920247 |
| Extended Simpson's Rule 64     | = 0.702223048723 |
| Error 64                       | = 0.184003876729 |
| Extended Simpson's Rule 80     | = 0.767628992292 |
| Error 80                       | = 0.118597933161 |
| Extended Simpson's Rule 96     | = 0.767628992292 |
| Error 96                       | = 0.118597933161 |