

Internet Math Puzzle

$$\frac{8^x - 2^x}{6^x - 3^x} = 2$$

Multiply by the preceding denominator:

$$f(x) = 8^x - 2^x - 2(6^x - 3^x)$$

$$f'(x) = 8^x \ln 8 - 2^x \ln 2 - 2[6^x \ln 6 - 3^x \ln 3]$$

By inspection and the Newton-Raphson iteration method using the function and its derivative, it is easy to show that $x = 1$.

$x_0 = 2.0$

maximum iterations = 100

$x_1 = 1$

iterations = 10

$x_0 = 1.75$

maximum iterations = 100

$x_1 = 1$

iterations = 100

$x_0 = 1.5$

maximum iterations = 100

$x_1 = 1$

iterations = 9

$x_0 = 1.25$

maximum iterations = 100

$x_1 = 1$

iterations = 8

$x_0 = 1$

maximum iterations = 100

$x_1 = 1$

iterations = 1

$x_0 = 0$