Online Function Minimization Problem © Wednesday August 28, 2024, by James Pate Williams, Jr. Requires Basic Differential Calculus and Elementary Numerical Analysis

The function to be minimized is:

$$f(x, y) = \sqrt{4 + x^2} + \sqrt{9 + y^2}$$

Subject to the linear equality constraint:

$$x + y = 12$$

Elimination of y via the constraint equation yields:

$$y = 12 - x$$

And with substitution we have a one-dimensional equation to be minimized.

$$h(x) = \sqrt{4 + x^2} + \sqrt{9 + 144 - 24x + x^2}$$

which is a one-dimensional equation. We now setup the problem to be solved numerically using the Newton-Raphson Method. We want to find the minimum of the previous equation using its first and second ordinary derivatives. Recall from basic calculus that the first derivative is zero at an extremum (minimum or maximum).

$$h'(x) = \frac{x}{\sqrt{4 + x^2}} + \frac{x - 12}{\sqrt{153 - 24x + x^2}}$$

$$h''(x) = \frac{1}{\sqrt{4 + x^2}} - \frac{x^2}{(4 + x^2)^{3/2}} + \frac{1}{\sqrt{153 - 24x + x^2}} - \frac{(x - 12)^2}{(153 - 24x + x^2)^{3/2}}$$

The Newton-Raphson iterative function is written:

$$x_{n+1} = x_n - \frac{h'(x)}{h''(x)}$$

Now we can write a simple C application to determine the minimum of the target function. The output from our little bit of source code is shown below:

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Minimize: f(x, y) = sqrt(4 + x^2) + sqrt(9 + y^2)
Subject to the equality constraint as follows:
x + y = 12
x0 = 2
x = 4.800000
y = 7.200000
f = 13.000000
h'(x)  = 0.000000
h''(x) = 0.047413
another run (n or y)? n
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