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1  /* Other online mathematics problems (c)
2  * August 27, 2024 by James Pate Williams, Jr.
3  * Find the root of  $f(t) = \cos(2t) + \sin(3t)$ 
4  *  $t = 0.628318530718$ 
5  *  $f(t) = 1.11022302e-16$ 
6  * Also find the root of the equation
7  *  $f(x) = \sqrt{1 + \sqrt{1 + x}} - x^{1/3}$ 
8  *  $x = 8.000000000000$ 
9  *  $f(x) = 0.00000000e+00$ 
10 * Solution of  $f(x) = 9^x + 12^x - 16^x$ 
11 *  $x = -16.387968065352$ 
12 *  $f(x) = 2.32137533e-16$ 
13 * Solution of  $f(x) = 8^x - 2^x - 2(6^x - 3^x)$ 
14 *  $x = 1.000000000000$ 
15 *  $f(x) = 0.00000000e+00$ 
16 */
17
18 #include <math.h>
19 #include <stdio.h>
20
21 typedef long double real;
22
23 real f1(real t) {
24     return cosl(2 * t) + cosl(3 * t);
25 }
26
27 real g(real t) {
28     return -2 * sinl(2 * t) - 3 * sinl(3 * t);
29 }
30
31 real f2(real x) {
32     return sqrtl(1 + sqrtl(1 + x)) - powl(x, 1.0 / 3.0);
33 }
34
35 real f3(real x) {
36     return powl(9, x) + powl(12, x) - powl(16, x);
37 }
38
39 real f4(real x) {
40     return powl(8, x) - powl(2, x) -
41         2 * (powl(6, x) - powl(3, x));
42 }
43
44 real CentralDifference(
45     real x, real h, real(*fx)(real)) {
46     return (fx(x + h) - fx(x - h)) / (2.0 * h);
47 }
48
49 real NewtonRaphson1(
50     real x0, real ftolerance, real xtolerance,
51     real(*fx)(real), real(*deriv)(real)) {
52     real x1 = 0.0;
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53
54     while (1) {
55         real f = fx(x0);
56         real dfdx = deriv(x0);
57         x1 = x0 - f / dfdx;
58
59         if (fabsl(x1 - x0) < xtolerance ||
60             fabsl(f) < ftolerance)
61             break;
62
63         x0 = x1;
64     }
65
66     return x1;
67 }
68
69 real NewtonRaphson2(
70     real x0, real ftolerance, real xtolerance,
71     real(*fx)(real),
72     real(*deriv)(real, real, real(*fx)(real))) {
73     real h = 2.0 / 10000.0, x1 = 0.0;
74
75     while (1) {
76         real f = fx(x0);
77         real dfdx = deriv(x0, h, fx);
78         x1 = x0 - f / dfdx;
79
80         if (fabsl(x1 - x0) < xtolerance ||
81             fabsl(f) < ftolerance)
82             break;
83
84         x0 = x1;
85     }
86
87     return x1;
88 }
89
90 int main(void) {
91     real ftolerance = 1.0e-15, xtolerance = 1.0e-15;
92     real t = 0, x = 0;
93
94     t = NewtonRaphson1(
95         0.5, ftolerance, xtolerance, f1, g);
96     printf_s("Solution of f(t) = cos(2t) + cos(3t)\n");
97     printf_s("t      = %14.12Lf\n", t);
98     printf_s("f(t) = %14.8e\n", f1(t));
99     x = NewtonRaphson2(
100         1.5, ftolerance, xtolerance, f2, CentralDifference);
101     printf_s("Solution of f(x) = sqrt(1 + sqrt(1 + x)) - x^1/3\n");
102     printf_s("x      = %14.12Lf\n", x);
103     printf_s("f(x) = %14.8e\n", f2(x));
104     x = NewtonRaphson2(
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105     1.0, ftolerance, xtolerance, f3, CentralDifference);
106     printf_s("Solution of  $f(x) = 9^x + 12^x - 16^x$ \n");
107     printf_s("x      = \t%14.12Lf\n", x);
108     printf_s("f(x) = \t %14.8e\n", f3(x));
109     x = NewtonRaphson2(
110         1.0, ftolerance, xtolerance, f4, CentralDifference);
111     printf_s("Solution of  $f(x) = 8^x - 2^x - 2(6^x - 3^x)$ \n");
112     printf_s("x      = \t%14.12Lf\n", x);
113     printf_s("f(x) = \t%14.8e\n", f4(x));
114     return 0;
115 }
```