Coupled First Order Ordinary Differential Equations © Wednesday, October 2, 2024, by James Pate Williams, Jr.

7.2: Coupled First-Order Equations - Mathematics LibreTexts

$$\dot{x}_1 = ax_1 + bx_2$$

$$\dot{x}_2 = cx_1 + dx_2$$

$$\dot{\vec{x}} = Ax$$

$$\frac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$\vec{x}(t) = \vec{v}e^{\lambda t}$$

$$\vec{x}(t) = \vec{v}e^{\lambda t}$$

$$\lambda\vec{v}e^{\lambda t} = A\vec{v}e^{\lambda t}$$

$$A\vec{v} = \lambda\vec{v}$$

$$det(A - \lambda I) = 0$$

$$det\begin{pmatrix} a-\lambda & b \\ c & d-\lambda \end{pmatrix} = 0$$

$$det\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \frac{1}{ad-bc}\begin{vmatrix} d & -b \\ -c & a \end{vmatrix}$$

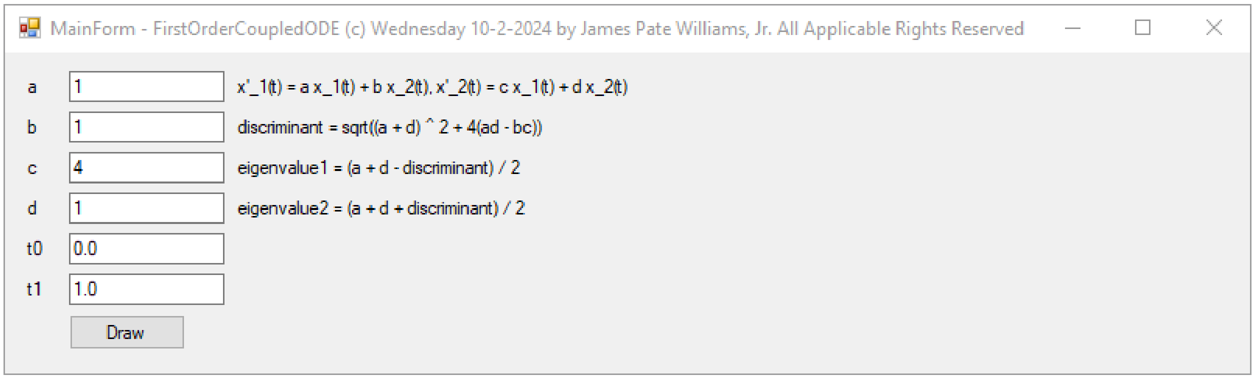
$$\frac{1}{ad-bc}\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

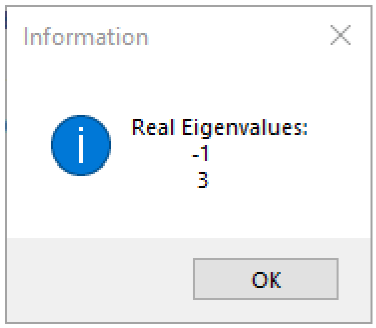
$$det\begin{pmatrix} a-\lambda & b \\ c & d-\lambda \end{pmatrix} = \frac{1}{(a-\lambda)(d-\lambda)}\begin{vmatrix} d-\lambda & -b \\ -c & a-\lambda \end{vmatrix} = 0$$

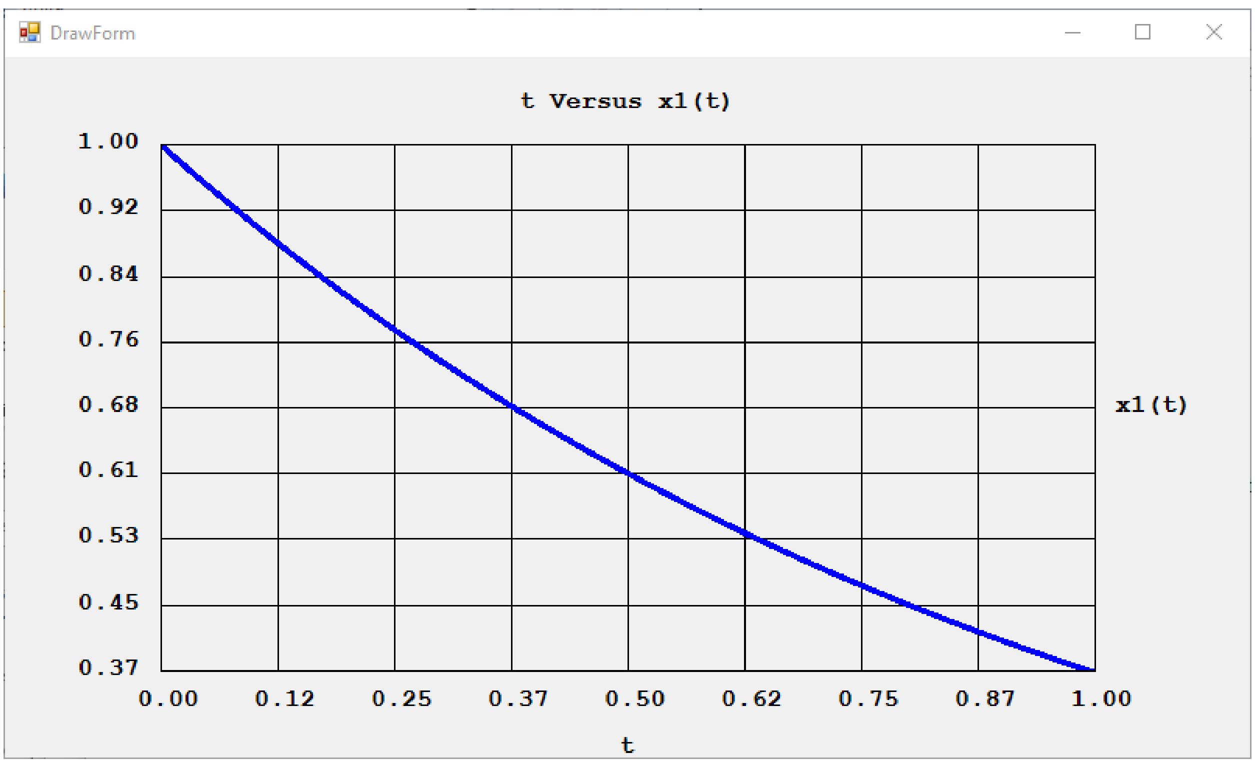
$$\begin{vmatrix} d-\lambda & -b \\ -c & a-\lambda \end{vmatrix} = (d-\lambda)(a-\lambda) - bc = 0$$

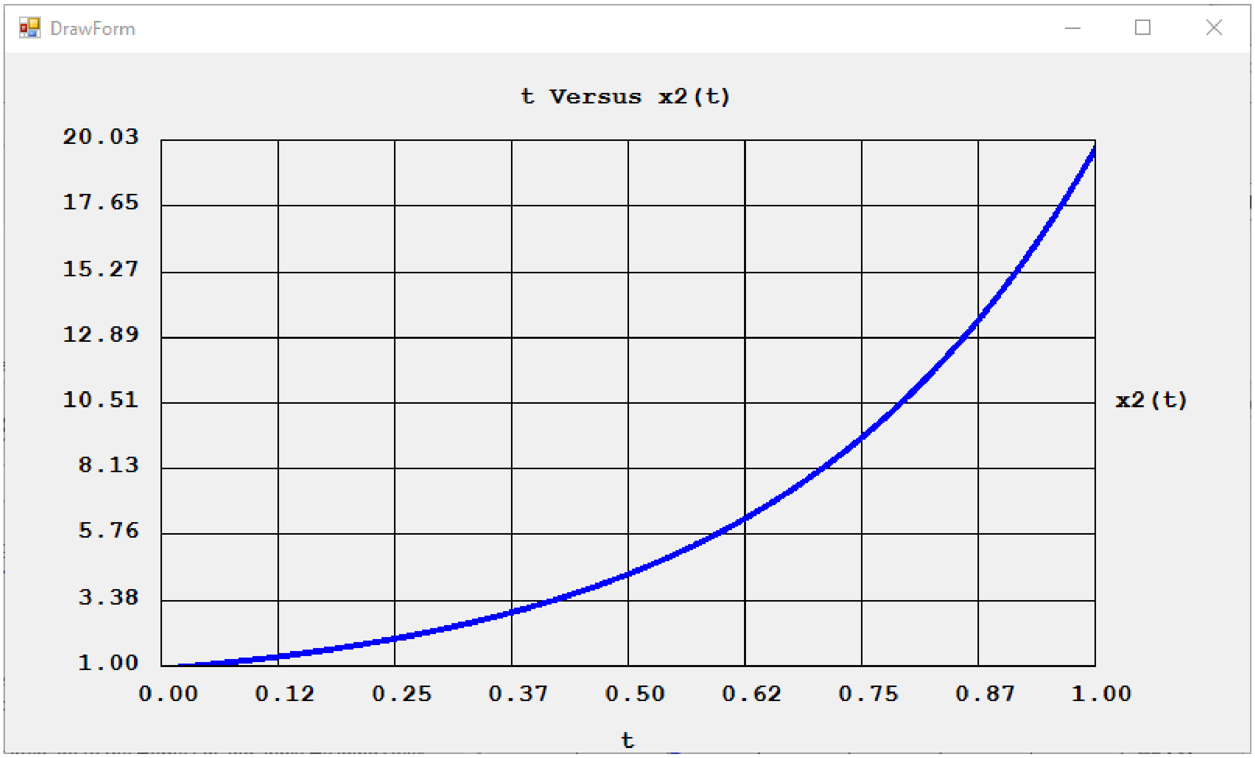
$$\lambda^2 - a\lambda - d\lambda + ad - cb = 0$$

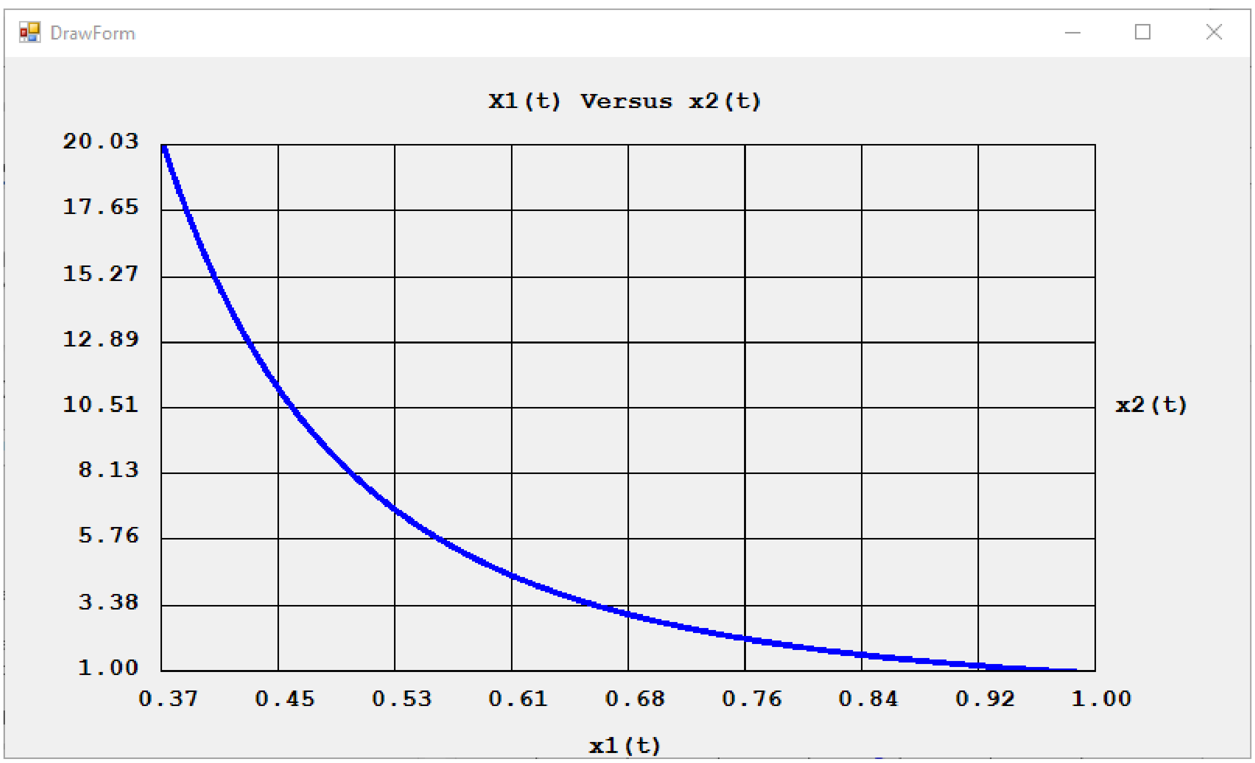
$$\lambda = \frac{a + d \pm \sqrt{(a+d)^2 + 4(ad-bc)}}{2}$$

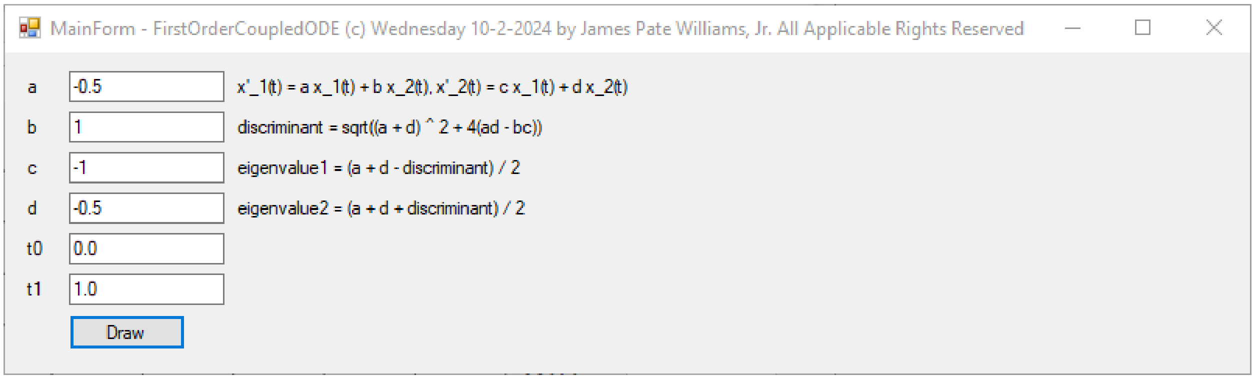

MainForm - FirstOrderCoupledODE (c) Wednesday 10-2-2024 by James Pate Williams, Jr. All Applicable Rights Reserved
a 1
b 1
c 4
d 1
t0 0.0
t1 1.0
x'_1(t) = a x_1(t) + b x_2(t), x'_2(t) = c x_1(t) + d x_2(t)
discriminant = sqrt((a + d) ^ 2 + 4(ad - bc))
eigenvalue1 = (a + d - discriminant) / 2
eigenvalue2 = (a + d + discriminant) / 2
Draw

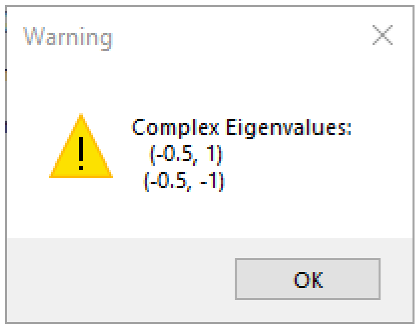

Information
Real Eigenvalues:
-1
3
OK


DrawForm
t Versus x1(t)
1.00
0.92
0.84
0.76
0.68
0.61
0.53
0.45
0.37
x1(t)
0.00
0.12
0.25
0.37
0.50
0.62
0.75
0.87
1.00
t


DrawForm
t Versus x2(t)
20.03
17.65
15.27
12.89
10.51
8.13
5.76
3.38
1.00
0.00
0.12
0.25
0.37
0.50
0.62
0.75
0.87
1.00
t
x2(t)


DrawForm
X1(t) Versus x2(t)
20.03
17.65
15.27
12.89
10.51
8.13
5.76
3.38
1.00
0.37
0.45
0.53
0.61
0.68
0.76
0.84
0.92
1.00
x1(t)
x2(t)


MainForm - FirstOrderCoupledODE (c) Wednesday 10-2-2024 by James Pate Williams, Jr. All Applicable Rights Reserved
a
-0.5
x'_1(t) = a x_1(t) + b x_2(t), x'_2(t) = c x_1(t) + d x_2(t)
b
1
discriminant = sqrt((a + d) ^ 2 + 4(ad - bc))
c
-1
eigenvalue1 = (a + d - discriminant) / 2
d
-0.5
eigenvalue2 = (a + d + discriminant) / 2
t0
0.0
t1
1.0
Draw


Warning
Complex Eigenvalues:
(-0.5, 1)
(-0.5, -1)
OK