Blog Entry © Saturday, January 25, 2025, by James Pate Williams, Jr. Graphs and Tables of Airy Functions

The Airy Functions, Ai(z) and Bi(z) are the two linearly independent solutions of the second order ordinary differential equation:

$$y''(z) - y(z)z = 0$$

We use Frobenius' infinite power series method to solve the real version of the equation:

$$y(x) = \sum_{n=0}^{\infty} a_n x^n$$

The initial conditions are y(0) = 0.35502805, y'(0) = -0.25881940.

$$y''(x) - y(x)x = \sum_{n=0}^{\infty} [n(n-1)a_n x^{n-2} - a_n x^{n+1}] = 0$$

$$3 \cdot 2a_3 - a_0 = 0$$

$$a_3 = \frac{a_0}{6} \cong 0.059171342$$

$$4 \cdot 3a_4 - a_1 = 0$$

$$a_4 = \frac{a_1}{12} \cong -0.02156828$$

$$5 \cdot 4a_5 - a_2 = 0$$

$$a_5 = \frac{a_2}{20} = 0$$

$$6 \cdot 5a_6 - a_3 = 0$$

$$a_6 = \frac{a_3}{30} \cong 0.00197238$$

$$7 \cdot 6a_7 - a_4 = 0$$

$$a_7 = \frac{a_4}{42} \cong -5.1353048 \times 10^{-4}$$

$$8 \cdot 7a_8 - a_5 = 0$$

$$a_8 = \frac{a_5}{56} = 0$$

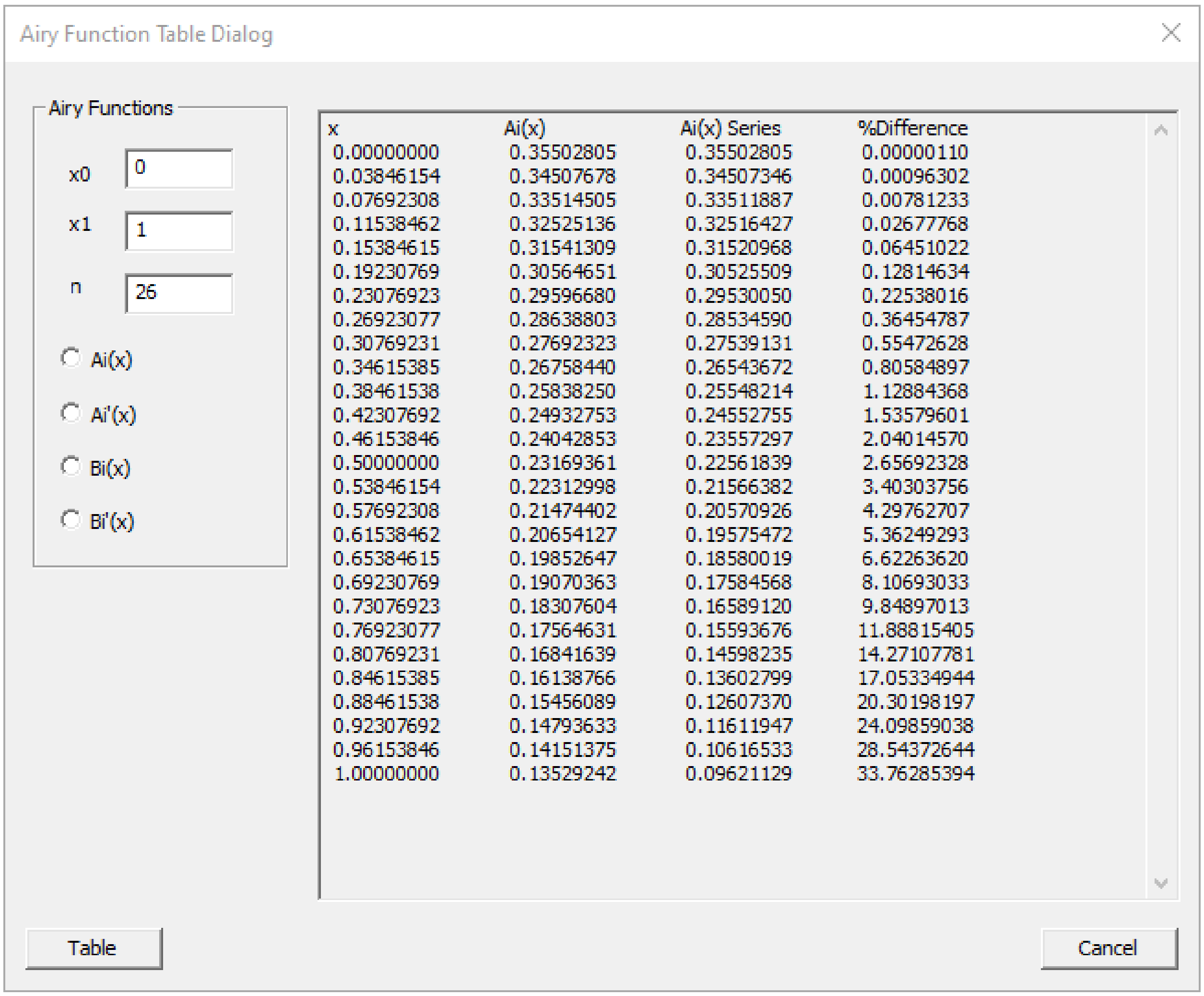
$$9 \cdot 8a_9 - a_6 = 0$$

$$a_9 = \frac{a_6}{72} \cong 2.7394167 \times 10^{-5}$$

$$10 \cdot 9a_{10} - a_7 = 0$$

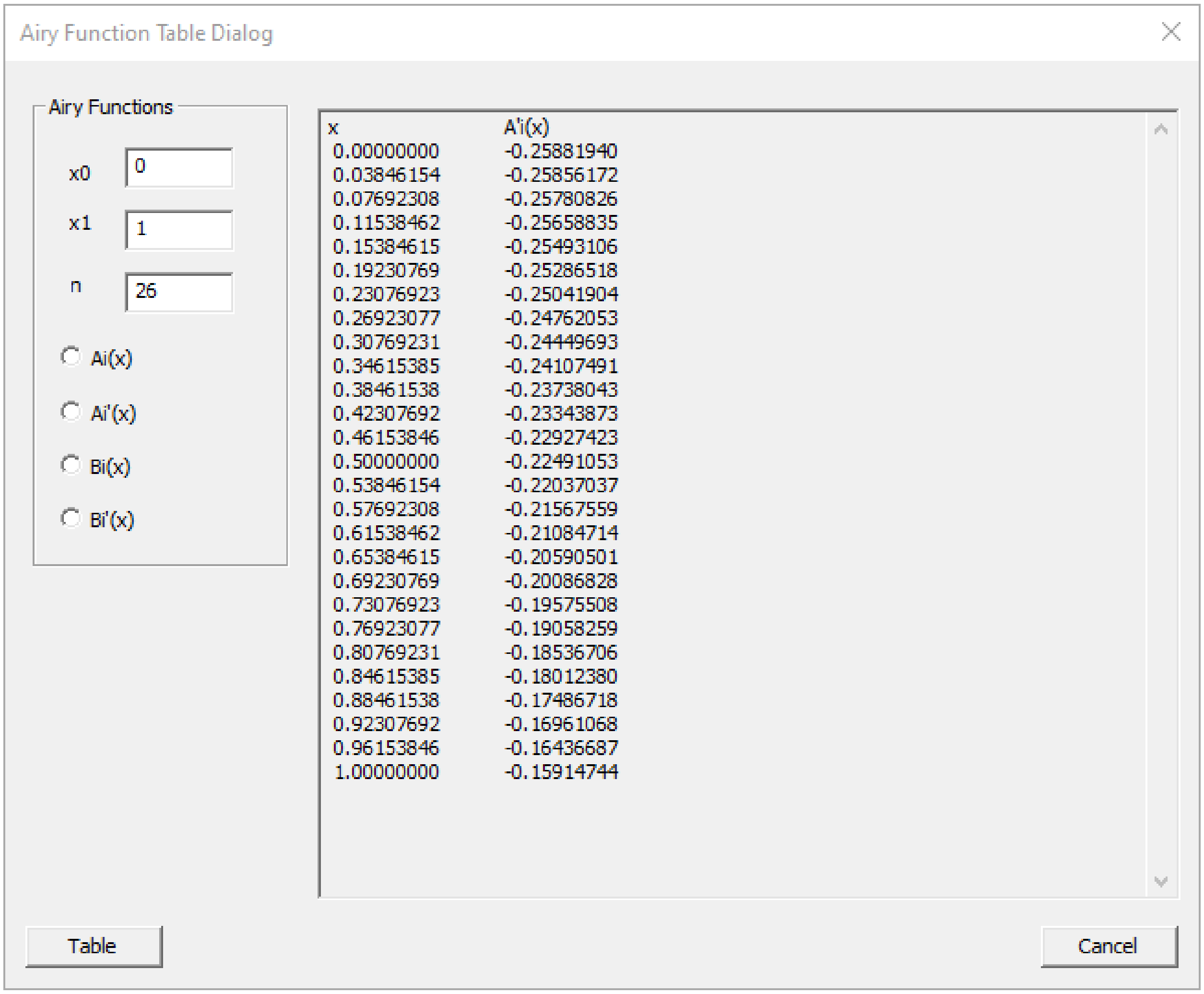
$$a_{10} = \frac{a_7}{90} \cong \frac{-0.00051353048}{90} \cong -5.7058942 \times 10^{-6}$$

$$y(x) \cong 0.35502805 - 0.25881940\mathrm{x} + \frac{0.00197238x^6}{6!} - 5.1353048 \times \frac{10^{-4}x^7}{7!} + 2.7394167 \times \frac{10^{-5}x^9}{9!} - 5.7058942 \times \frac{10^{-6}x^{10}}{10!} + \cdots$$

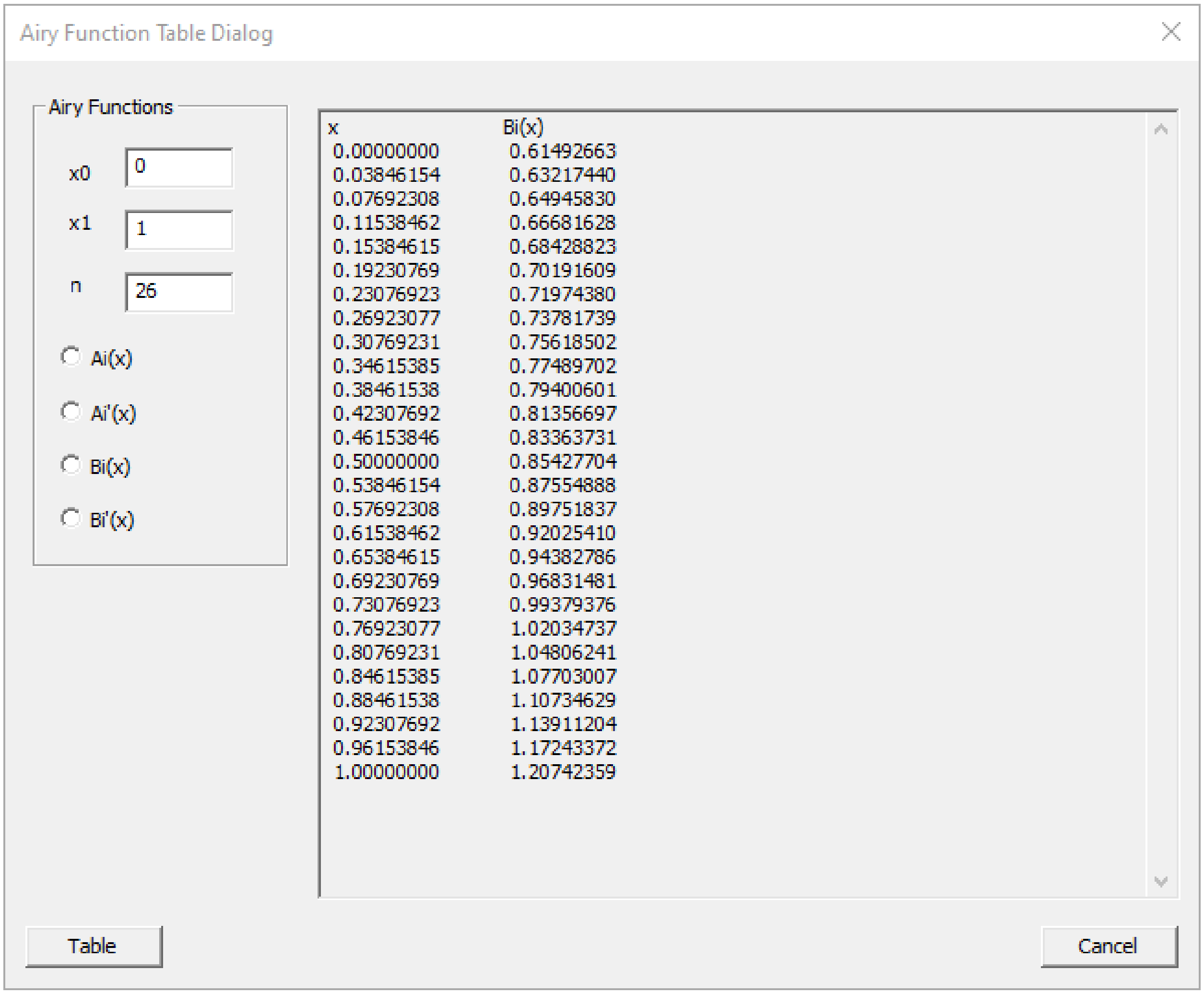


x	Ai(x)	Ai(x) Series	%Difference
0.00000000	0.35502805	0.35502805	0.00000110
0.03846154	0.34507678	0.34507346	0.00096302
0.07692308	0.33514505	0.33511887	0.00781233
0.11538462	0.32525136	0.32516427	0.02677768
0.15384615	0.31541309	0.31520968	0.06451022
0.19230769	0.30564651	0.30525509	0.12814634
0.23076923	0.29596680	0.29530050	0.22538016
0.26923077	0.28638803	0.28534590	0.36454787
0.30769231	0.27692323	0.27539131	0.55472628
0.34615385	0.26758440	0.26543672	0.80584897
0.38461538	0.25838250	0.25548214	1.12884368
0.42307692	0.24932753	0.24552755	1.53579601
0.46153846	0.24042853	0.23557297	2.04014570
0.50000000	0.23169361	0.22561839	2.65692328
0.53846154	0.22312998	0.21566382	3.40303756
0.57692308	0.21474402	0.20570926	4.29762707
0.61538462	0.20654127	0.19575472	5.36249293
0.65384615	0.19852647	0.18580019	6.62263620
0.69230769	0.19070363	0.17584568	8.10693033
0.73076923	0.18307604	0.16589120	9.84897013
0.76923077	0.17564631	0.15593676	11.88815405
0.80769231	0.16841639	0.14598235	14.27107781
0.84615385	0.16138766	0.13602799	17.05334944
0.88461538	0.15456089	0.12607370	20.30198197
0.92307692	0.14793633	0.11611947	24.09859038
0.96153846	0.14151375	0.10616533	28.54372644
1.00000000	0.13529242	0.09621129	33.76285394

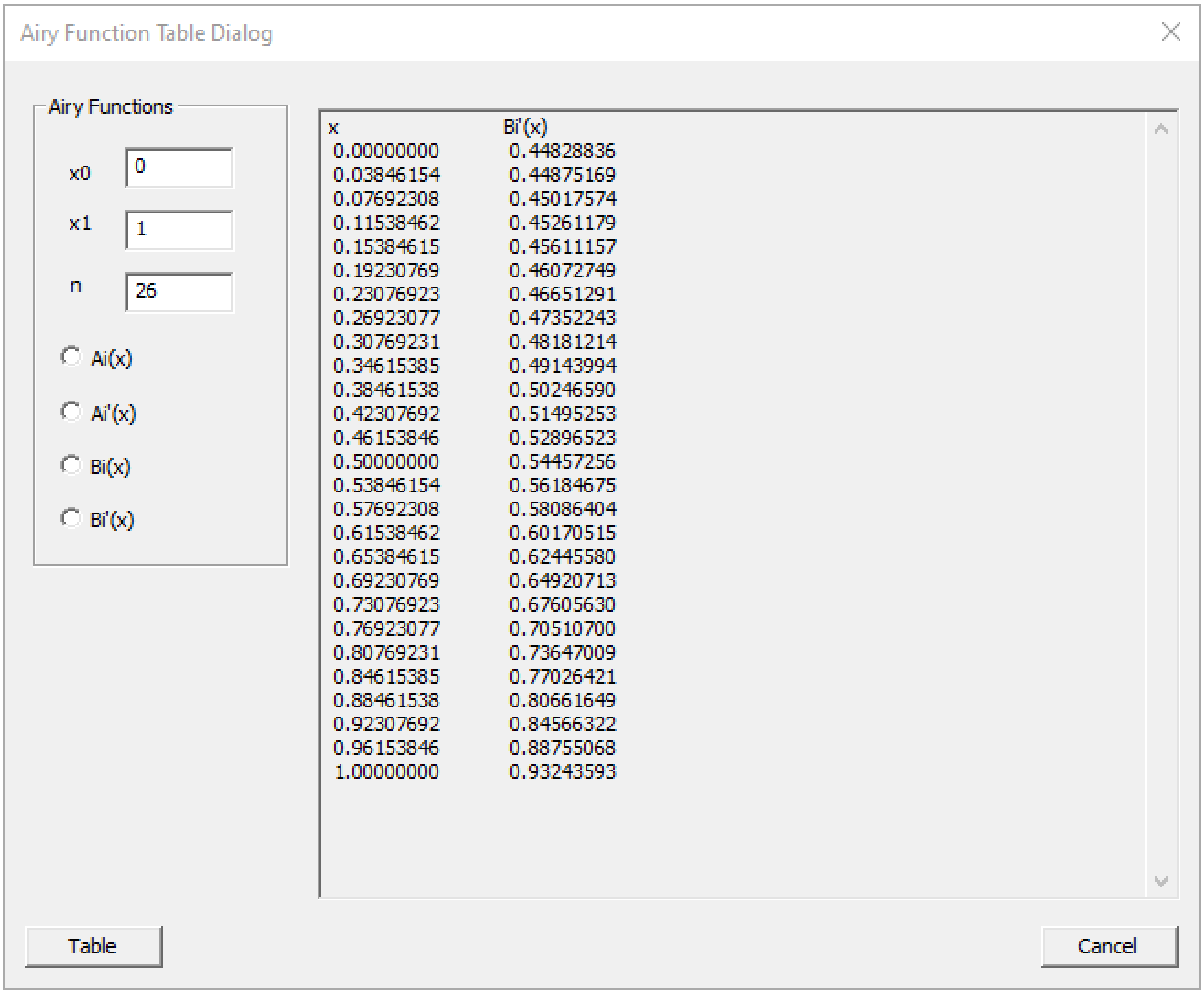
The series approximation is accurate for about x < 0.4.

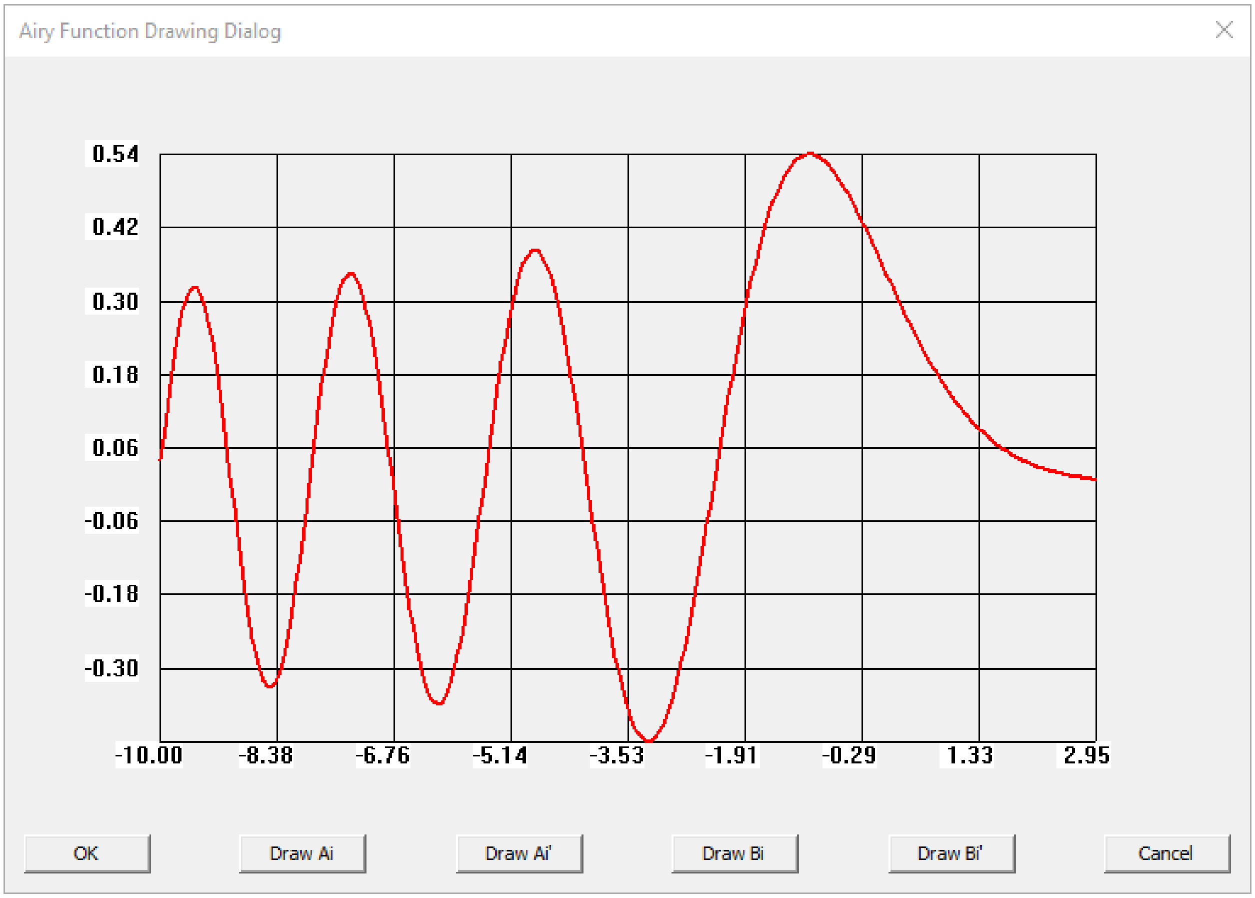


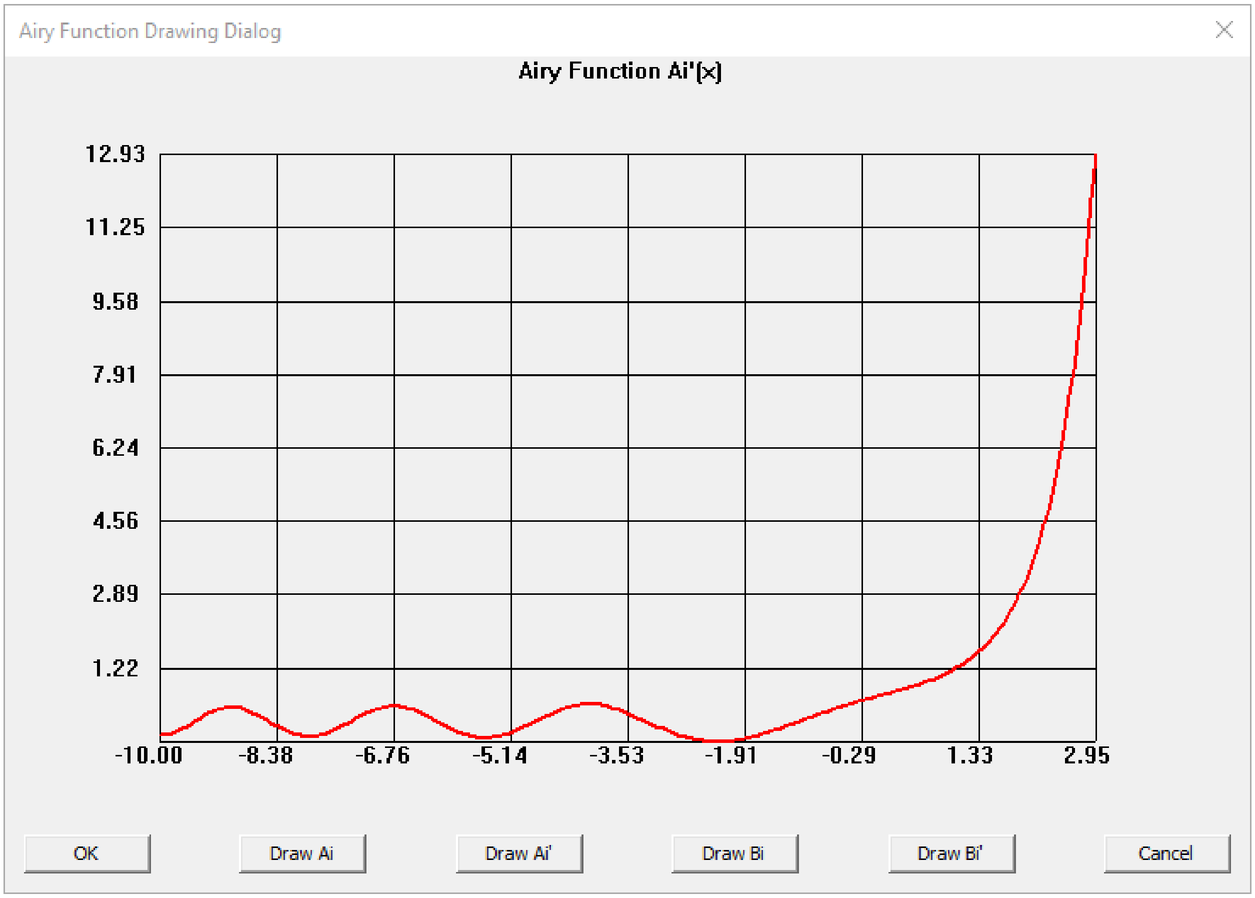
x	A'i(x)
0.00000000	-0.25881940
0.03846154	-0.25856172
0.07692308	-0.25780826
0.11538462	-0.25658835
0.15384615	-0.25493106
0.19230769	-0.25286518
0.23076923	-0.25041904
0.26923077	-0.24762053
0.30769231	-0.24449693
0.34615385	-0.24107491
0.38461538	-0.23738043
0.42307692	-0.23343873
0.46153846	-0.22927423
0.50000000	-0.22491053
0.53846154	-0.22037037
0.57692308	-0.21567559
0.61538462	-0.21084714
0.65384615	-0.20590501
0.69230769	-0.20086828
0.73076923	-0.19575508
0.76923077	-0.19058259
0.80769231	-0.18536706
0.84615385	-0.18012380
0.88461538	-0.17486718
0.92307692	-0.16961068
0.96153846	-0.16436687
1.00000000	-0.15914744

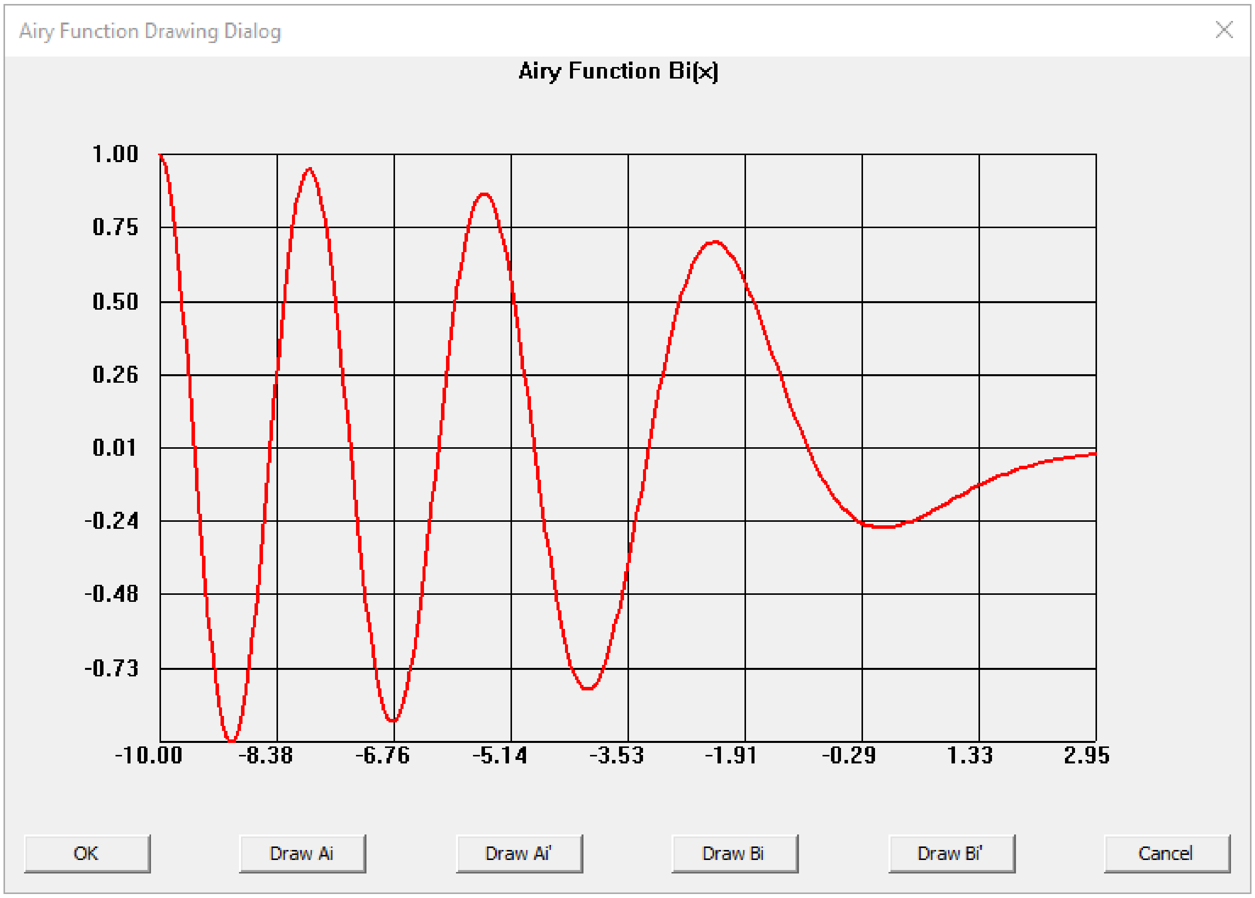


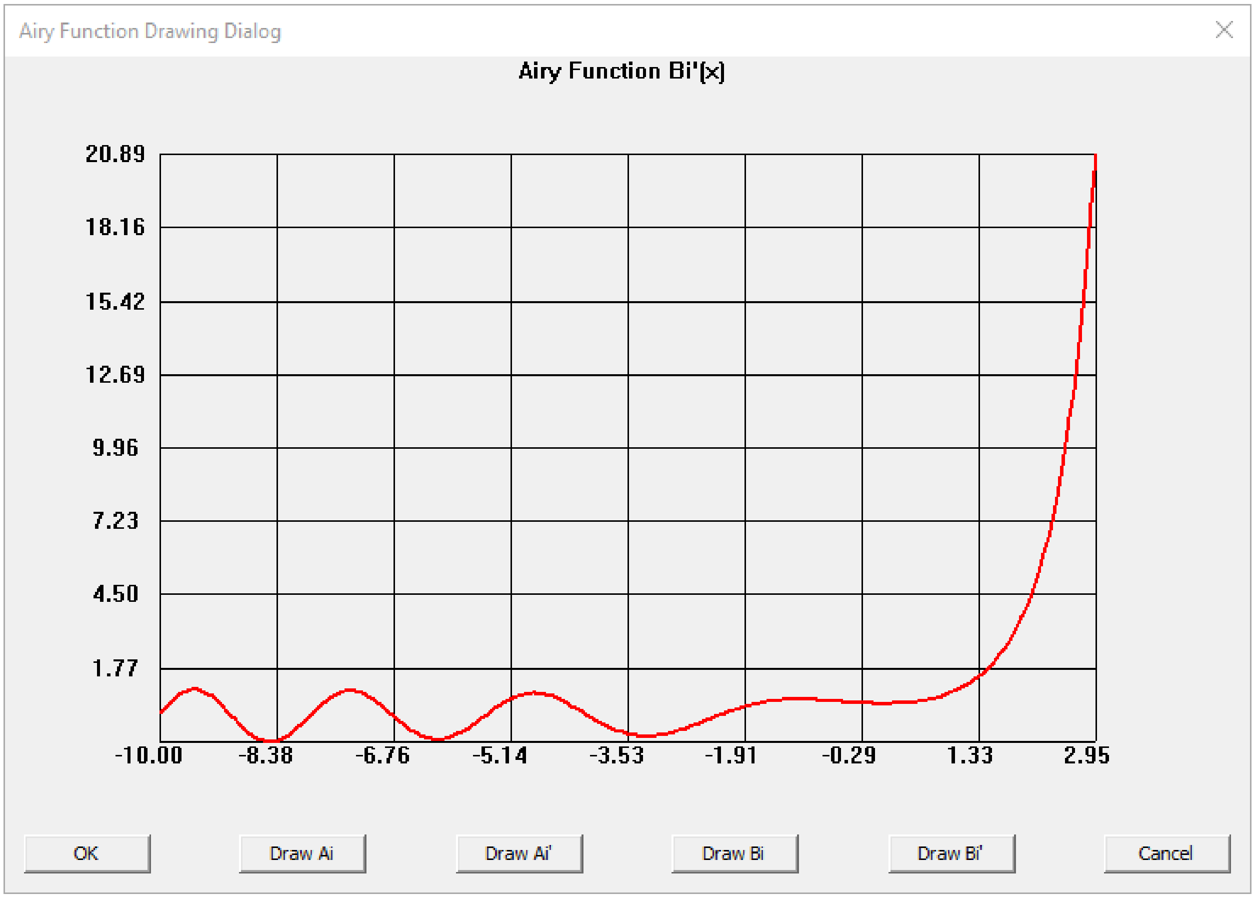
x	Bi(x)
0.00000000	0.61492663
0.03846154	0.63217440
0.07692308	0.64945830
0.11538462	0.66681628
0.15384615	0.68428823
0.19230769	0.70191609
0.23076923	0.71974380
0.26923077	0.73781739
0.30769231	0.75618502
0.34615385	0.77489702
0.38461538	0.79400601
0.42307692	0.81356697
0.46153846	0.83363731
0.50000000	0.85427704
0.53846154	0.87554888
0.57692308	0.89751837
0.61538462	0.92025410
0.65384615	0.94382786
0.69230769	0.96831481
0.73076923	0.99379376
0.76923077	1.02034737
0.80769231	1.04806241
0.84615385	1.07703007
0.88461538	1.10734629
0.92307692	1.13911204
0.96153846	1.17243372
1.00000000	1.20742359


Airy Function Table Dialog
Airy Functions
x0
0
x1
1
n
26
Ai(x)
Ai'(x)
Bi(x)
Bi'(x)
x Bi'(x)
0.00000000 0.44828836
0.03846154 0.44875169
0.07692308 0.45017574
0.11538462 0.45261179
0.15384615 0.45611157
0.19230769 0.46072749
0.23076923 0.46651291
0.26923077 0.47352243
0.30769231 0.48181214
0.34615385 0.49143994
0.38461538 0.50246590
0.42307692 0.51495253
0.46153846 0.52896523
0.50000000 0.54457256
0.53846154 0.56184675
0.57692308 0.58086404
0.61538462 0.60170515
0.65384615 0.62445580
0.69230769 0.64920713
0.73076923 0.67605630
0.76923077 0.70510700
0.80769231 0.73647009
0.84615385 0.77026421
0.88461538 0.80661649
0.92307692 0.84566322
0.96153846 0.88755068
1.00000000 0.93243593
Table
Cancel


Airy Function Drawing Dialog
0.54
0.42
0.30
0.18
0.06
-0.06
-0.18
-0.30
-10.00
-8.38
-6.76
-5.14
-3.53
-1.91
-0.29
1.33
2.95
OK
Draw Ai
Draw Ai'
Draw Bi
Draw Bi'
Cancel


Airy Function Drawing Dialog
Airy Function Ai'(x)
12.93
11.25
9.58
7.91
6.24
4.56
2.89
1.22
-10.00
-8.38
-6.76
-5.14
-3.53
-1.91
-0.29
1.33
2.95
OK
Draw Ai
Draw Ai'
Draw Bi
Draw Bi'
Cancel


Airy Function Drawing Dialog
Airy Function Bi(x)
1.00
0.75
0.50
0.26
0.01
-0.24
-0.48
-0.73
-10.00
-8.38
-6.76
-5.14
-3.53
-1.91
-0.29
1.33
2.95
OK
Draw Ai
Draw Ai'
Draw Bi
Draw Bi'
Cancel


Airy Function Drawing Dialog
Airy Function Bi'(x)
20.89
18.16
15.42
12.69
9.96
7.23
4.50
1.77
-10.00
-8.38
-6.76
-5.14
-3.53
-1.91
-0.29
1.33
2.95
OK
Draw Ai
Draw Ai'
Draw Bi
Draw Bi'
Cancel