Exercises from ***Elementary Statistical Physics*** by C. Kittel © 1958 by James Pate Williams, Jr. © Tuesday, February 25, 2025

Exercise 1.1. Using the Hamiltonian (1.28) for a particle in a magnetic field, find the Hamilton equations of motion. Show that the result for the time derivative of the momentum in the uniform magnetic field

$$A_x = -\frac{1}{2}Hy, A_x = \frac{1}{2}Hx, A_z = 0$$

is just the Lorentz force equation.

$$H = \vec{p}\cdot\vec{v} - \frac{1}{2}Mv^2 + e\varphi - \frac{e}{c}\vec{v}\cdot\vec{A}, (1.28)$$

$$\frac{\partial H}{\partial p_i} = \dot{q_\iota}, \frac{\partial H}{\partial q_i} = -\dot{p_\iota}, (1.10)$$

$$\frac{\partial H}{\partial p_x} = v_x, \frac{\partial H}{\partial p_y} = v_y, \frac{\partial H}{\partial p_z} = v_z, \frac{\partial H}{\partial x} = e\frac{\partial \varphi}{\partial x} - \frac{e}{c}v_xA_x, \frac{\partial H}{\partial y} = e\frac{\partial \varphi}{\partial y} - \frac{e}{c}v_yA_y, \frac{\partial H}{\partial z} = e\frac{\partial \varphi}{\partial z} - \frac{e}{c}v_zA_z$$

$$\dot{p_x} = -e\frac{\partial \varphi}{\partial x} + \frac{e}{c}v_xA_x, \dot{p_y} = -e\frac{\partial \varphi}{\partial y} + \frac{e}{c}v_yA_y, \dot{p_z} = -e\frac{\partial \varphi}{\partial z} + \frac{e}{c}v_zA_z$$

$$\dot{\vec{p}} = -e\vec{\nabla}\varphi + \frac{e}{c}\vec{v}\cdot\vec{A}$$

Exercise 1.2. Find the Hamiliton equations of motion of a free particle in cylindrical coordinates.

$$x = r\cos\theta, y = r\sin\theta, z = z$$

$$\dot{x} = \dot{r}\cos\theta - r\dot{\theta}\sin\theta$$

$$\dot{y} = \dot{r}\sin\theta + r\dot{\theta}\cos\theta$$

$$\dot{x}^2 = (\dot{r}\cos\theta)^2 - 2\dot{r}r\dot{\theta}\cos\theta\sin\theta + \left(r\dot{\theta}\sin\theta\right)^2$$

$$\dot{y}^2 = (\dot{r}\sin\theta)^2 + 2\dot{r}r\dot{\theta}\sin\theta\cos\theta + \left(r\dot{\theta}\cos\theta\right)^2$$

$$L = T = \frac{1}{2}M(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) = \frac{1}{2}M\left[(\dot{r}\cos\theta)^2 + (\dot{r}\sin\theta)^2 + \left(r\dot{\theta}\sin\theta\right)^2 + \left(r\dot{\theta}\cos\theta\right)^2 + \dot{z}^2\right]$$
$$= \frac{1}{2}M\left(\dot{r}^2 + r^2\dot{\theta}^2 + \dot{z}^2\right)$$

$$p_r = \frac{\partial L}{\partial \dot{r}} = \frac{\partial T}{\partial \dot{r}} = M\dot{r}, p_\theta = \frac{\partial L}{\partial \dot{\theta}} = \frac{\partial T}{\partial \dot{\theta}} = Mr^2\theta, p_z = \frac{\partial L}{\partial \dot{z}} = \frac{\partial T}{\partial \dot{z}} = M\dot{z}$$

$$H = L = T$$

$$H = p_r\dot{r} + p_\theta\dot{\theta} + p_z\dot{z} - \frac{1}{2}M\left(\dot{r}^2 + r^2\dot{\theta}^2 + \dot{z}^2\right)$$

$$\frac{\partial H}{\partial p_r} = \dot{r}, \frac{\partial H}{\partial p_\theta} = \dot{\theta}, \frac{\partial H}{\partial p_z} = \dot{z}$$

Exercise 2.1. Consider a simple linear harmonic oscillator; show that the orbit in phase space is an ellipse.

$$q = \sqrt{\frac{J}{\pi m\omega}}\sin\omega t\,, p = \sqrt{\frac{mJ\omega}{\pi}}\cos\omega t\,, J = \frac{2\pi\alpha}{\omega}, \alpha = H$$

q Versus p for the Classical Linear Harmonic Oscillator