

Fermi-Thomas Equation without Exchange © Friday, February 28, 2025, by James Pate Williams, Jr., BA, BS, Master of Software Engineering, PhD Computer Science

Based on the original technical report by R. P. Feynman, N. Metropolis, and E. Teller:

osti.gov/servlets/purl/4417654/

The Fermi-Thomas second order ordinary non-linear differential equation for $T = 0$ is:

$$\Phi''(\chi) = \frac{\Phi^{3/2}(\chi)}{\sqrt{\chi}}$$

Following the technical report, we expand the potential energy around the origin as follows:

$$\Phi(\chi) = 1 + a_1\chi + \sum_{n=2}^N a_n \chi^{(n+1)/2}$$

$$\Phi'(\chi) = a_1 + \sum_{n=2}^N [(n+1)/2] a_n \chi^{(n+1)/2-1}$$

$$\Phi''(\chi) = \sum_{n=2}^N [(n+1)/2][(n+1)/2-1] a_n \chi^{(n+1)/2-2}$$

We use an elitist evolutionary hill climber with roulette-wheel selection to compute the finite series expansion coefficients. The population was 25 and 1,000,000 generations were utilized. A selected candidate solution for each generation was mutated by adding plus or minus 0.1 to the chosen coefficients. The pseudo-random generator (PRNG) chosen was the C++ Mersenne-Twister PRNG with a constant seed of 1 and range -15 to 15. We use $N = 20$ and 30. We found that the potential energy did not converge toward zero with decreasing independent variable, so to have more physical realistic results, we divided the potential by its derivative. The two tables are presented next.

N = 20

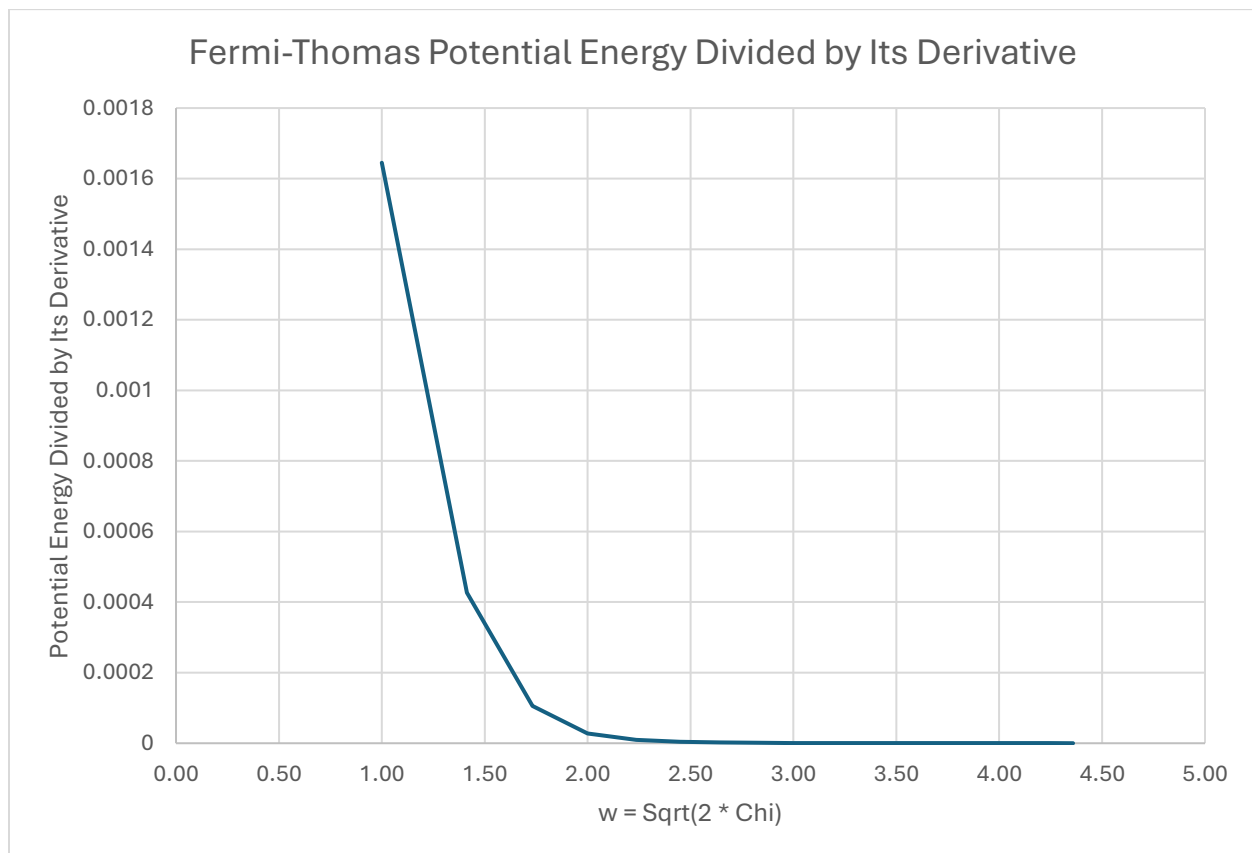
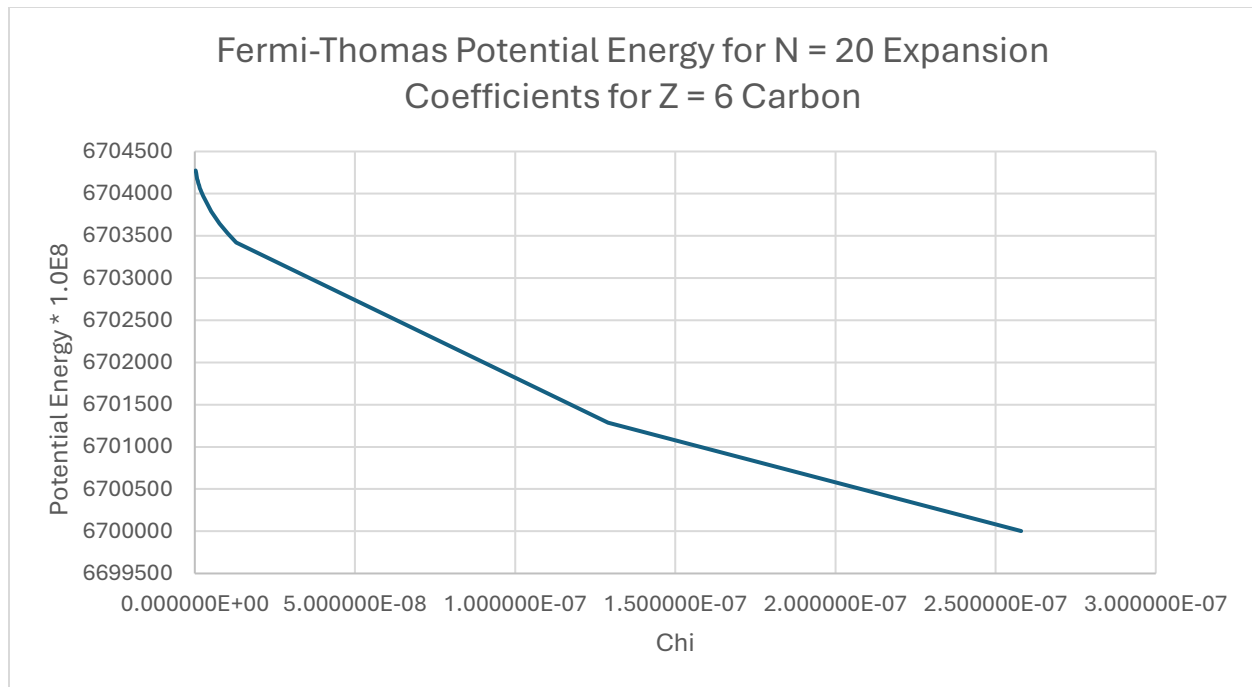
Chi	Phi/dPhi/dChi	dPhi/dChi	d2Phi/dChi2
1.289836e-10	0.059936998294895	1.501576701436910e+01	8.635380725656955e+05
2.579673e-10	0.067042745992143	1.491585687681953e+01	6.059376413419956e+05
3.869509e-10	0.067042430321170	1.491592713720455e+01	4.947419183531483e+05
5.159345e-10	0.067042164222815	1.491598636899739e+01	4.284560806170527e+05
6.449182e-10	0.067041929804096	1.491603855291207e+01	3.832204133414846e+05
7.739018e-10	0.067041717888400	1.491608573050110e+01	3.498288309959896e+05
9.028854e-10	0.067041523024400	1.491612911454030e+01	3.238769083155259e+05
1.031869e-09	0.067041341660254	1.491616949522021e+01	3.029576657569647e+05
1.160853e-09	0.067041171328844	1.491620742140906e+01	2.856299506296364e+05
1.289836e-09	0.067041010233877	1.491624329273448e+01	2.709712187813375e+05
1.418820e-09	0.067040857019296	1.491627741088301e+01	2.583598816950065e+05
1.547804e-09	0.067040710631581	1.491631001023512e+01	2.473598045665043e+05
1.676787e-09	0.067040570232921	1.491634127718129e+01	2.376547378170521e+05
1.805771e-09	0.067040435144023	1.491637136284618e+01	2.290090241427977e+05
1.934755e-09	0.067040304805053	1.491640039177820e+01	2.212429800617728e+05
2.579673e-09	0.067039710223969	1.491653283014034e+01	1.915990525426797e+05
5.159345e-09	0.067037872649822	1.491694228101236e+01	1.354744561483409e+05
7.739018e-09	0.067036463358411	1.491725645032900e+01	1.106103374681816e+05
1.031869e-08	0.067035275762436	1.491752129828407e+01	9.578837428156706e+04
1.289836e-08	0.067034229842050	1.491775462717157e+01	8.567337221735896e+04
1.289836e-07	0.067012884719527	1.492253211673268e+01	2.707706055358327e+04
2.579673e-07	0.067000025518133	1.492542490377209e+01	1.913985219187345e+04

Runtime (s) = 24.098

N = 30

Chi	Phi/dPhi/dChi	dPhi/dChi	d2Phi/dChi2
1.289836e-10	0.060746109736198	1.481576360724024e+01	8.503304711346861e+05
2.579673e-10	0.067042745992143	1.491585687681953e+01	6.059376413419956e+05
3.869509e-10	0.067042430321170	1.491592713720455e+01	4.947419183531483e+05
5.159345e-10	0.067042164222815	1.491598636899739e+01	4.284560806170527e+05
6.449182e-10	0.067041929804096	1.491603855291207e+01	3.832204133414846e+05
7.739018e-10	0.067041717888400	1.491608573050110e+01	3.498288309959896e+05
9.028854e-10	0.067041523024400	1.491612911454030e+01	3.238769083155259e+05
1.031869e-09	0.067041341660254	1.491616949522021e+01	3.029576657569647e+05
1.160853e-09	0.067041171328844	1.491620742140906e+01	2.856299506296364e+05
1.289836e-09	0.067041010233877	1.491624329273448e+01	2.709712187813375e+05
1.418820e-09	0.067040857019296	1.491627741088301e+01	2.583598816950065e+05
1.547804e-09	0.067040710631581	1.491631001023512e+01	2.473598045665043e+05
1.676787e-09	0.067040570232921	1.491634127718129e+01	2.376547378170521e+05
1.805771e-09	0.067040435144023	1.491637136284618e+01	2.290090241427977e+05
1.934755e-09	0.067040304805053	1.491640039177820e+01	2.212429800617728e+05
2.579673e-09	0.067039710223969	1.491653283014034e+01	1.915990525426797e+05
5.159345e-09	0.067037872649822	1.491694228101236e+01	1.354744561483409e+05
7.739018e-09	0.067036463358411	1.491725645032900e+01	1.106103374681816e+05
1.031869e-08	0.067035275762436	1.491752129828407e+01	9.578837428156706e+04
1.289836e-08	0.067034229842050	1.491775462717157e+01	8.567337221735896e+04
1.289836e-07	0.067012884719527	1.492253211673268e+01	2.707706055358327e+04
2.579673e-07	0.067000025518133	1.492542490377209e+01	1.913985219187345e+04

Runtime (s) = 25.863



For the previous chart a range of -10 to 10 was used for the PRNG.