Problems from the Textbook: ***Mathematical Methods in the Physical Sciences Second Edition*** , Solutions Provided by James Pate Williams, Jr.

PROBLEMS, SECTION 2

$$\sum_{n=1}^{\infty}\frac{n}{2^n}=\frac{1}{2}+\frac{2}{4}+\frac{3}{8}+\frac{4}{16}+\cdots$$

$$\sum_{n=1}^{\infty}\frac{(-1)^n}{n}=-1+\frac{1}{2}-\frac{1}{3}+\frac{1}{4}-\cdots$$

$$\sum_{n=1}^{\infty}\frac{n}{n+5}=\frac{1}{6}+\frac{2}{7}+\frac{3}{8}+\frac{4}{9}+\cdots$$

$$\sum_{n=1}^{\infty}\frac{\sqrt{n}}{n+1}=\frac{1}{2}+\frac{\sqrt{2}}{3}+\frac{\sqrt{3}}{4}+\frac{\sqrt{4}}{5}+\cdots$$

$$\sum_{n=1}^{\infty}\frac{2n(2n+1)}{3n+5}=\frac{2\cdot 3}{8}+\frac{4\cdot 5}{11}+\frac{6\cdot 7}{14}+\frac{8\cdot 9}{17}+\cdots=\frac{6}{8}+\frac{20}{11}+\frac{42}{14}+\frac{72}{17}+\cdots$$

$$\sum_{n=1}^{\infty}\frac{(n!)^2}{(2n)!}=\frac{1}{2!}+\frac{(2)^2}{4!}+\frac{(6)^2}{6!}+\frac{(24)^2}{8!}+\cdots=\frac{1}{2}+\frac{4}{24}+\frac{36}{720}+\frac{576}{40{,}320}+\cdots$$

$$\frac{1}{3}+\frac{2}{5}+\frac{4}{7}+\frac{8}{9}+\frac{16}{11}+\cdots=\sum_{n=0}^{\infty}\frac{2^n}{2n+3}$$

$$\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\frac{1}{4\cdot 5}+\frac{1}{5\cdot 6}+\cdots=\sum_{n=1}^{\infty}\frac{1}{(n+1)(n+2)}$$

$$\frac{1}{4}-\frac{1}{9}+\frac{1}{16}-\frac{1}{25}+\cdots=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{(n+1)^2}$$

$$\frac{1}{7}+\frac{2}{9}+\frac{3}{11}+\frac{4}{13}+\cdots=\sum_{n=1}^{\infty}\frac{n}{2n+5}$$

$$\frac{1}{4}-\frac{1}{8}+\frac{1}{16}-\frac{1}{32}+\cdots=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{2^{n+1}}$$

$$\frac{\ln 2}{2}-\frac{\ln 3}{3}+\frac{\ln 4}{4}-\frac{\ln 5}{5}+\cdots=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}\ln(n+1)}{n+1}$$

PROBLEMS, SECTION 4

$$S=\sum_{n=1}^{\infty}\frac{1}{2^n}$$

$$|S-S_4|=\left|S-\sum_{n=1}^{4}\frac{1}{2^n}\right|=\left|1-\frac{1}{2}-\frac{1}{4}-\frac{1}{8}-\frac{1}{16}\right|=|1-0.9375|=0.0625$$

$$|S-S_5|=\left|1-0.9375-\frac{1}{32}\right|=|0.0625-0.03125|=0.03125$$

$$|S-S_6|=\left|0.03125-\frac{1}{64}\right|=0.016525$$

$$|S-S_7|=\left|0.01625-\frac{1}{128}\right|=0.0084375$$

$$S=\sum_{n=1}^{\infty}\frac{1}{5^n}<\sum_{n=1}^{\infty}\frac{1}{2^n}$$

$$|S-S_4|=\left|S-\sum_{n=1}^{4}\frac{1}{5^n}\right|=\left|0.25-\frac{1}{5}-\frac{1}{25}-\frac{1}{125}-\frac{1}{625}\right|=|0.25-0.2496|=0.0004$$

$$|S-S_5|=\left|0.0004-\frac{1}{3{,}125}\right|=0.00008$$

$$S=\sum_{n=1}^{\infty}\frac{1}{n(n+1)}=\frac{1}{2}+\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\cdots$$

$$\frac{1}{n(n+1)}=\frac{1}{n}-\frac{1}{n+1}$$

$$\sum_{n=1}^{\infty}\frac{1}{n(n+1)}<\sum_{n=1}^{\infty}\frac{1}{n^2}$$

$$|S-S_4|=|1-0.883333\ldots|=0.1666\ldots$$

$$S=\sum_{n=1}^{\infty}\frac{1}{n!}=1+\frac{1}{2}+\frac{1}{6}+\frac{1}{24}+\cdots$$

$$\frac{1}{n!} < \frac{1}{2^n} \forall n \geq 3$$

$$S = \sum_{n=1}^{\infty} \frac{1}{n2^n} = \frac{1}{2} + \frac{1}{8} + \frac{1}{24} + \frac{1}{64} + \cdots$$

$$\frac{1}{n2^n} < \frac{1}{2^n} \forall n \geq 2$$

$$S = \sum_{n=1}^{\infty} \frac{1}{2^n + 3^n} = \frac{1}{2+3} + \frac{1}{4+9} + \frac{1}{8+27} + \frac{1}{16+81} + \cdots = \frac{1}{5} + \frac{1}{13} + \frac{1}{35} + \frac{1}{97} + \cdots$$

$$2^n + 3^n > 2^n \forall n \geq 1$$

$$\therefore \frac{1}{2^n + 3^n} < \frac{1}{2^n} \forall n \geq 1$$

$$S = \sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \frac{1}{36} + \cdots$$

$$T = \sum_{n=1}^{\infty} \frac{1}{2^n} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} + \cdots$$

$$\therefore \frac{1}{n^2} \geq \frac{1}{2^n}$$

$$S = \sum_{n=1}^{\infty} \frac{1}{n^2} = \sum_{n=0}^{\infty} \frac{1}{(n+1)^2}$$

$$\frac{1}{(n+1)^2} < \frac{1}{n(n+1)}$$

$$S = \sum_{n=2}^{\infty} \frac{1}{n^2 - 1} = \frac{1}{3} + \frac{1}{8} + \frac{1}{15} + \frac{1}{24} + \cdots$$

$$\frac{1}{n^2 - 1} = \frac{1}{(n+1)(n-1)} \forall n \geq 2$$

PROBLEMS, SECTION 5

$$\frac{1}{2} - \frac{4}{5} + \frac{9}{10} - \frac{16}{17} + \frac{25}{26} - \frac{36}{37} + \cdots = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} n^2}{n^2 + 1}$$

$$\lim_{n\to\infty}\left|\frac{(-1)^{n+1}n^2}{n^2+1}\right| = \lim_{n\to\infty}\left(\frac{2n}{2n}\right) = 1 \therefore \textit{the series diverges}$$

$$\sqrt{2}+\frac{\sqrt{3}}{2}+\frac{\sqrt{4}}{3}+\frac{\sqrt{5}}{4}+\frac{\sqrt{6}}{5}+\cdots = \sum_{n=1}^{\infty}\frac{\sqrt{n+1}}{n}$$

$$\lim_{n\to\infty}\frac{\sqrt{n+1}}{n} = \lim_{n\to\infty}\frac{\frac{1}{2\sqrt{n+1}}}{1} = 0$$

$$\sum_{n=1}^{\infty}\frac{n+3}{n^2+10n}$$

$$\lim_{n\to\infty}\frac{n+3}{n^2+10n} = \lim_{n\to\infty}\frac{1}{2n+10} = 0$$

$$\sum_{n=1}^{\infty}\frac{(-1)^n n^2}{(n+1)^2}$$

$$\lim_{n\to\infty}\left|\frac{(-1)^n n^2}{(n+1)^2}\right| = \frac{2n}{2n+2} = \frac{2}{2} = 1 \therefore \textit{the series diverges}$$

$$\sum_{n=1}^{\infty}\frac{n!}{n!+1}$$

$$\lim_{n\to\infty}\frac{n!}{n!+1} \approx \lim_{n\to\infty}\frac{n\ln n - n}{n\ln n - n + 1} = \frac{\ln n + 1 - 1}{\ln n + 1 - 1} = 1 \therefore \textit{the series diverges}$$

$$\sum_{n=1}^{\infty}\frac{n!}{(n+1)!}$$

$$\lim_{n\to\infty}\frac{n!}{(n+1)!} \approx \lim_{n\to\infty}\frac{n\ln n - n}{(n+1)\ln(n+1)-(n+1)} = \lim_{n\to\infty}\frac{\ln n + 1 - 1}{\ln(n+1)+1-1} = \lim_{n\to\infty}\frac{\ln n}{\ln(n+1)}$$

$$= \lim_{n\to\infty}\frac{\frac{1}{n}}{\frac{1}{n+1}} = \lim_{n\to\infty}\frac{n+1}{n} = 1 \therefore \textit{the series diverges}$$

$$\sum_{n=1}^{\infty}\frac{(-1)^n n}{\sqrt{n^3+1}}$$

$$\lim_{n\to\infty}\left|\frac{(-1)^n n}{\sqrt{n^3+1}}\right| = \lim_{n\to\infty} n(n^3+1)^{-1/2} = \lim_{n\to\infty}\left|(n^3+1)^{-1/2} - \frac{1}{2}\cdot 3n^3(n^3+1)^{-3/2}\right| > 0$$

$\therefore$ *the series diverges*

$$\sum_{n=1}^{\infty}\frac{\ln n}{n}$$

$$\lim_{n\to\infty}\frac{\ln n}{n} = \lim_{n\to\infty}\frac{\frac{1}{n}}{1} = \lim_{n\to\infty}\frac{1}{n} = 0$$

$$\sum_{n=1}^{\infty}\frac{3^n}{2^n+3^n}$$

$$\lim_{n\to\infty}\frac{3^n}{2^n+3^n} = \lim_{n\to\infty}\frac{3^n\ln 3}{2^n\ln 2 + 3^n\ln 3} = 0$$

$$\sum_{n=2}^{\infty}\left(1-\frac{1}{n^2}\right) = \sum_{n=2}^{\infty}\frac{n^2-1}{n^2}$$

$$\lim_{n\to\infty}\frac{n^2-1}{n^2} = \lim_{n\to\infty}\frac{2n}{2n} = 1 \therefore \textit{the series diverges}$$