Two Examples of First Order Systems of Ordinary Differential Equations © Tuesday, April 22, 2025, by James Pate Williams, Jr.

The first system is from [1]. A second order system of two ordinary differential equations is converted to four first order ordinary differential equations:

$$\frac{d^2z}{dx^2} = z^2 - y + e^x$$

$$\frac{d^2y}{dx^2} = z - y^2 - e^x$$

$$z(0) = z'(0) = 0, y(0) = 1, y'(0) = -2$$

The first order system is [2]:

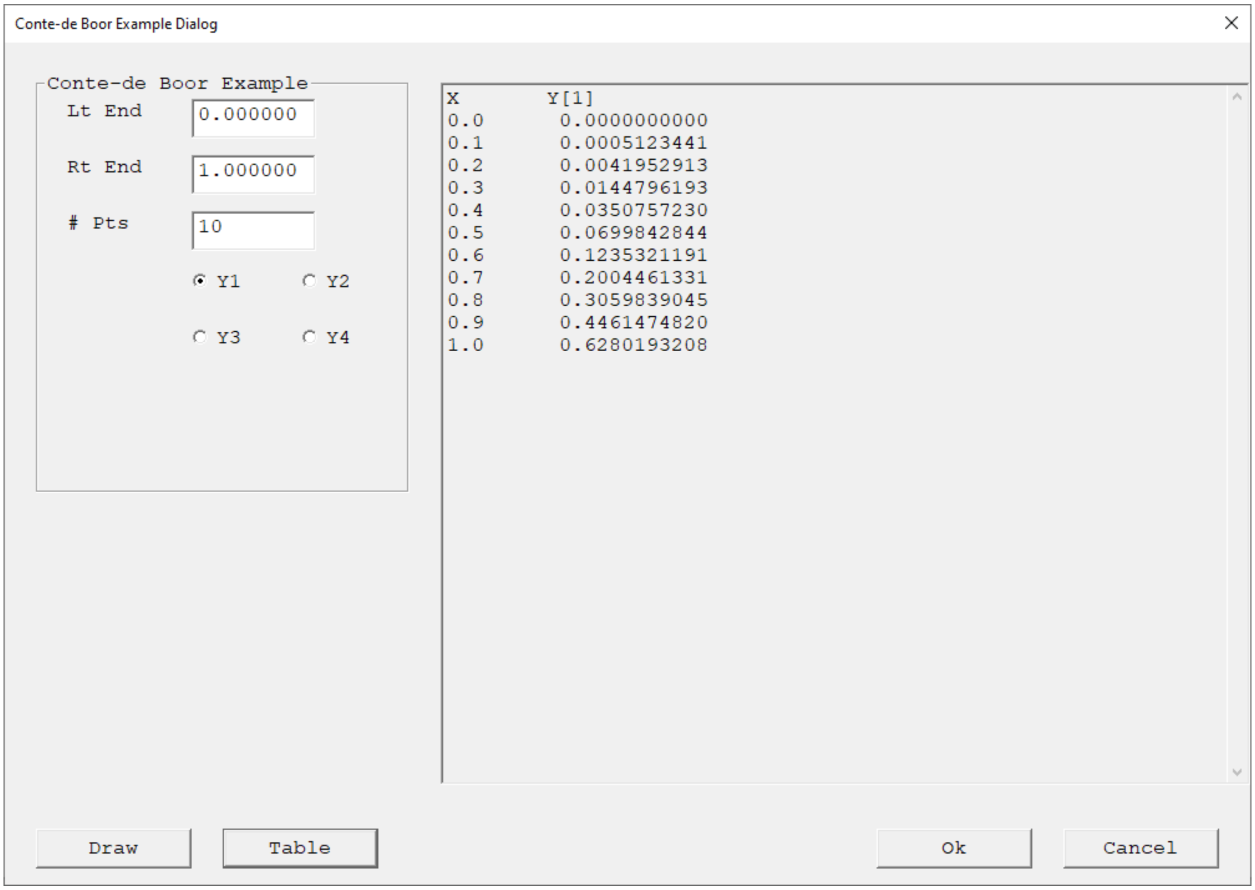
$$y_1'(x) = y_3(x), y_1(0) = 0$$

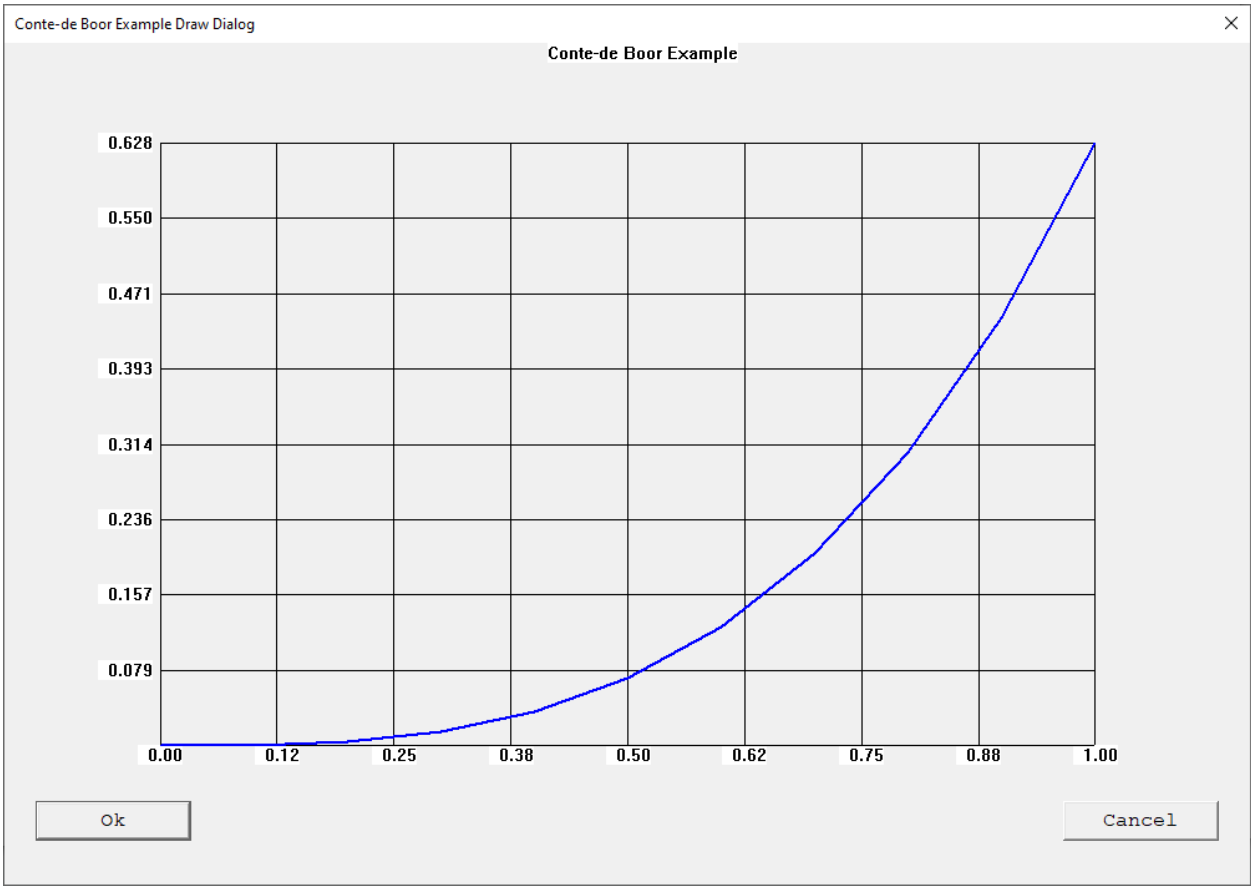
$$y_2'(x) = y_4(x), y_2(0) = 1$$

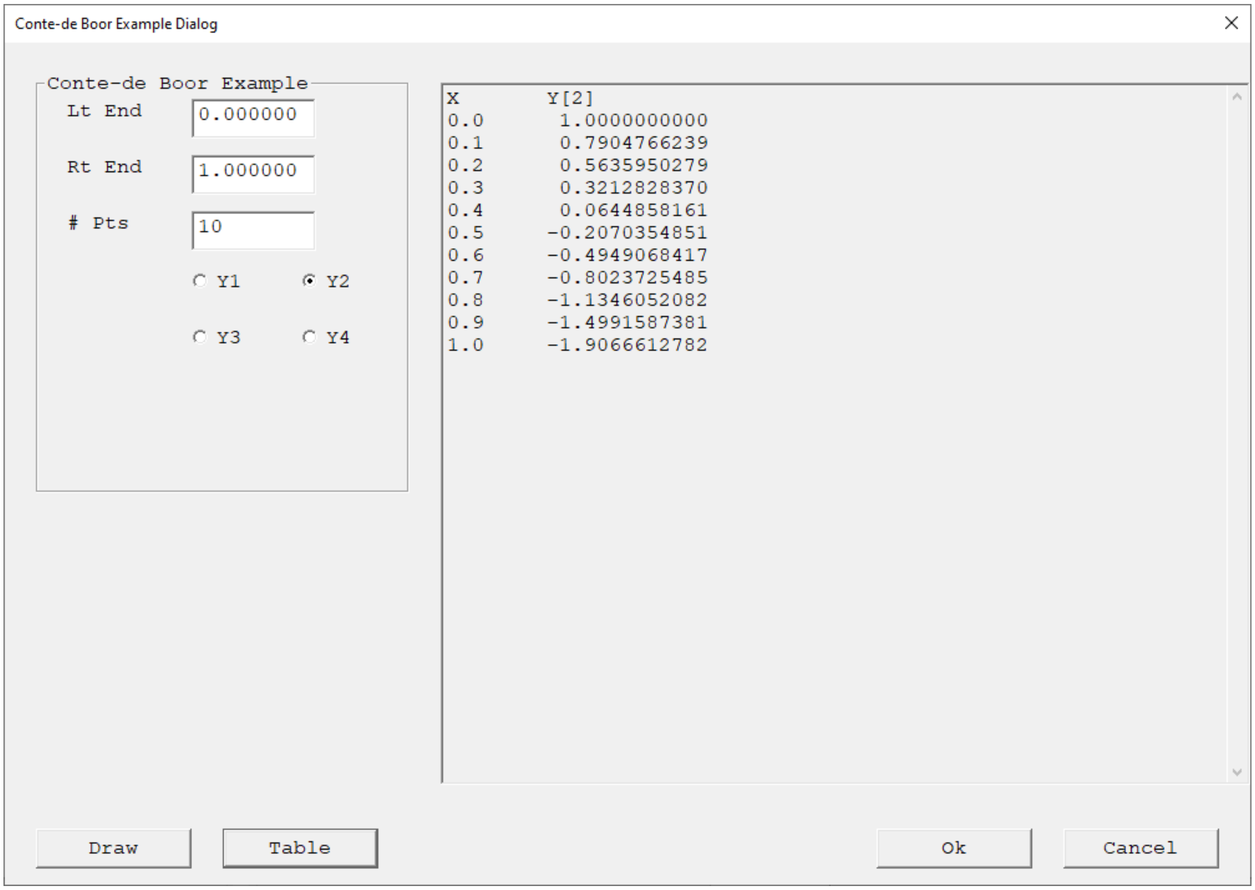
$$y_3'(x) = y_1^2(x) - y_2(x) + e^x, y_3(0) = 0$$

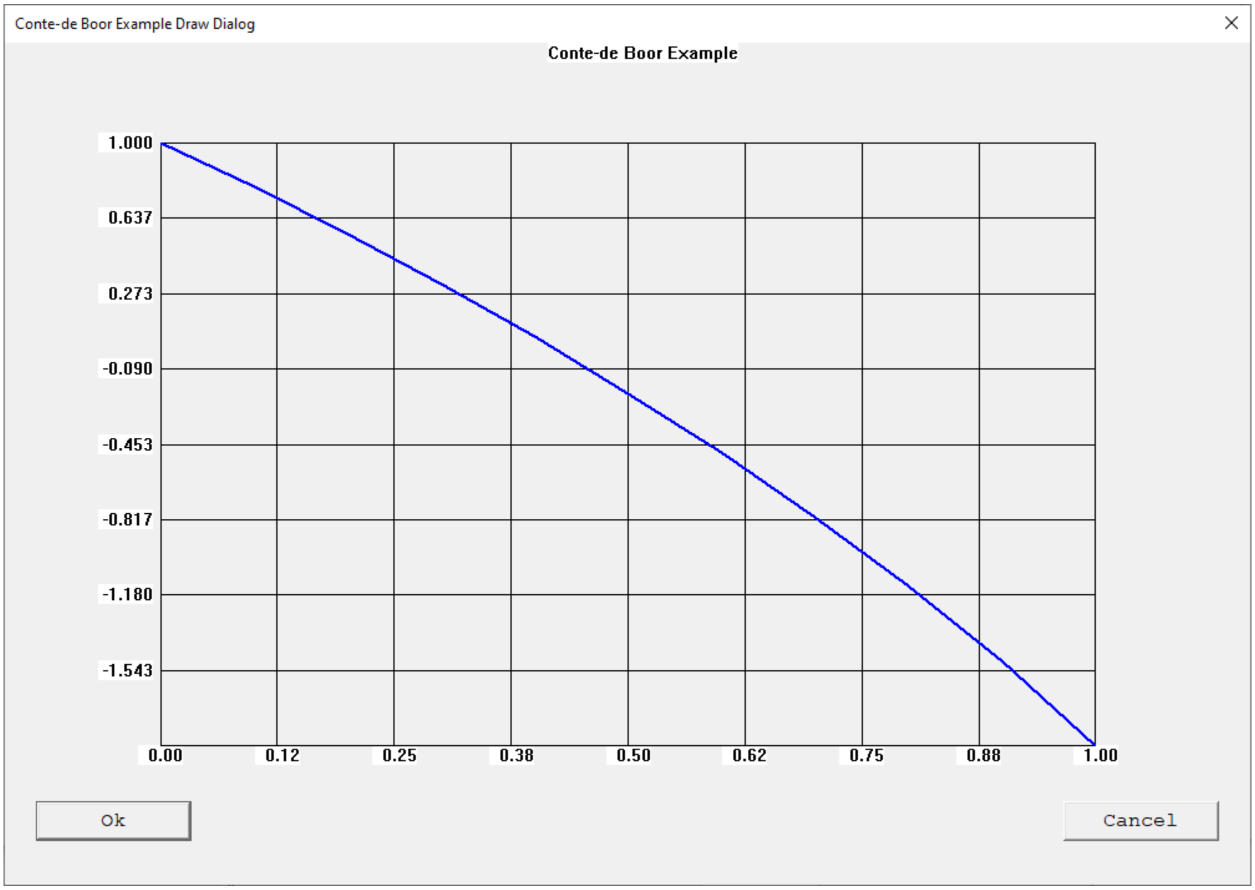
$$y_4'(x) = y_1(x) - y_2^2(x) - e^x, y_4(0) = -2$$

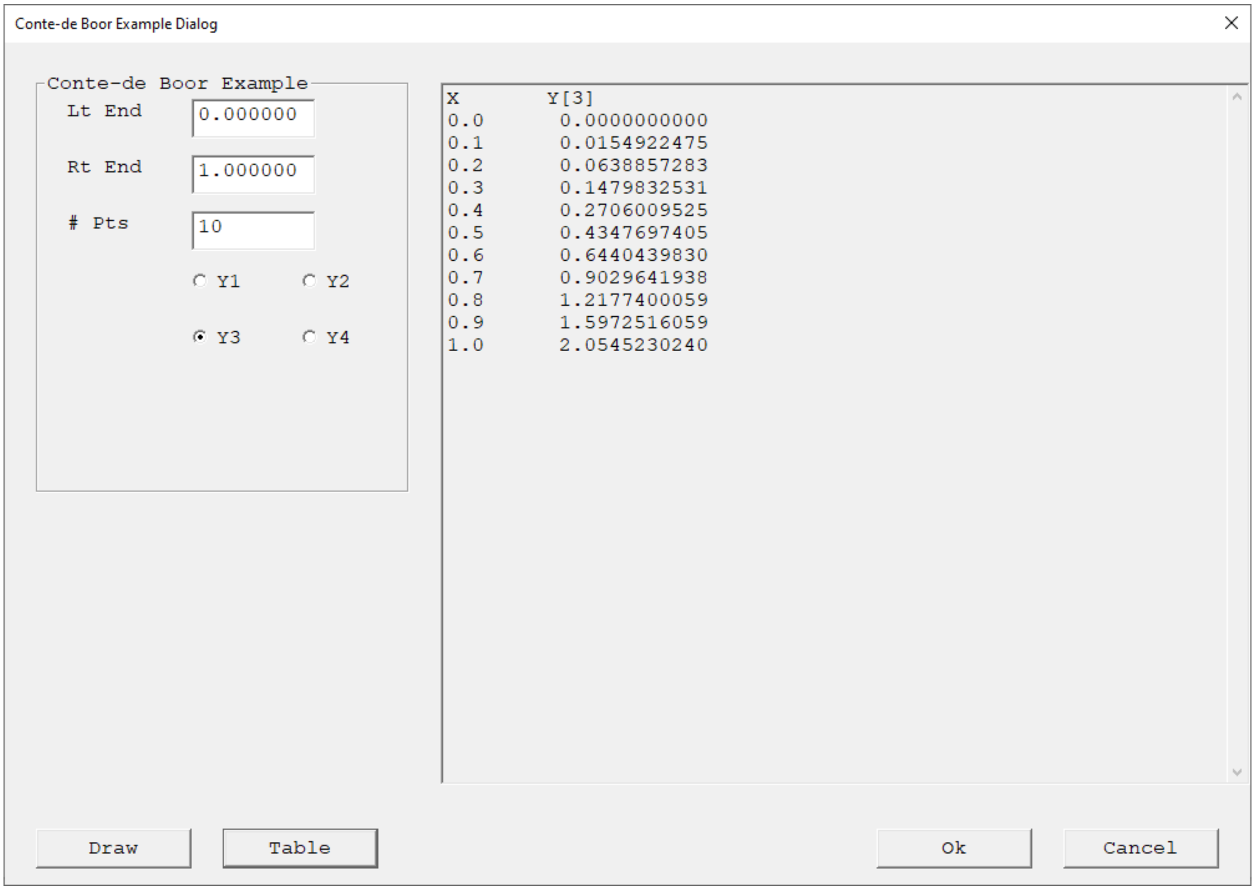
Conte and de Boor use the FORTRAN subroutine DVERK from the IMSL library to solve the first order system of four equations. We use the C function DIFFSYS from [3].

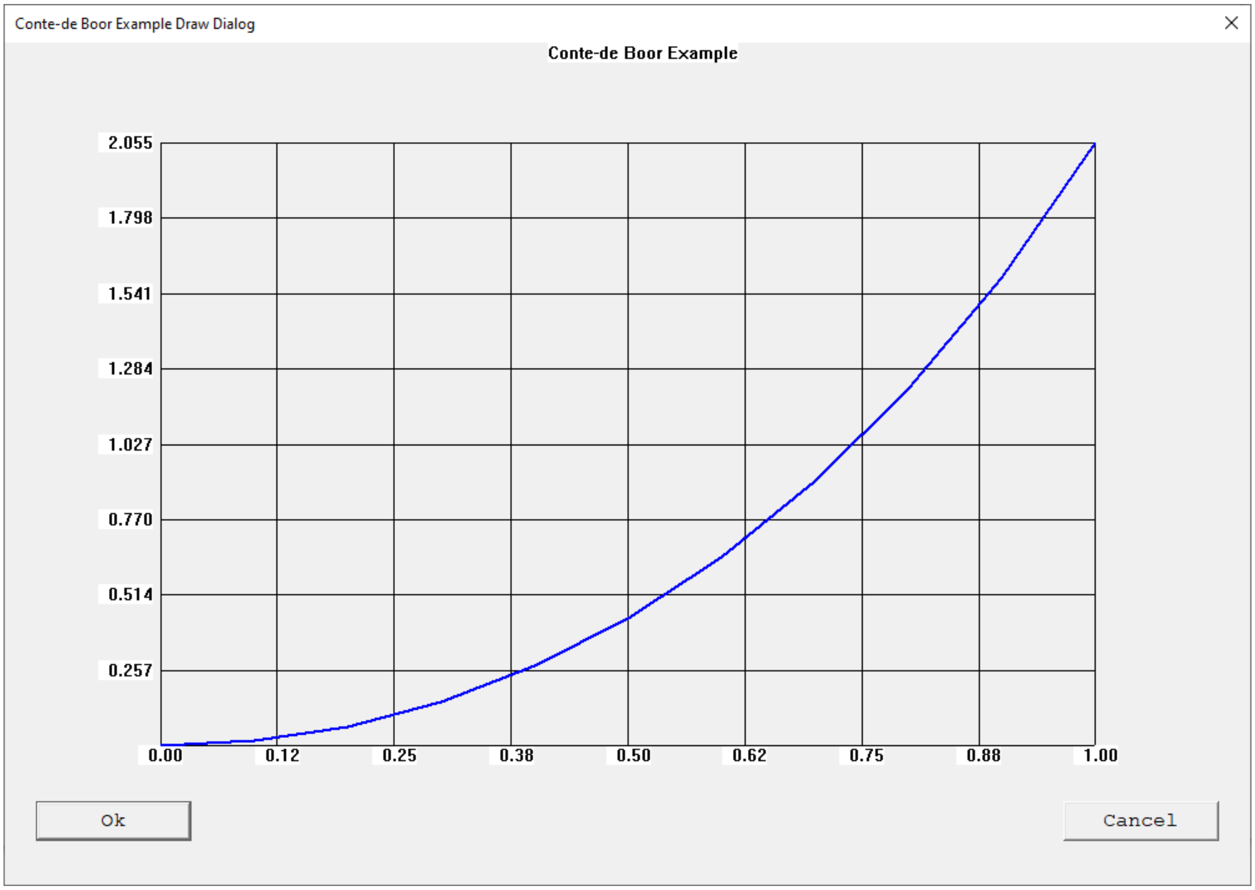

Conte-de Boor Example Dialog
Conte-de Boor Example
Lt End
0.000000
Rt End
1.000000
Pts
10
Y1
Y2
Y3
Y4
X Y[1]
0.0 0.0000000000
0.1 0.0005123441
0.2 0.0041952913
0.3 0.0144796193
0.4 0.0350757230
0.5 0.0699842844
0.6 0.1235321191
0.7 0.2004461331
0.8 0.3059839045
0.9 0.4461474820
1.0 0.6280193208
Draw
Table
Ok
Cancel

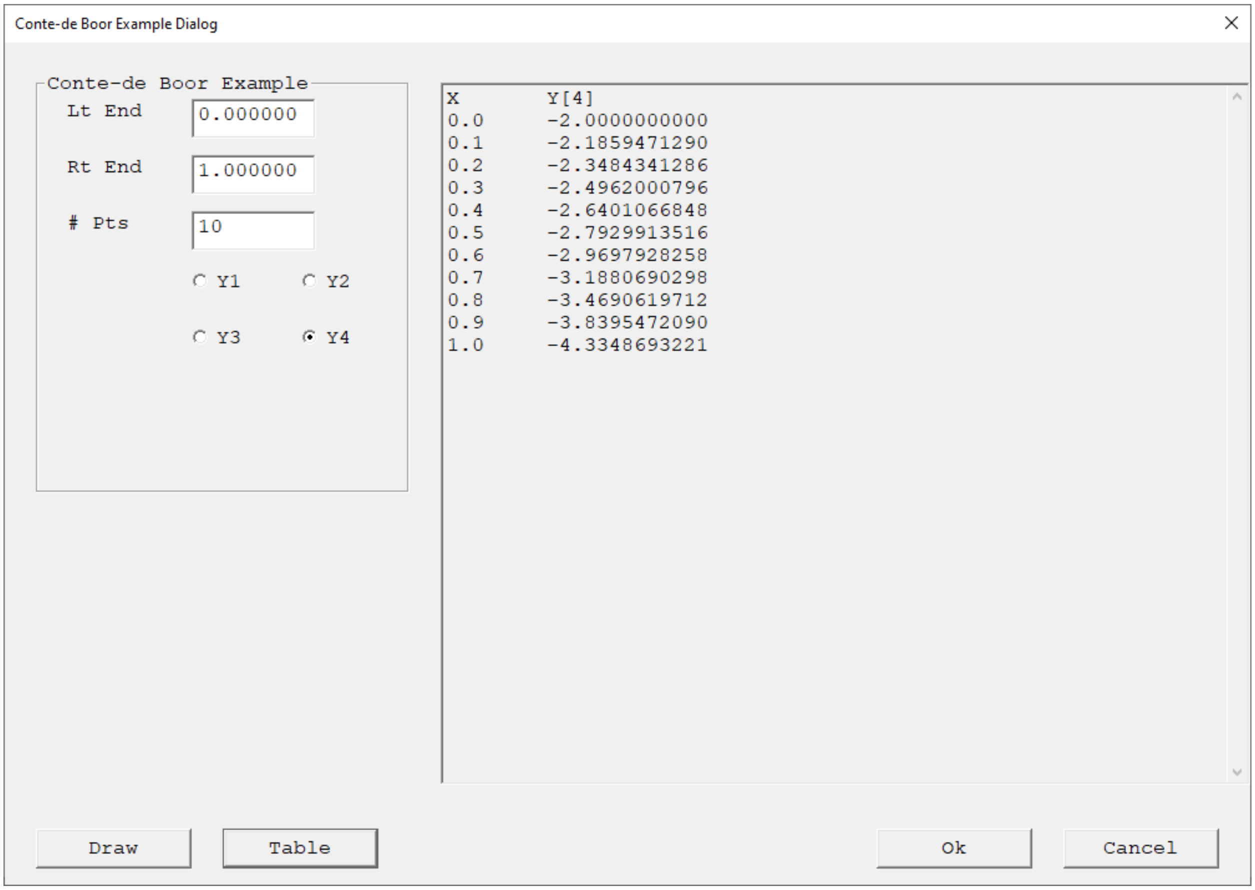

Conte-de Boor Example Draw Dialog
Conte-de Boor Example
0.628
0.550
0.471
0.393
0.314
0.236
0.157
0.079
0.00
0.12
0.25
0.38
0.50
0.62
0.75
0.88
1.00
Ok
Cancel

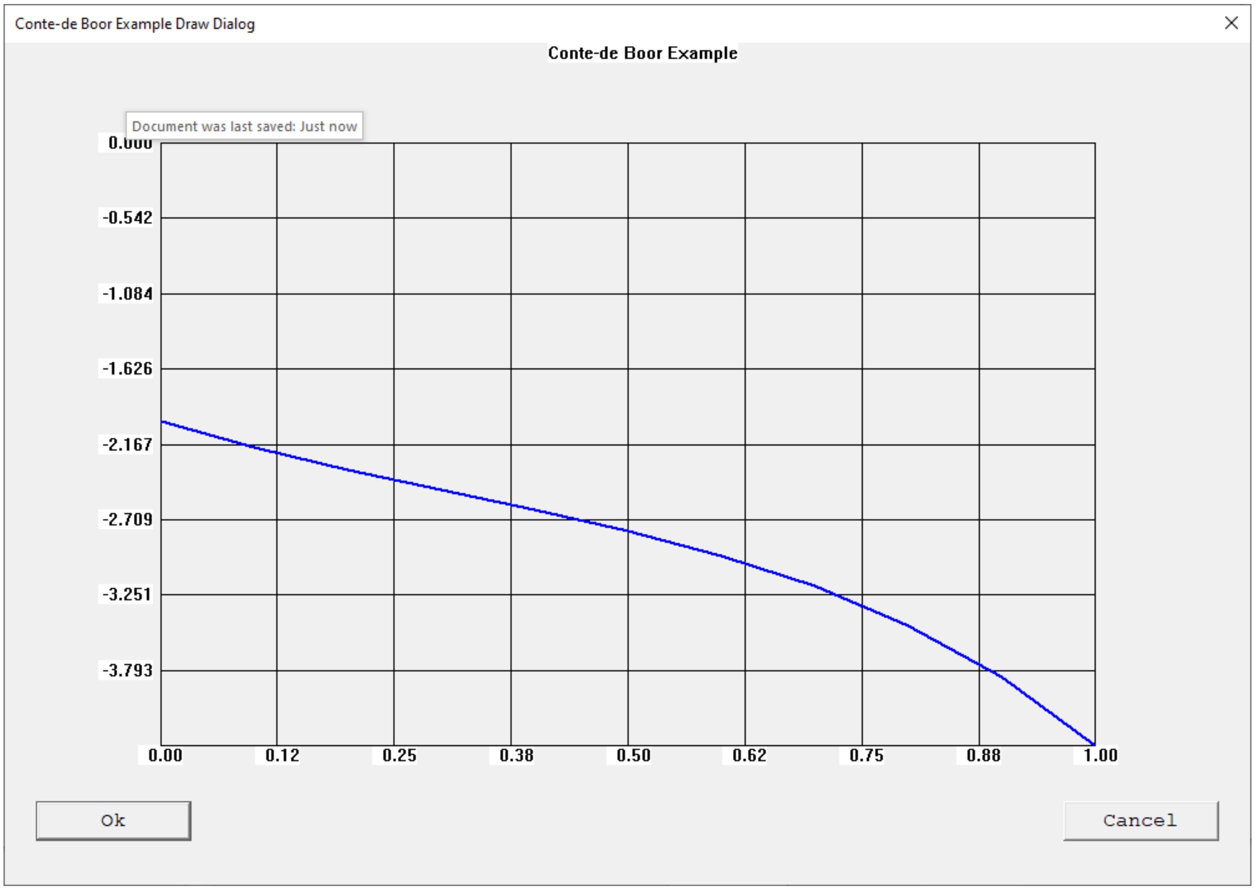

Conte-de Boor Example Dialog
Conte-de Boor Example
Lt End
0.000000
Rt End
1.000000
Pts
10
Y1
Y2
Y3
Y4
X Y[2]
0.0 1.0000000000
0.1 0.7904766239
0.2 0.5635950279
0.3 0.3212828370
0.4 0.0644858161
0.5 -0.2070354851
0.6 -0.4949068417
0.7 -0.8023725485
0.8 -1.1346052082
0.9 -1.4991587381
1.0 -1.9066612782
Draw
Table
Ok
Cancel


Conte-de Boor Example Draw Dialog
Conte-de Boor Example
1.000
0.637
0.273
-0.090
-0.453
-0.817
-1.180
-1.543
0.00
0.12
0.25
0.38
0.50
0.62
0.75
0.88
1.00
Ok
Cancel


Conte-de Boor Example Dialog
Conte-de Boor Example
Lt End
0.000000
Rt End
1.000000
Pts
10
Y1
Y2
Y3
Y4
X Y[3]
0.0 0.0000000000
0.1 0.0154922475
0.2 0.0638857283
0.3 0.1479832531
0.4 0.2706009525
0.5 0.4347697405
0.6 0.6440439830
0.7 0.9029641938
0.8 1.2177400059
0.9 1.5972516059
1.0 2.0545230240
Draw
Table
Ok
Cancel


Conte-de Boor Example Draw Dialog
Conte-de Boor Example
2.055
1.798
1.541
1.284
1.027
0.770
0.514
0.257
0.00
0.12
0.25
0.38
0.50
0.62
0.75
0.88
1.00
Ok
Cancel


Conte-de Boor Example Dialog
Conte-de Boor Example
Lt End 0.000000
Rt End 1.000000
Pts 10
Y1
Y2
Y3
Y4
X Y[4]
0.0 -2.0000000000
0.1 -2.1859471290
0.2 -2.3484341286
0.3 -2.4962000796
0.4 -2.6401066848
0.5 -2.7929913516
0.6 -2.9697928258
0.7 -3.1880690298
0.8 -3.4690619712
0.9 -3.8395472090
1.0 -4.3348693221
Draw
Table
Ok
Cancel



The second example is a predator-prey model where two predators, X and Y, are preying on one prey, Z [4]:

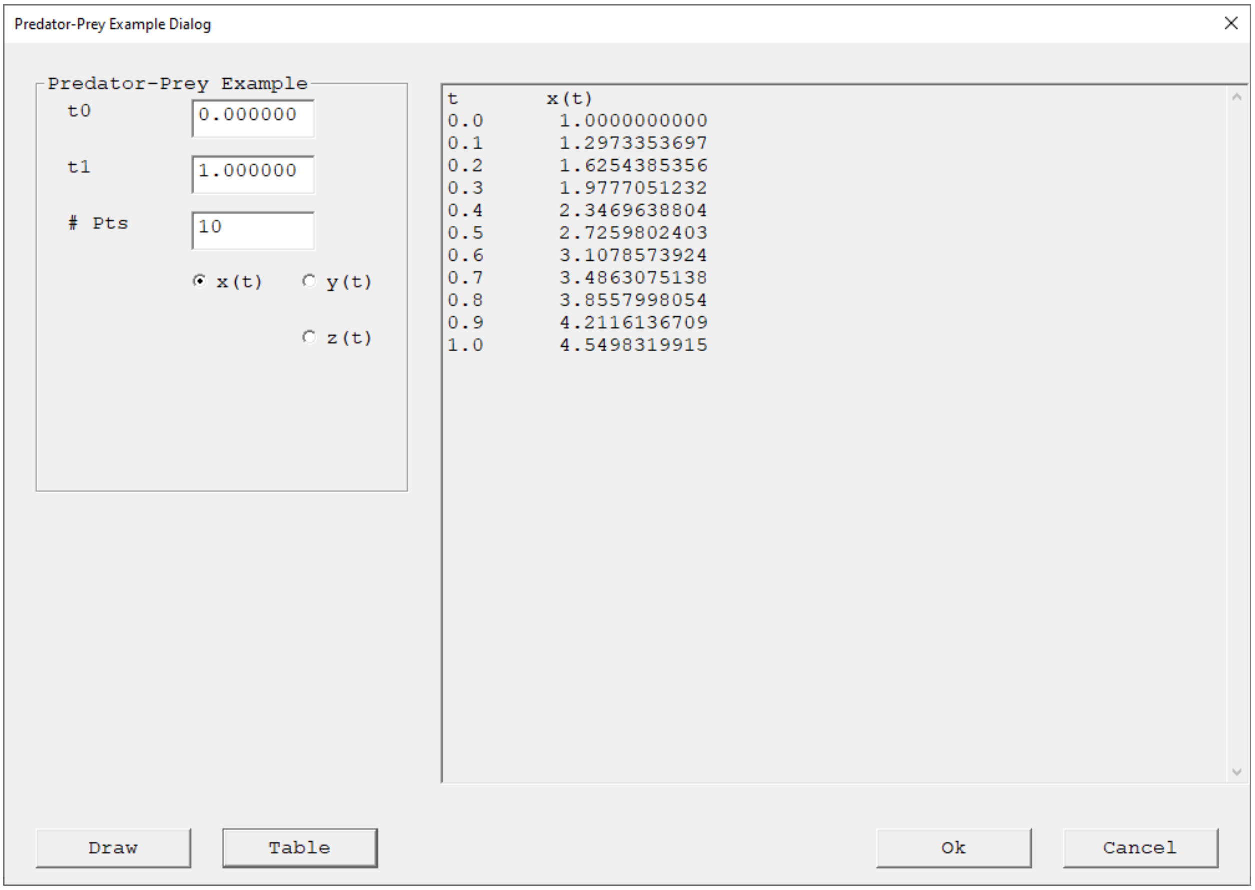
$$x'(t) = -0.2x(t) + 0.5z(t)x(t)$$

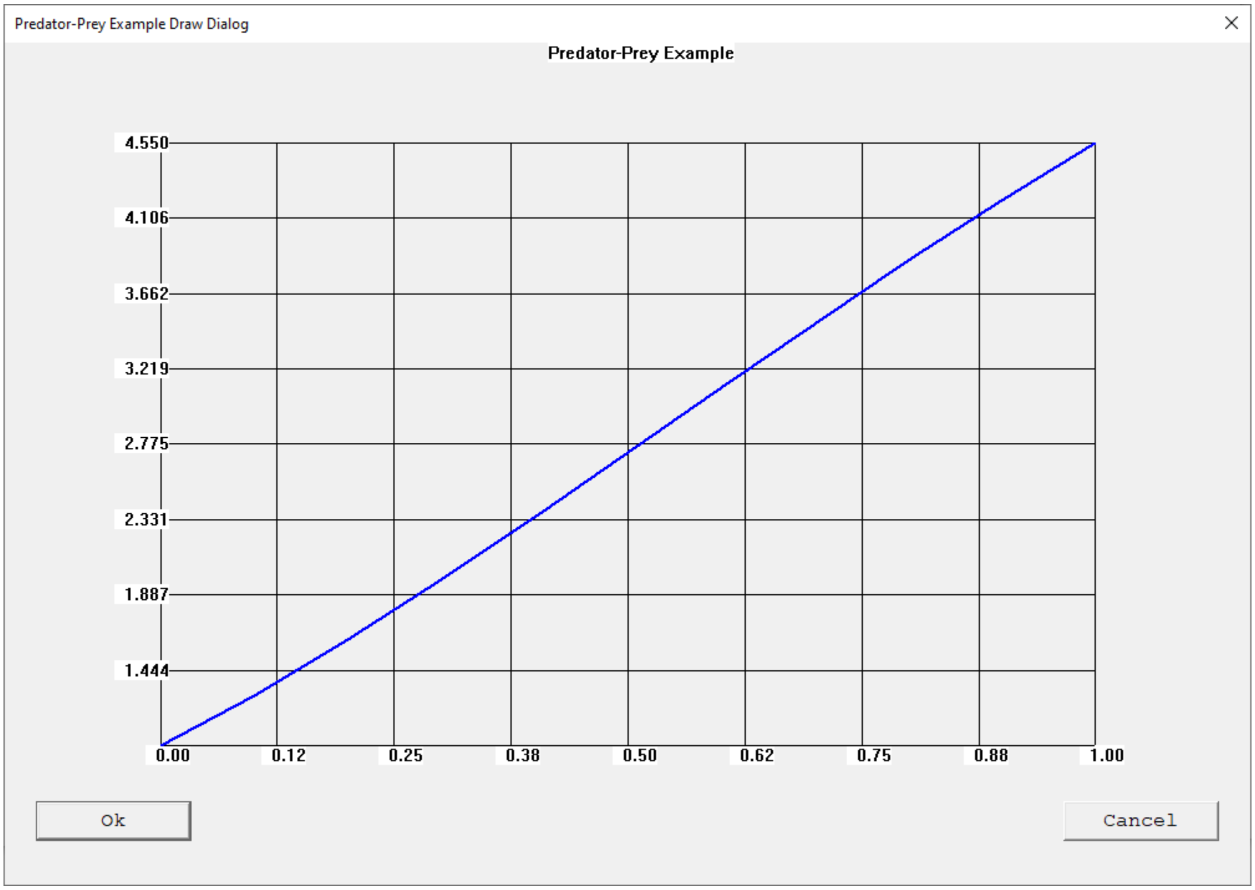
$$y'(t) = -0.1x(t) - 0.5x(t)y(t) + 0.4y(t)z(t)$$

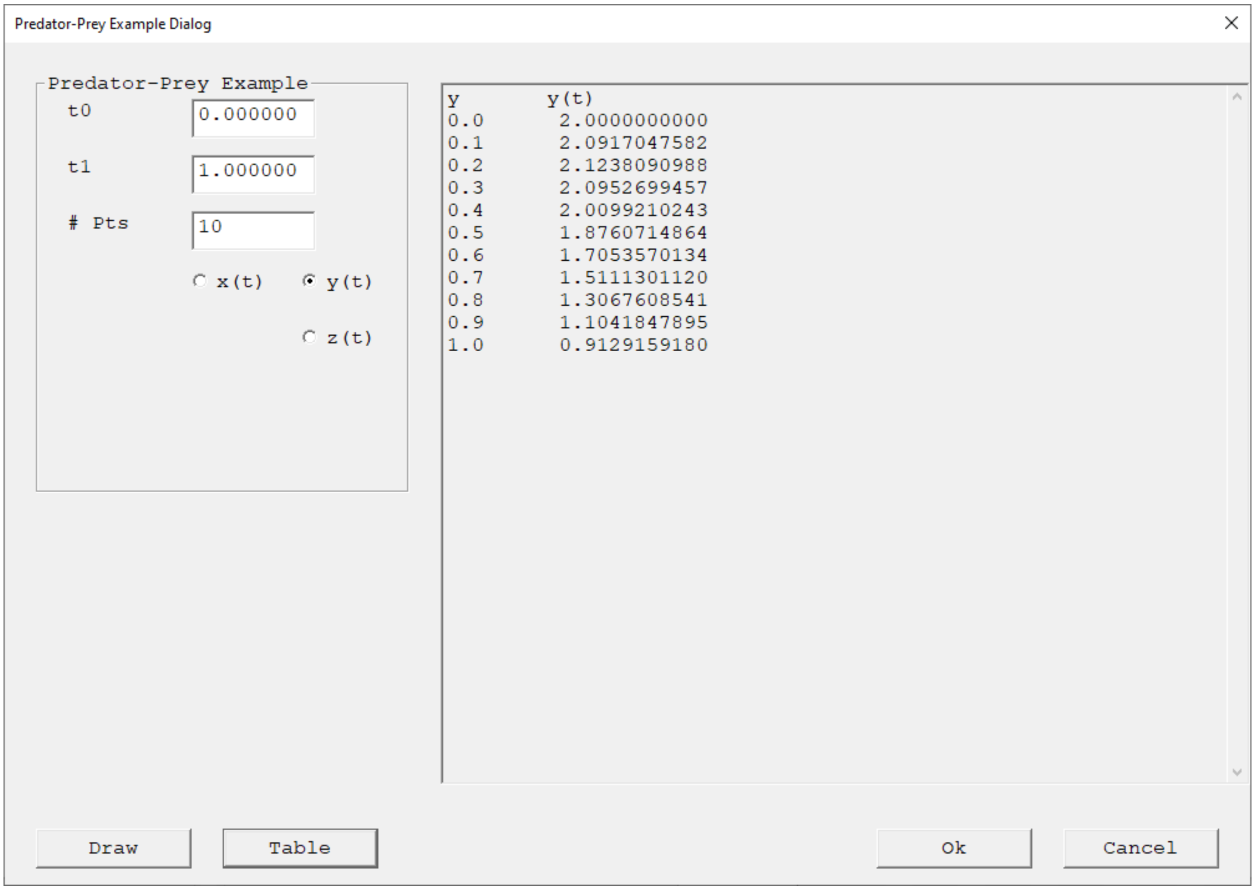
$$z'(t) = 0.4z(t)[1 - z(t)] - 0.2z(t)[x(t) + y(t)]$$

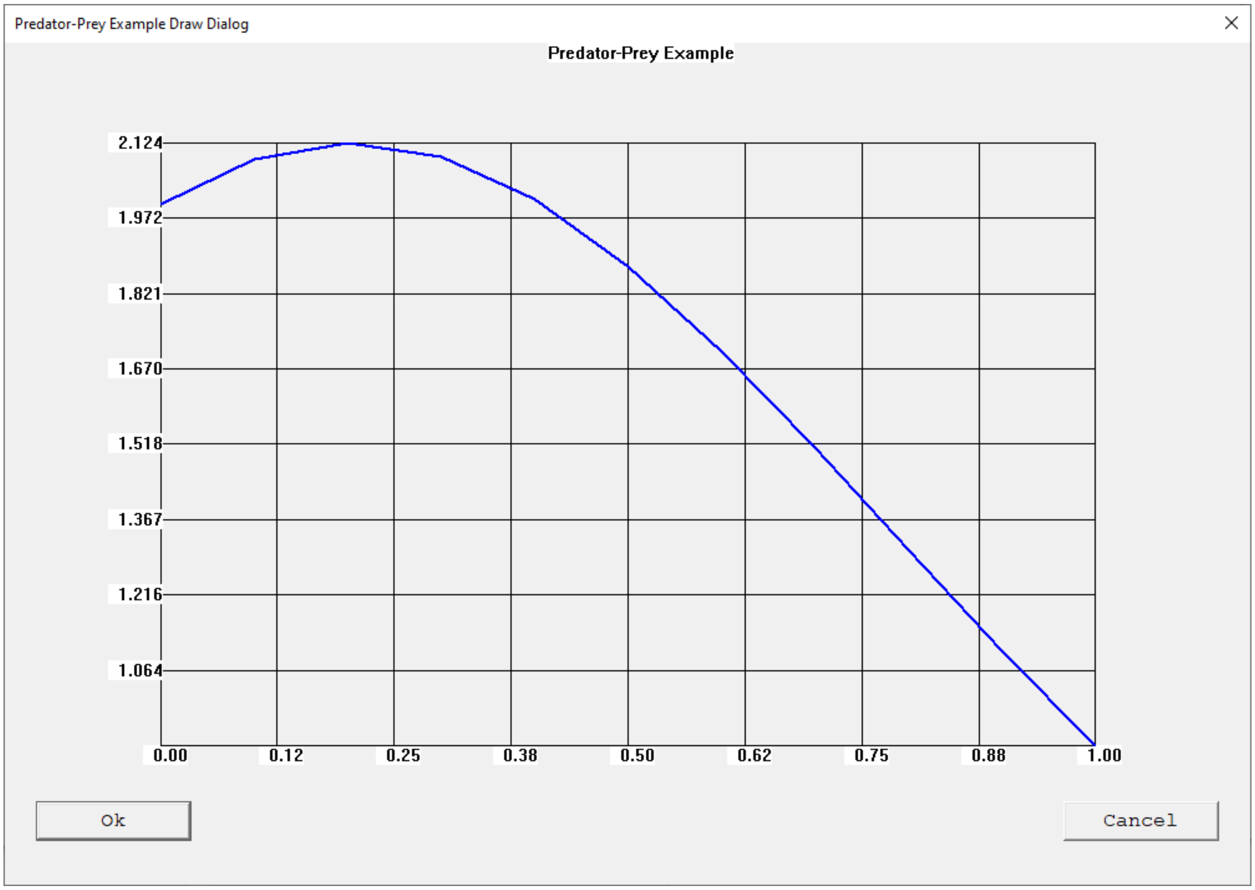
I added the initial conditions:

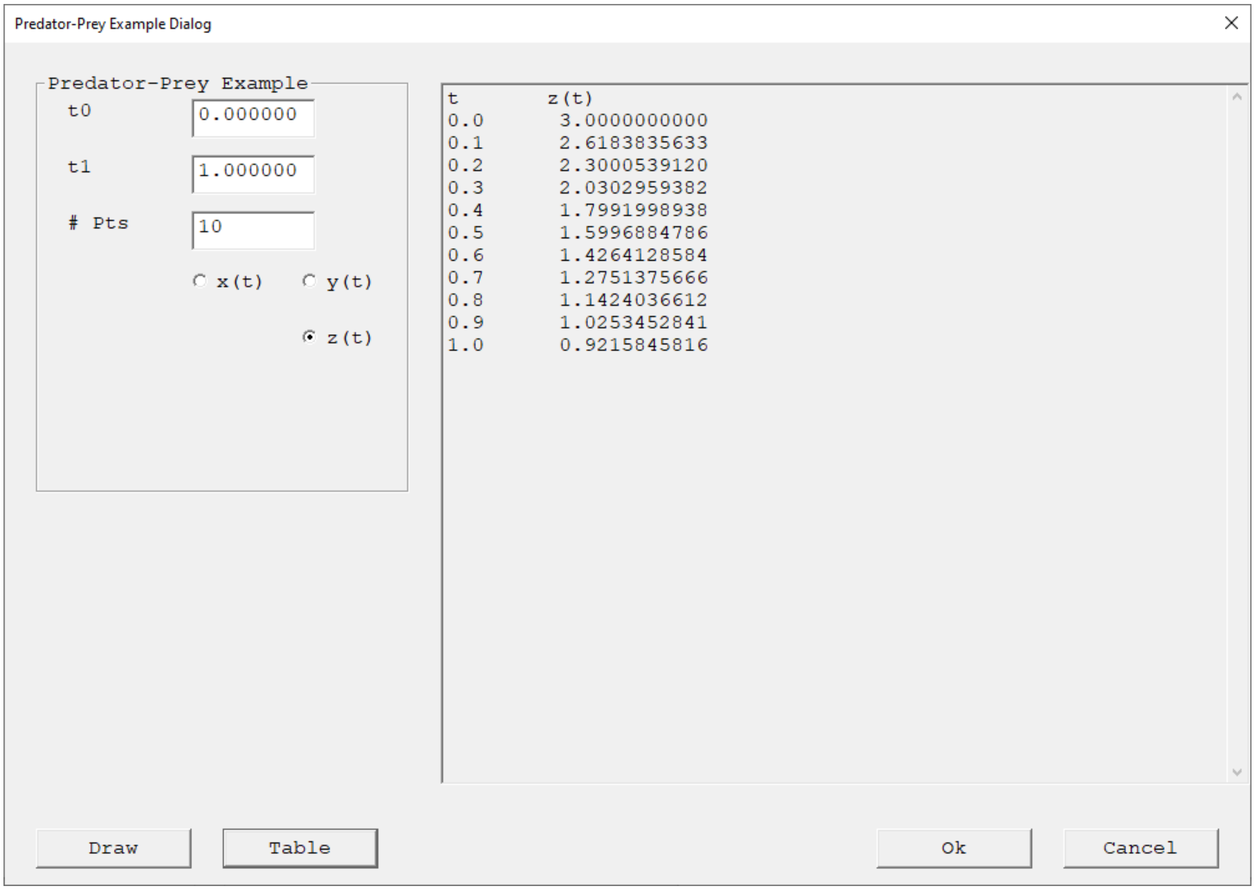
$$x(0) = 1, y(0) = 2, z(0) = 3$$


Predator-Prey Example Dialog
Predator-Prey Example
t0
0.000000
t1
1.000000
Pts
10
x(t)
y(t)
z(t)
t x(t)
0.0 1.0000000000
0.1 1.2973353697
0.2 1.6254385356
0.3 1.9777051232
0.4 2.3469638804
0.5 2.7259802403
0.6 3.1078573924
0.7 3.4863075138
0.8 3.8557998054
0.9 4.2116136709
1.0 4.5498319915
Draw
Table
Ok
Cancel


Predator-Prey Example Draw Dialog
Predator-Prey Example
4.550
4.106
3.662
3.219
2.775
2.331
1.887
1.444
0.00
0.12
0.25
0.38
0.50
0.62
0.75
0.88
1.00
Ok
Cancel


Predator-Prey Example Dialog
Predator-Prey Example
t0
0.000000
t1
1.000000
Pts
10
x(t)
y(t)
z(t)
y y(t)
0.0 2.0000000000
0.1 2.0917047582
0.2 2.1238090988
0.3 2.0952699457
0.4 2.0099210243
0.5 1.8760714864
0.6 1.7053570134
0.7 1.5111301120
0.8 1.3067608541
0.9 1.1041847895
1.0 0.9129159180
Draw
Table
Ok
Cancel


Predator-Prey Example Draw Dialog
Predator-Prey Example
2.124
1.972
1.821
1.670
1.518
1.367
1.216
1.064
0.00
0.12
0.25
0.38
0.50
0.62
0.75
0.88
1.00
Ok
Cancel


Predator-Prey Example Dialog
Predator-Prey Example
t0
0.000000
t1
1.000000
Pts
10
x(t)
y(t)
z(t)
t z(t)
0.0 3.0000000000
0.1 2.6183835633
0.2 2.3000539120
0.3 2.0302959382
0.4 1.7991998938
0.5 1.5996884786
0.6 1.4264128584
0.7 1.2751375666
0.8 1.1424036612
0.9 1.0253452841
1.0 0.9215845816
Draw
Table
Ok
Cancel

References

[1] S. Conte and C. de Boor, "COMPUTER RESULTS FOR EXAMPLE 8.9," in *Elementary Numerical Analysis An Algorithmic Approach Third Edition*, New York, McGraw-Hill Book Company, 1980, p. 401.

[2] S. Conte and C. de Boor, "Example 8.9," in *Elementary Numerical Analysis an Algorithmic Approach*, New York, McGraw-Hill Book Company, 1980, p. 400.

[3] H. T. Lau, "G. diffsys," in *A Numerical Library in C for Scientists and Engineers*, Boca Raton, CRC Press, 1995, pp. 391-394.

[4] V. W. Noonburg, "Exercise 5.," in *Ordinary Differntial Equations from Calculus to Dynamical Systems*, Washington D. C., Mathematical Association of America, 2014, p. 203.