

Blog Entry © Thursday, June 5, 2025, by James Pate Williams, Jr., Analytic, Numeric, and Siacci's Method for Solving Ballistic Trajectory Problems (Point-Mass Projectile Motion)

First, we use the applicable methods, Analytic and Numeric, to find the projectile flight parameters for a Major League Baseball [1] [2].

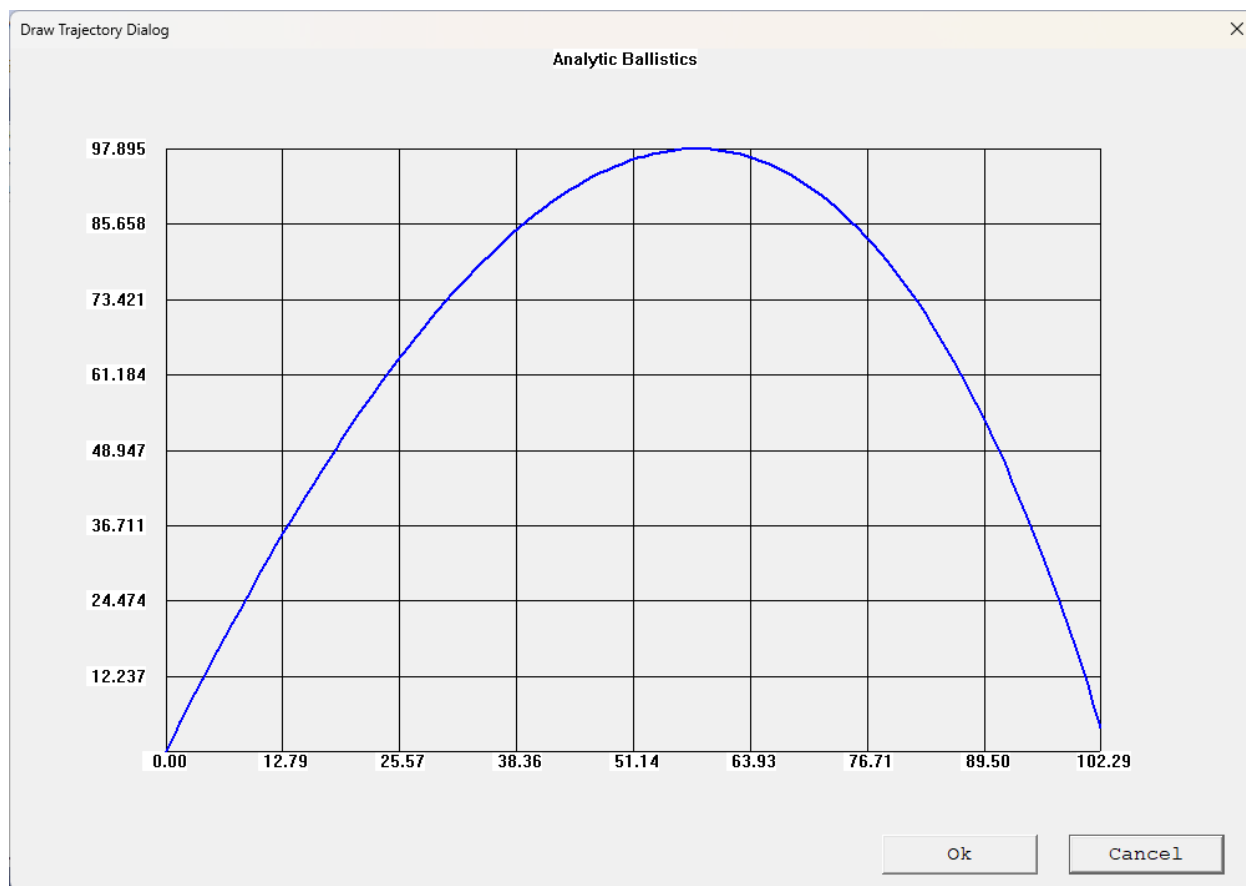
c = 0.3517 drag coefficient
d = 2.9 inches
e = 45 degrees elevation
v = 131.2336 feet / second
w = 0.3125 pounds

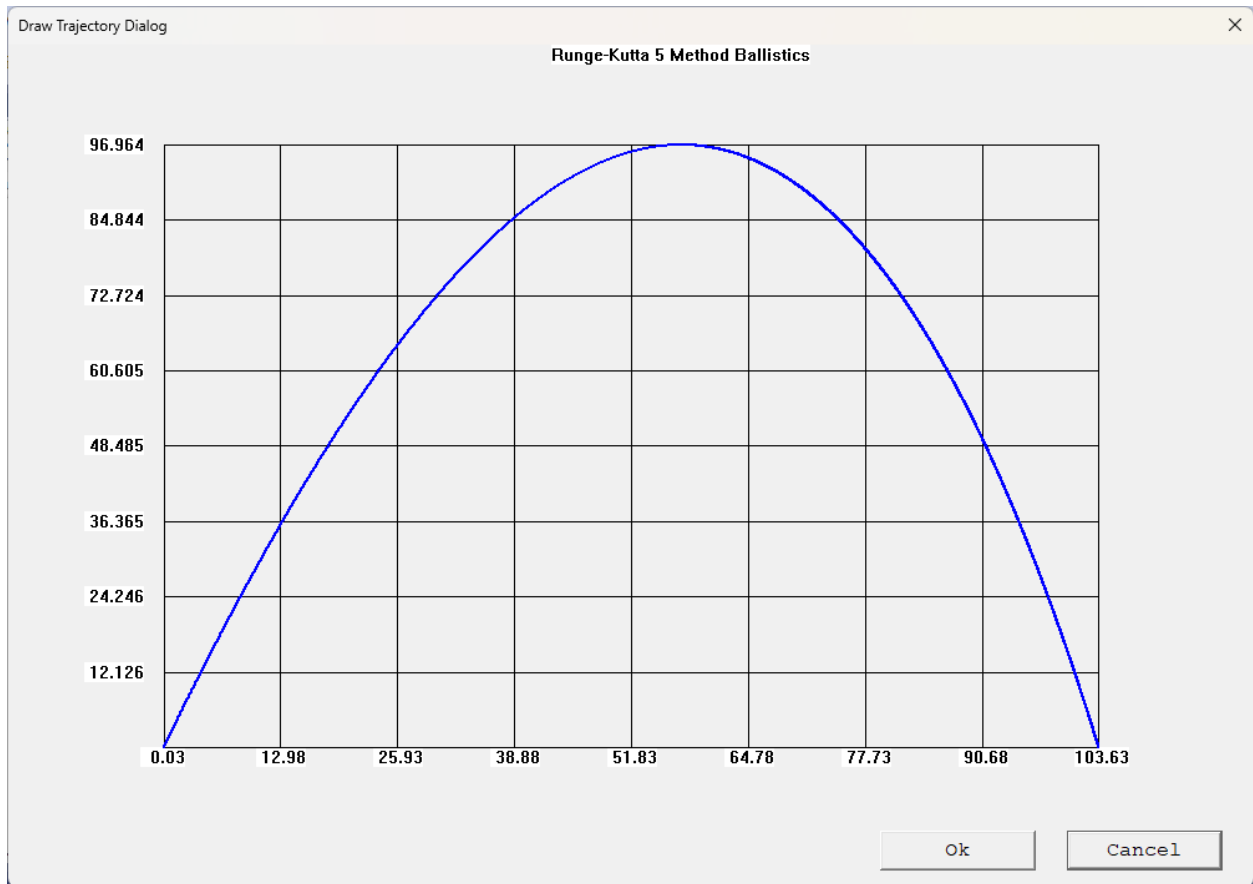
Analytic

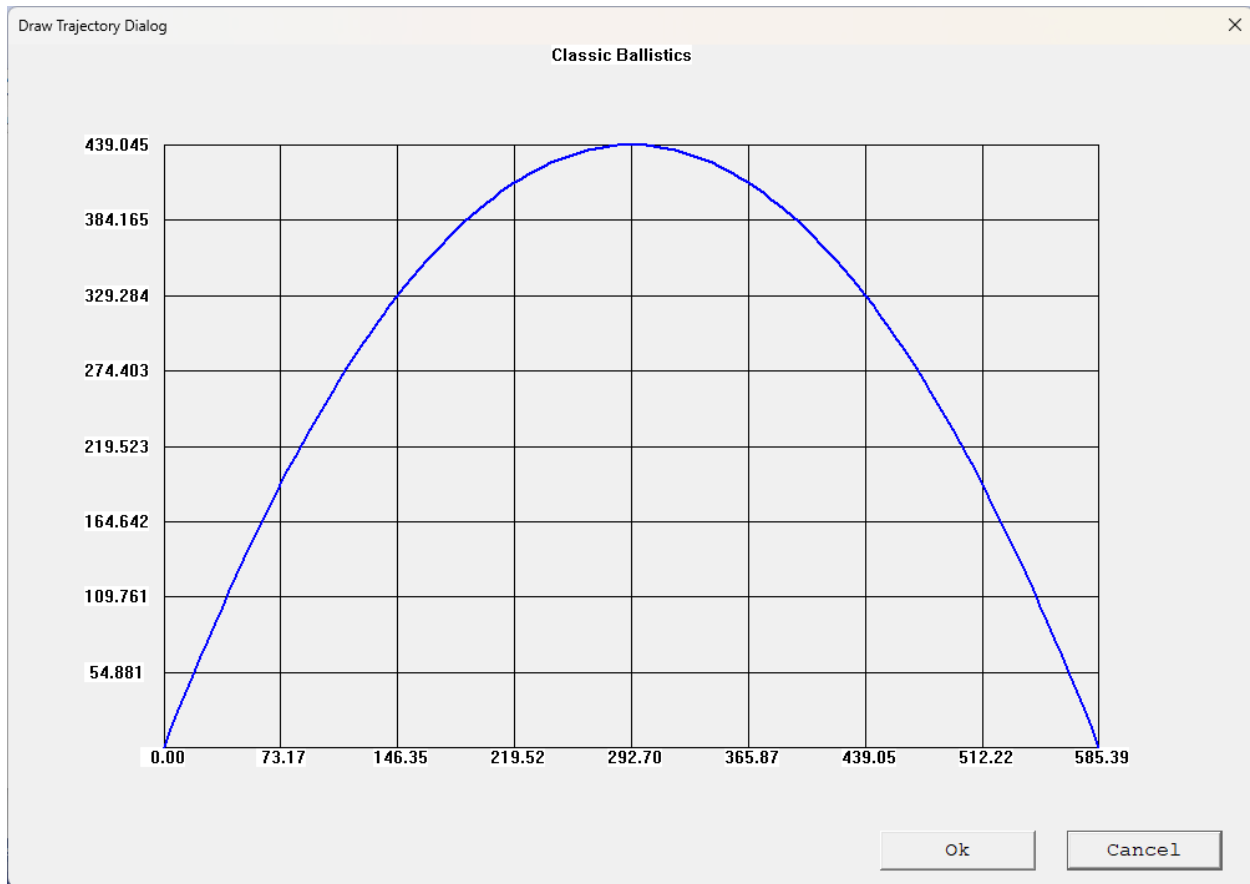
H = 97.894704	; height at apex (apogee or summit)
T = 4.933689	; time of flight in seconds
Va = 62.685616	; velocity at apex in feet per seconds
L = 103.090437	; range in yards
Ta = 2.281915	; time to apex in seconds
xa = 58.000023	; range at apex
Theta1 = -58.851954	; angle of obliquity or striking angle
V1 = 84.433847	; terminal velocity in feet per seconds

Numeric (5th Order Runge-Kutta Method) [3]

H = 96.963770
T = 4.892000
Va = 62.687013
L = 103.633078
Ta = 2.297000
xa = 19.112292
Theta1 = -57.552318
V1 = 82.896348







Now we are exploring the USN Fast Battleship 16-Inch 50 Calibers Rifles:

$c = 0.197$ drag coefficient

$d = 16$ inches

$e = 45$ degrees elevation

$v = 2500$ feet / second

$w = 2700$ pounds

Analytic

$H = 38456.604351$

$T = 97.786208$

$V_a = 1298.547890$

$L = 42326.691213$

$T_a = 45.923274$

$x_a = 23293.352539$

$\text{Theta1} = -56.269903$

$V1 = 1752.644186$

Numeric

$$H = 37911.442582$$

$$T = 96.842000$$

$$V_a = 1298.549748$$

$$L = 42300.616007$$

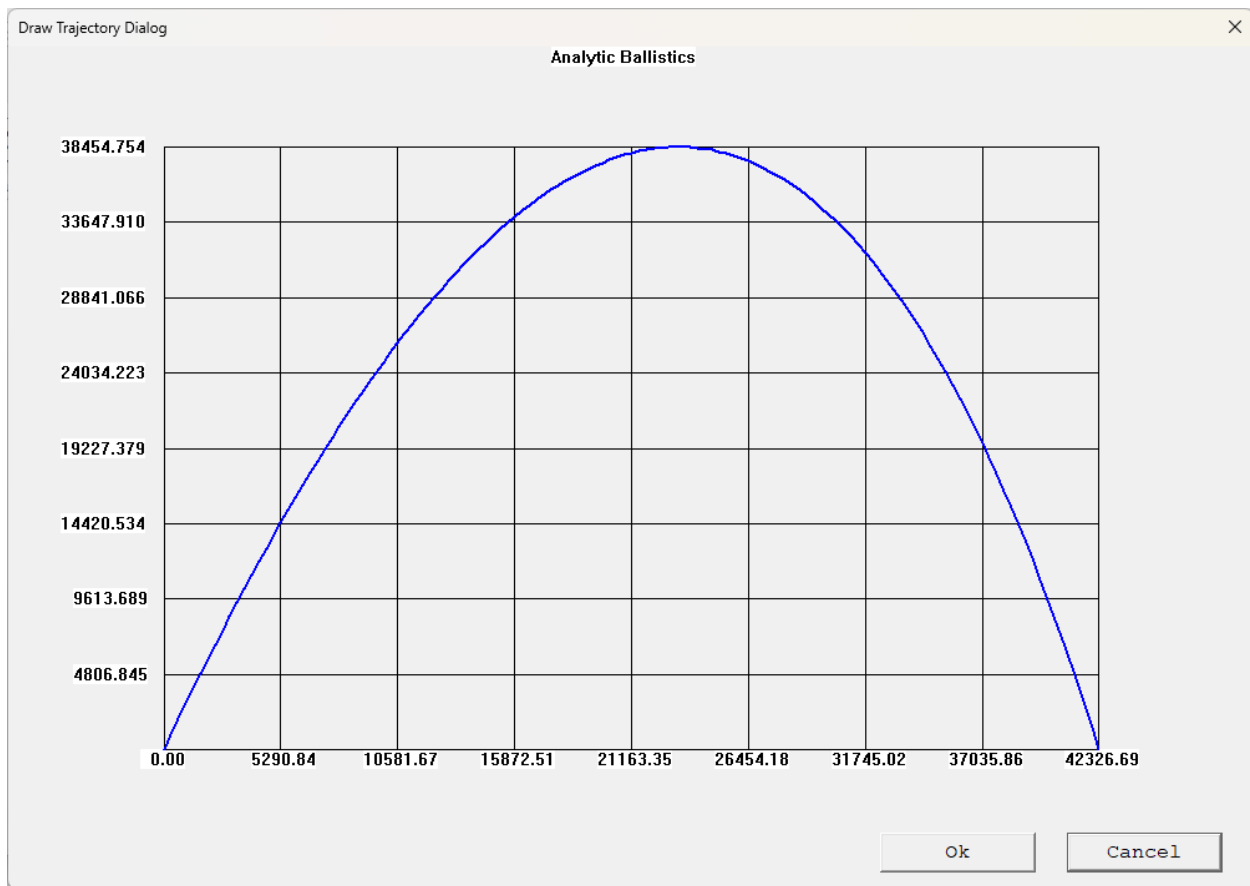
$$T_a = 46.091000$$

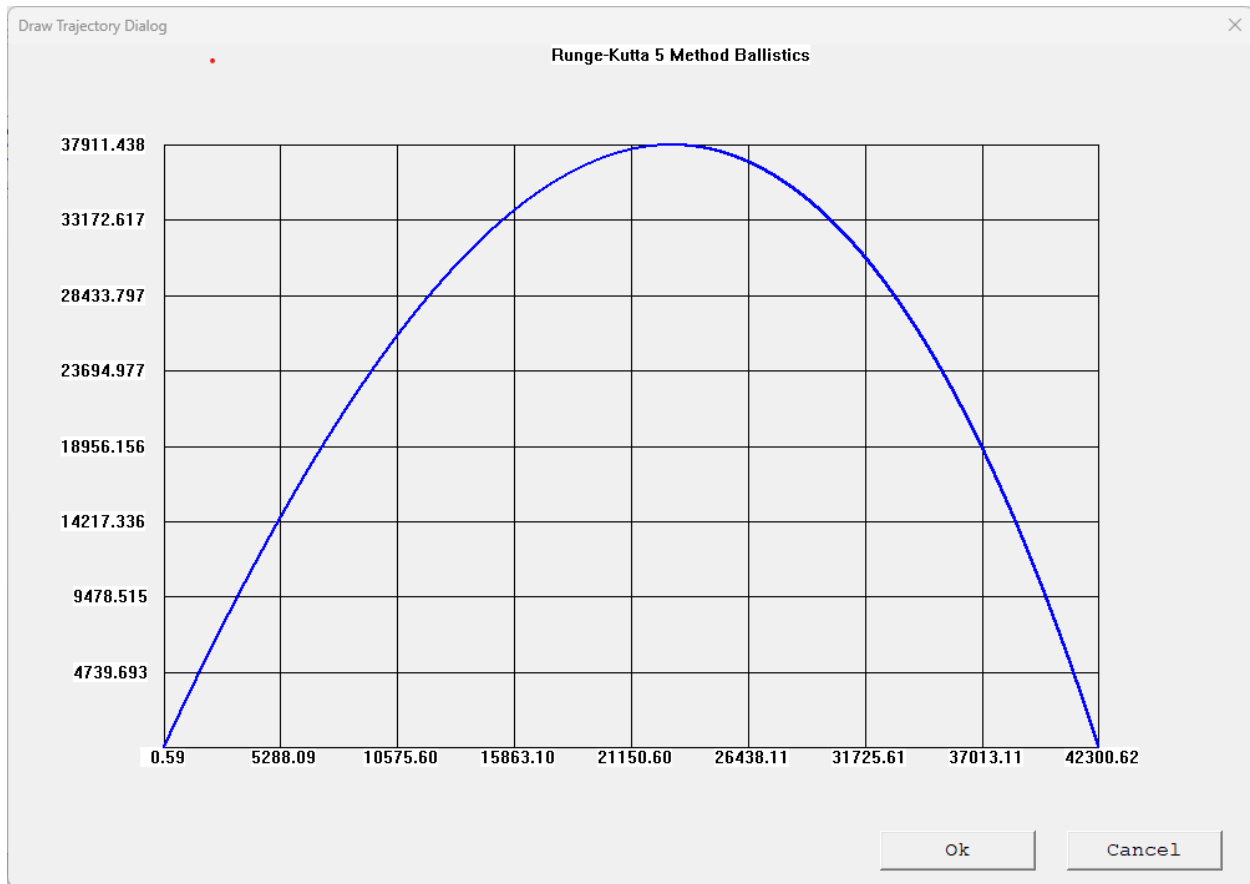
$$x_a = 22960.706653$$

$$\text{Theta1} = -55.167828$$

$$V1 = 1724.935268$$

According to the reference [3] the range of the Iowa class naval artillery is 42,345 yards. In my previous work, I used Mayevski's seven velocity zones.





We investigate very low angles of elevation so we can include Siacci's Method [4]:

Analytic

$H = 29.393196$

$T = 2.703433$

$V_a = 2467.810549$

$L = 2223.853146$

$T_a = 1.347402$

$x_a = 1117.262179$

$\Theta_1 = -1.019376$

$V_1 = 2436.989433$

Numeric

H = 29.331687
 T = 2.701000
 Va = 2467.843103
 L = 2221.116212
 Ta = 1.346000
 xa = 1115.170010
 Theta1 = -1.016713
 V1 = 2437.089309

Siacci's Method

Range	Start	Angle	Final	Angle	Time	V1	Y-Max			
Yards	Degrees		Minutes	Degrees			Minutes	Sec	FPS	Feet
1000	0	26.6	0	27	1.20	2482	6			
1100	0	29.3	0	30	1.33	2480	7			
1200	0	32.0	0	32	1.45	2478	8			
1300	0	34.7	0	35	1.57	2476	10			
1400	0	37.3	0	38	1.69	2474	11			
1500	0	40.1	0	40	1.81	2473	13			
1600	0	42.8	0	43	1.93	2471	15			
1700	0	45.5	0	46	2.05	2469	17			
1800	0	48.1	0	49	2.17	2467	19			
1900	0	50.8	0	52	2.30	2465	21			
2000	0	53.5	0	54	2.42	2463	23			
2100	0	56.2	0	57	2.54	2462	26			
2200	0	58.9	0	60	2.66	2460	28			
2300	1	1.6	1	3	2.78	2458	31			

Detailed Deviations from Ordnance Pamphlet 770

Range	Start	Angle	Final	Angle	Time	V1	Y-Max			
Yards	Degrees		Minutes	Degrees			Minutes	Sec	FPS	Feet
0	0	0.3	0	1	0	-27	1			
0	0	0.3	0	1	0	-30	-1			
0	0	0.3	0	1	0	-32	-2			
0	0	0.3	0	1	0	-36	-3			
0	0	0.4	0	1	0	-38	-5			
0	0	0.4	0	1	0	-41	0			
0	0	0.5	0	1	0	-43	-2			
0	0	0.6	0	2	0	-45	-3			
0	0	0.8	0	1	0	-49	-5			
0	0	0.9	0	1	0	-51	-8			
0	0	1.0	0	1	0	-54	1			
0	0	1.1	0	1	0	-56	-1			
0	1	-58.8	1	-58	0	-59	-4			
0	0	1.3	0	2	0	-61	-7			

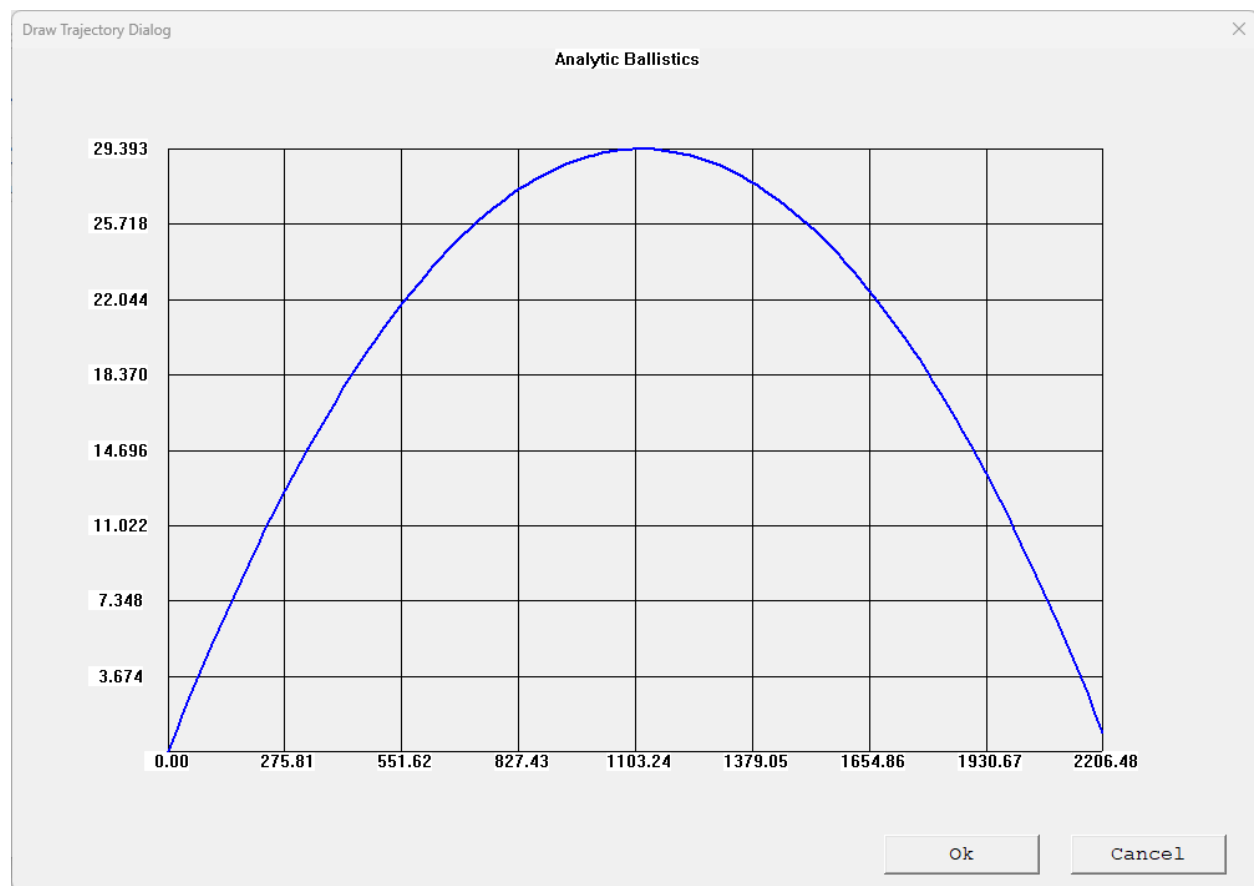
Total 71

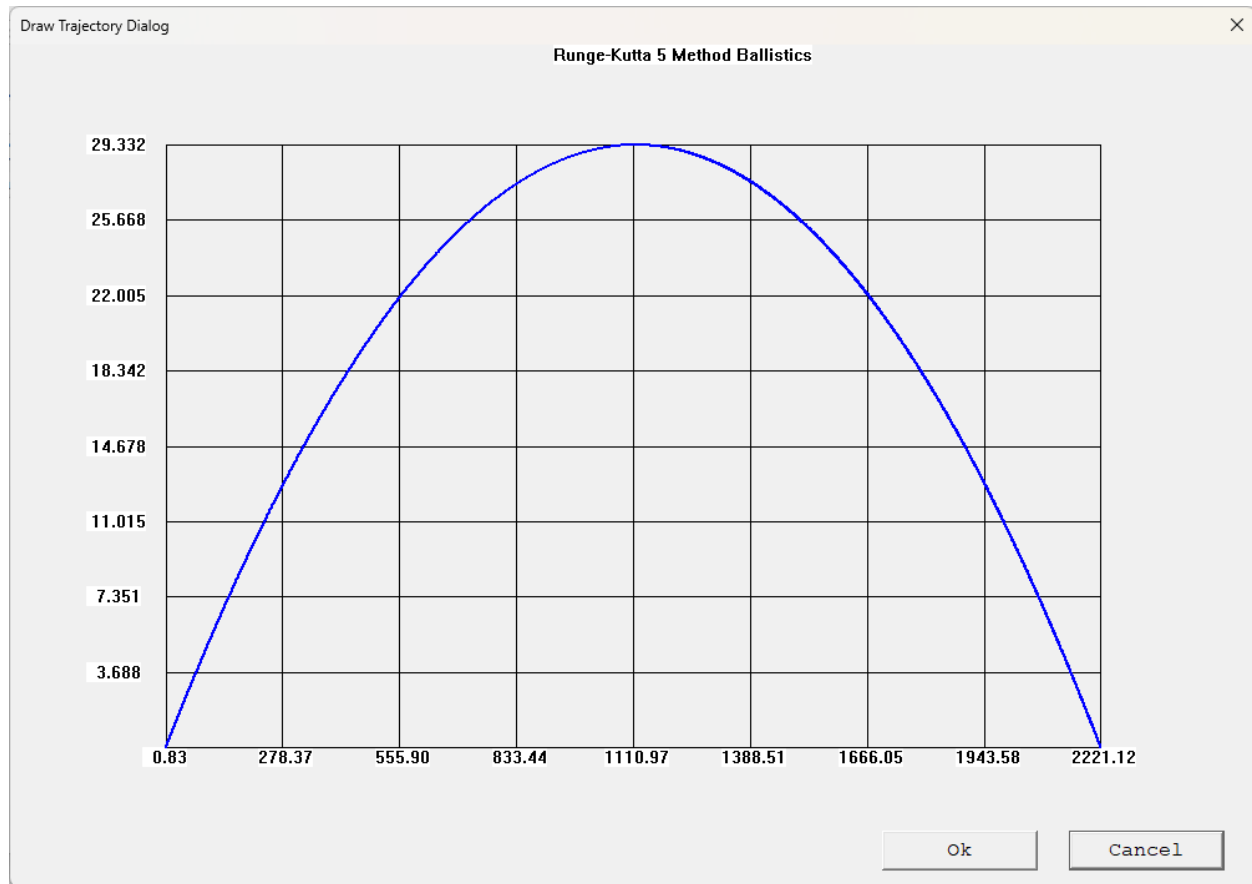
Out of 112

Percent Error 63.39

Err CntAvg Abs Dev

Range	0	0.000
Degrees	1	1.000
Minutes	14	4.789
Degrees	1	1.000
Minutes	14	5.214
Time	14	0.019
V1	14	44.429
Y-Max	13	3.308





References

- [1] P. S. Chudinov, V. A. Eltyshev and Y. A. Barykin, "Simple analytical description of projectile motion," Lat. Am. J. Phys. Educ. Vol. 7, No. 3, Sept., 2013, 1 September 2013. [Online]. Available: http://www.lajpe.org/sep13/04-LAJPE-782_Chudinov.pdf. [Accessed 5 June 2025].
- [2] A. Nathan and D. Kagen, "Drag Dashboard," MLB Advanced Media, LP, 5 June 2025. [Online]. Available: <https://baseballsavant.mlb.com/drag-dashboard>. [Accessed 5 June 2025].
- [3] H. T. Lau, A Numerical Library in C for Scientists and Engineers, Boca Raton: CRC Press, 1995.

- [4] United States Navy Ordnance Bureau, "Ordnance Pamphlet 770," United States Navy, 1 October 1941. [Online]. Available: <https://eugeneleeslover.com/USN-GUNS-AND-RANGE-TABLES/OP-770-1.html>. [Accessed 5 June 2025].
- [5] E. E. Herrmann, "Appendix A The Practical Application of Siacci's Method to the Solution of the Trajectory. Ingall's Method.," in *Exterior ballistics*, 1935, Annapolis, United States Naval Academy, 1935, pp. 247-267.