

Blog Entry © Tuesday, July 29, 2025, Double and Triple Monte Carlo Integration by James Pate Williams, Jr.

Double integrals from **Quantum Mechanics Third Edition** © 1968 by Leonard I. Schiff

First-order Stark effect in hydrogen:

$$\langle 10|H'|10\rangle = \frac{eE}{16a_0^4} \int_0^\infty \int_{-1}^1 r^4 \left(2 - \frac{r}{a_0}\right) e^{-r/a_0} w^2 dw dr = -3eEa_0$$

Perturbation electron-electron repulsion integral:

$$(4\pi)^2 \int_0^\infty \left[\int_0^{r_1} \frac{1}{r_1} e^{-2Z(r_1+r_2)/a_0} r_2^2 dr_2 + \int_{r_1}^\infty \frac{1}{r_2} e^{-2Z(r_1+r_2)/a_0} r_2^2 dr_2 \right] r_1^2 dr_1 = \frac{5\pi^2 a_0^5}{8Z^5}$$

N = 1000000	integral = -6.0069965950	% error = 0.1166099164
N = 2000000	integral = -5.9804109806	% error = 0.3264836571
N = 3000000	integral = -5.9934479275	% error = 0.1092012078
N = 4000000	integral = -6.0194674492	% error = 0.3244574863
N = 5000000	integral = -6.0101824901	% error = 0.1697081685
N = 6000000	integral = -5.9953764106	% error = 0.0770598226
N = 7000000	integral = -5.9942825100	% error = 0.0952915004
N = 8000000	integral = -6.0018197498	% error = 0.0303291641
N = 9000000	integral = -6.0086805313	% error = 0.1446755209
exact value = -6.0000000000		
N = 1000000	integral = 0.1877049421	% error = 2.6253470193
N = 2000000	integral = 0.1909598216	% error = 0.9368312434
N = 3000000	integral = 0.1906877161	% error = 1.0779898948
N = 4000000	integral = 0.1950775031	% error = 1.1992755973
N = 5000000	integral = 0.1908750288	% error = 0.9808187139
N = 6000000	integral = 0.1927364511	% error = 0.0151789927
N = 7000000	integral = 0.1901243369	% error = 1.3702509899
N = 8000000	integral = 0.1907041809	% error = 1.0694485276
N = 9000000	integral = 0.1924049217	% error = 0.1871646228
exact value = 0.1927657110		

D:\Perturbation\x64\Release\Perturbation.exe (process 33884) exited with code 0 (0x0).
Press any key to close this window . . .

Monte Carlo integration of the triple integral:

[The basics of Monte Carlo integration | by victor_ | TDS Archive | Medium](#)

[int sin^2 x + y sin z dx dy dz , x=0..pi, y=0..1, z=0..pi - Wolfram|Alpha](#)

and $x + y * z * z$

[15.4: Triple Integrals - Mathematics LibreTexts](#)

```
integrand pow(sin(x), 2.0) + y * sin(z)
x = 0 to pi, y = 0 to 1, z = 0 to pi
N = 1000000      integral = 8.0811406303      % error = 0.0587610711
N = 2000000      integral = 8.0714086885      % error = 0.0617375163
N = 3000000      integral = 8.0722518304      % error = 0.0512979349
N = 4000000      integral = 8.0756280725      % error = 0.0094941081
N = 5000000      integral = 8.0790904421      % error = 0.0333761295
N = 6000000      integral = 8.0750036649      % error = 0.0172253747
N = 7000000      integral = 8.0766069861      % error = 0.0026265673
N = 8000000      integral = 8.0770368959      % error = 0.0079496082
N = 9000000      integral = 8.0757802791      % error = 0.0076095214
```

exact value = 8.0763948541

```
integrand x + y * z * z
x = -1 to 5, y = 2 to 4, z = 0 to 1
N = 1000000      integral = 36.0475736432      % error = 0.1321490088
N = 2000000      integral = 36.0205388220      % error = 0.0570522834
N = 3000000      integral = 35.9801347087      % error = 0.0551813647
N = 4000000      integral = 35.9925082278      % error = 0.0208104784
N = 5000000      integral = 36.0055176142      % error = 0.0153267061
N = 6000000      integral = 36.0227086755      % error = 0.0630796542
N = 7000000      integral = 36.0028979264      % error = 0.0080497956
N = 8000000      integral = 36.0001388562      % error = 0.0003857116
N = 9000000      integral = 35.9904145824      % error = 0.0266261599
```

exact value = 36.0000000000

D:\MonteCarlo3d\x64\Release\MonteCarlo3d.exe (process 35268) exited with code 0 (0x0).

Press any key to close this window . . .