Some Relativity Mathematics by James Pate Williams, Jr.

From Appendix B Equation (B3) of the paper: https://arxiv.org/pdf/2506.09946

$$P_n(x) = \left[n(x+1) + x^2 \frac{d}{dx}\right] P_{n-1}(x) = n(x+1)P_{n-1}(x) + x^2 \frac{dP_{n-1}(x)}{dx}, x \geq 1$$

$$P_0(x) = 1$$

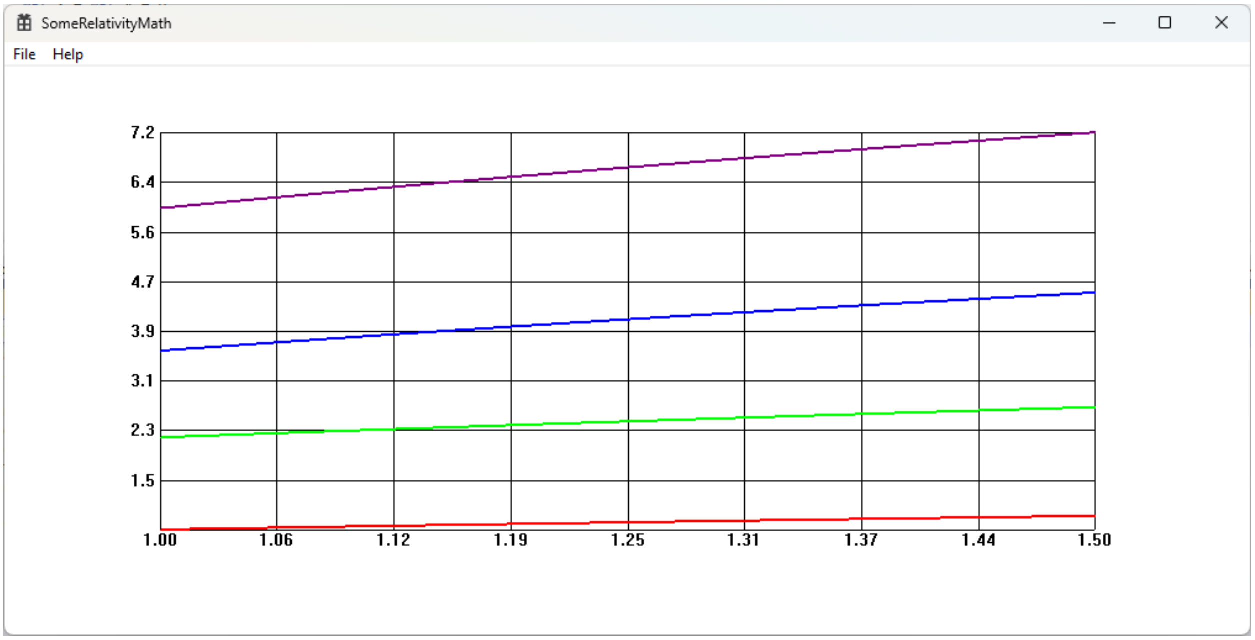
$$P_1(x) = x + 1$$

$$\begin{aligned} P_2(x) &= 2(x+1)P_1(x) + x^2 \frac{dP_1(x)}{dx} = 2(x+1)^2 + x^2 = 2(x^2 + 2x + 1) + x^2 \\ &= 3x^2 + 4x + 2 \end{aligned}$$

$$\begin{aligned} P_3(x) &= 3(x+1)P_2(x) + x^2 \frac{dP_2(x)}{dx} = 3(x+1)(3x^2 + 4x + 2) + x^2(6x + 4) \\ &= 3(3x^3 + 4x^2 + 2x) + 3(3x^2 + 4x + 2) + 6x^3 + 4x^2 \\ &= 15x^3 + (12 + 9 + 4)x^2 + (6 + 12)x + 6 = 15x^3 + 25x^2 + 18x + 6 \end{aligned}$$

$$\begin{aligned} P_4(x) &= 4(x+1)P_3(x) + x^2 \frac{dP_3(x)}{dx} \\ &= 4(x+1)(15x^3 + 25x^2 + 18x + 6) + x^2(45x^2 + 50x + 18) \\ &= 4(15x^4 + 25x^3 + 18x^2 + 6x) + 4(15x^3 + 25x^2 + 18x + 6) + 45x^4 + 50x^3 \\ &+ 18x^2 \\ &= 60x^4 + 100x^3 + 72x^2 + 24x + 60x^3 + 100x^2 + 72x + 24 + 45x^4 + 50x^3 \\ &= 105x^4 + (100 + 60 + 50)x^3 + 172x^2 + (24 + 72)x + 24 \\ &= 105x^4 + 210x^3 + 172x^2 + 96x + 24 \end{aligned}$$

I have graphed the logarithms of the polynomials for 1<=n<=4 using a Win32 C/C++ Desktop application in the Release x64 Configuration. I used the Microsoft Visual Studio 2022 Community Version.



n = 4 purple, n = 3 blue, n = 2 green, n = 1red