

Blog Entry © Friday, August 29, 2025, by James Pate Williams, Jr., Eigenvalue and Eigenvector Calculators and Linear System of Equations Solver

References: 1. ***Elementary Numerical Analysis an Algorithmic Approach Third Edition*** © 1980 by S. D. Conte and Carl de Boor and 2. ***A Numerical Library in C for Scientists and Engineers*** © 1995 by H. T. Lau, Ph.D.

1. Eigenvalue and Eigenvector Calculators
 - a. Complex Eigenvalues and Eigenvectors
 - b. Real Eigenvalues and Eigenvectors
2. Linear System of Equations Solvers
 - a. Classical Iterative Methods
 - I. Conjugate Gradient Method
 - II. Gauss-Seidel Iteration
 - III. Gradient Method
 - IV. Jacobi Iteration
 - V. Modified Richardson
 - VI. Successive Overrelaxation (SOR)
 - b. Direct Methods
 - I. Gaussian Elimination
 - II. LU Decomposition

The screenshot shows a 'Data Input Dialog' window. On the left, there are input fields for 'n:' (value 2), 'Max Iterations:' (value 125), and 'Tolerance:' (value 1.0e-12). Below these are 'OK' and 'Cancel' buttons. The main area on the right displays the results of the calculations. It starts with 'Original Matrix (and Perhaps Right-Hand Side):' followed by a 2x2 matrix of values in scientific notation. Below that, it shows 'Eigenvalues:' with two values, and 'Eigenvectors:' with two pairs of values, each pair representing the components of an eigenvector.

Original Matrix (and Perhaps Right-Hand Side):			
(+1.0000000000e+00,	+2.0000000000e+00)	(+2.0000000000e+00,	+3.0000000000e+00)
(+3.0000000000e+00,	+3.0000000000e+00)	(+4.0000000000e+00,	+4.0000000000e+00)

Eigenvalues:	
(+6.2571528869e+00,	+5.0930638884e+00)
(-1.2571528869e+00,	+9.0693611160e-01)

Eigenvectors:			
(+4.5798519659e-01,	+2.2178696124e-01)	(-8.2351181315e-01,	-1.9391722859e-02)
(+6.0871179696e-01,	+0.0000000000e+00)	(+3.5393167490e-01,	-1.9717100422e-01)

Original Matrix (and Perhaps Right-Hand Side):

(+1.0000000000e+00,	+2.0000000000e+00)	(+2.0000000000e+00,	+3.0000000000e+00)
(+3.0000000000e+00,	+3.0000000000e+00)	(+4.0000000000e+00,	+4.0000000000e+00)

Eigenvalues:

(+6.2571528869e+00,	+5.0930638884e+00)
(-1.2571528869e+00,	+9.0693611160e-01)

Eigenvectors:

(+4.5798519659e-01, +2.2178696124e-01) (-8.2351181315e-01, -1.9391722859e-02)
(+6.0871179696e-01, +0.0000000000e+00) (+3.5393167490e-01, -1.9717100422e-01)

The 'Data Input Dialog' window shows the following settings and results:

- n: 3
- Max Iterations: 125
- Tolerance: 1.0e-12

Original Matrix (and Perhaps Right-Hand Side):

+1.0000000000e+00	-1.0000000000e+00	+0.0000000000e+00
-1.0000000000e+00	+2.0000000000e+00	-1.0000000000e+00
+0.0000000000e+00	-1.0000000000e+00	+3.0000000000e+00

Eigenvalues:

+2.6794919243e-01	+2.0000000000e+00	+3.7320508076e+00
-------------------	-------------------	-------------------

Eigenvectors:

+1.0000000000e+00	+1.0000000000e+00	+2.6794919243e-01
+7.3205080757e-01	-1.0000000000e+00	-7.3205080757e-01
+2.6794919243e-01	-1.0000000000e+00	+1.0000000000e+00

Original Matrix (and Perhaps Right-Hand Side):

+1.0000000000e+00	-1.0000000000e+00	+0.0000000000e+00
-1.0000000000e+00	+2.0000000000e+00	-1.0000000000e+00
+0.0000000000e+00	-1.0000000000e+00	+3.0000000000e+00

Eigenvalues:

+2.6794919243e-01	+2.0000000000e+00	+3.7320508076e+00
-------------------	-------------------	-------------------

Eigenvectors:

+1.0000000000e+00	+1.0000000000e+00	+2.6794919243e-01
+7.3205080757e-01	-1.0000000000e+00	-7.3205080757e-01
+2.6794919243e-01	-1.0000000000e+00	+1.0000000000e+00

The 'Data Input Dialog' window shows the following settings and results:

- n: 2
- Max Iterations: 125
- Tolerance: 1.0e-12

Original Matrix (and Perhaps Right-Hand Side):

+5.0000000000e+00	+1.0000000000e+00	+3.0000000000e+00
+1.0000000000e+00	+8.0000000000e+00	+2.0000000000e+00

Conjugate Gradient Iteration Solution Vector:

Tolerance: 1.0000000000e-12

+5.6410256410e-01
+1.7948717949e-01

Operations:

number of fabs function calls =	0
number of pow function calls =	0
number of sqrt function calls =	0
number of add operations =	16
number of sub operations =	2
number of div operations =	0
number of mul operations =	20
number of iterations =	1

Original Matrix (and Perhaps Right-Hand Side):

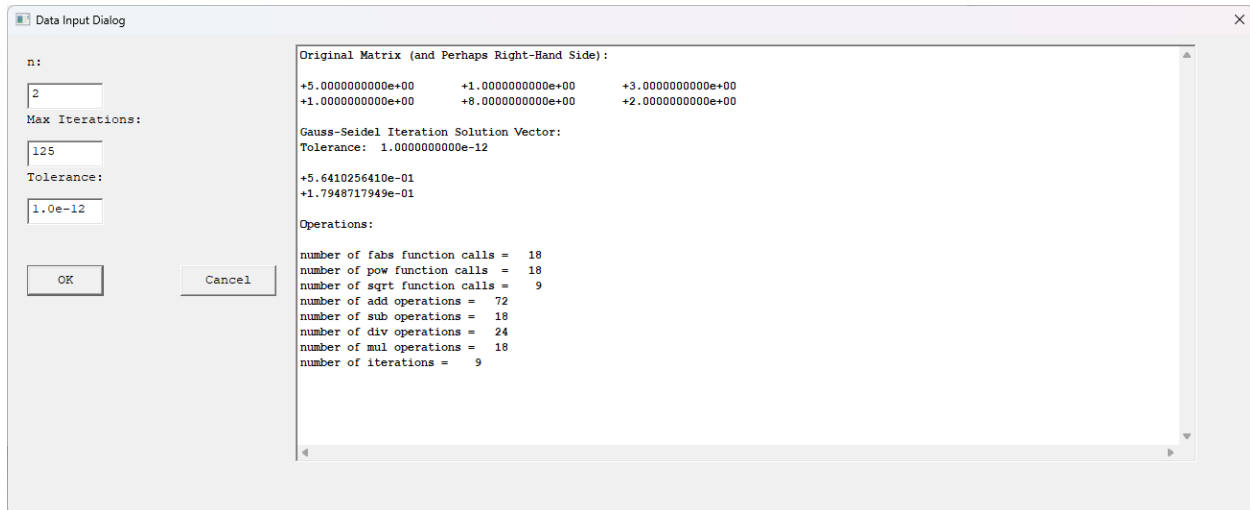
+5.0000000000e+00	+1.0000000000e+00	+3.0000000000e+00
+1.0000000000e+00	+8.0000000000e+00	+2.0000000000e+00

Conjugate Gradient Iteration Solution Vector:
Tolerance: 1.0000000000e-12

+5.6410256410e-01
+1.7948717949e-01

Operations:

number of fabs function calls = 0
number of pow function calls = 0
number of sqrt function calls = 0
number of add operations = 16
number of sub operations = 2
number of div operations = 0
number of mul operations = 20
number of iterations = 1



Original Matrix (and Perhaps Right-Hand Side):

+5.0000000000e+00	+1.0000000000e+00	+3.0000000000e+00
+1.0000000000e+00	+8.0000000000e+00	+2.0000000000e+00

Gauss-Seidel Iteration Solution Vector:
Tolerance: 1.0000000000e-12

+5.6410256410e-01
+1.7948717949e-01

Operations:

number of fabs function calls = 18
number of pow function calls = 18
number of sqrt function calls = 9
number of add operations = 72
number of sub operations = 18
number of div operations = 24
number of mul operations = 18
number of iterations = 9

Data Input Dialog

n:

2

Max Iterations:

125

Tolerance:

1.0e-12

OK

Cancel

Original Matrix (and Perhaps Right-Hand Side):

+5.000000000e+00

+1.000000000e+00

+3.000000000e+00

+1.000000000e+00

+8.000000000e+00

+2.000000000e+00

Gradient Method Solution Vector:

Tolerance: 1.000000000e-12

+5.6410256410e-01

+1.7948717949e-01

Operations:

number of fabs function calls = 0

number of pow function calls = 44

number of sqrt function calls = 22

number of add operations = 374

number of sub operations = 88

number of div operations = 22

number of mul operations = 308

number of iterations = 22

Original Matrix (and Perhaps Right-Hand Side):

+5.000000000e+00	+1.000000000e+00	+3.000000000e+00
+1.000000000e+00	+8.000000000e+00	+2.000000000e+00

Gradient Method Solution Vector:

Tolerance: 1.000000000e-12

+5.6410256410e-01

+1.7948717949e-01

Operations:

```

number of fabs function calls = 0
number of pow function calls = 44
number of sqrt function calls = 22
number of add operations = 374
number of sub operations = 88
number of div operations = 22
number of mul operations = 308
number of iterations = 22

```

Data Input Dialog

n:

2

Max Iterations:

125

Tolerance:

1.0e-12

OK

Cancel

Original Matrix (and Perhaps Right-Hand Side):

+5.000000000e+00

+1.000000000e+00

+3.000000000e+00

+1.000000000e+00

+8.000000000e+00

+2.000000000e+00

Jacobi Iteration Solution Vector:

Tolerance: 1.000000000e-12

+5.6410256410e-01

+1.7948717949e-01

Operations:

number of fabs function calls = 250

number of pow function calls = 250

number of sqrt function calls = 125

number of add operations = 500

number of sub operations = 500

number of div operations = 250

number of mul operations = 250

number of iterations = 125

Original Matrix (and Perhaps Right-Hand Side):

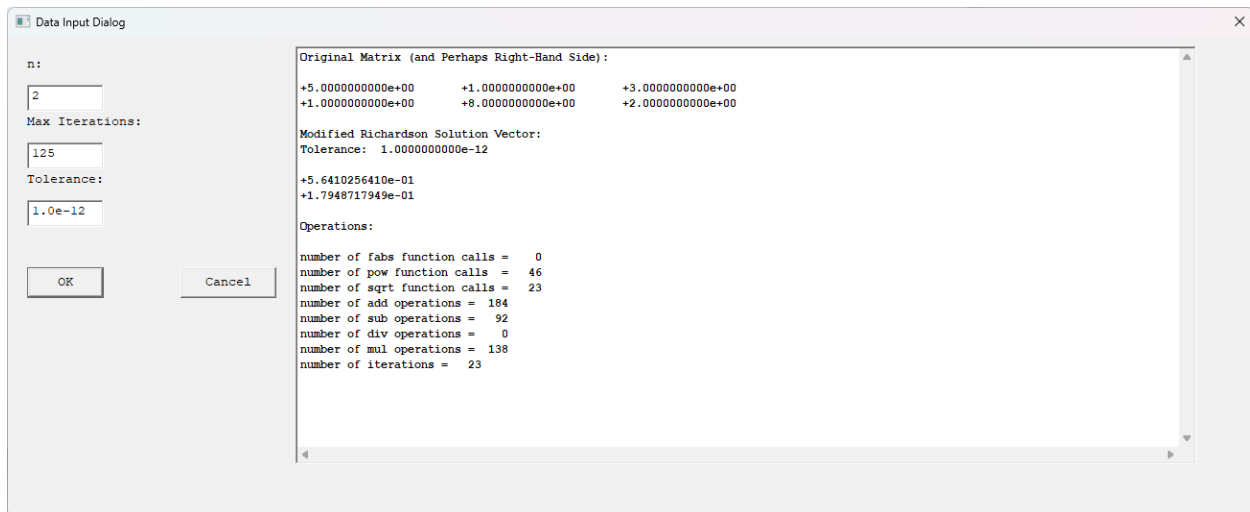
+5.000000000e+00	+1.000000000e+00	+3.000000000e+00
+1.000000000e+00	+8.000000000e+00	+2.000000000e+00

Jacobi Iteration Solution Vector:
Tolerance: 1.0000000000e-12

+5.6410256410e-01
+1.7948717949e-01

Operations:

number of fabs function calls = 250
number of pow function calls = 250
number of sqrt function calls = 125
number of add operations = 500
number of sub operations = 500
number of div operations = 250
number of mul operations = 250
number of iterations = 125



Original Matrix (and Perhaps Right-Hand Side):

+5.0000000000e+00	+1.0000000000e+00	+3.0000000000e+00
+1.0000000000e+00	+8.0000000000e+00	+2.0000000000e+00

Modified Richardson Solution Vector:
Tolerance: 1.0000000000e-12

+5.6410256410e-01
+1.7948717949e-01

Operations:

number of fabs function calls = 0
number of pow function calls = 46
number of sqrt function calls = 23
number of add operations = 184
number of sub operations = 92
number of div operations = 0
number of mul operations = 138
number of iterations = 23

Data Input Dialog

n:

Max Iterations:

Tolerance:

OK Cancel

Original Matrix (and Perhaps Right-Hand Side):

```
+5.0000000000e+00    +1.0000000000e+00    +3.0000000000e+00
+1.0000000000e+00    +8.0000000000e+00    +2.0000000000e+00
```

SOR Method Solution Vector:

Tolerance: 1.0000000000e-12

```
+5.6410256410e-01
+1.7948717949e-01
```

Operations:

```
number of fabs function calls = 0
number of pow function calls = 250
number of sqrt function calls = 125
number of add operations = 4
number of sub operations = 1000
number of div operations = 250
number of mul operations = 3
number of iterations = 125
```

Original Matrix (and Perhaps Right-Hand Side):

```
+5.0000000000e+00    +1.0000000000e+00    +3.0000000000e+00
+1.0000000000e+00    +8.0000000000e+00    +2.0000000000e+00
```

SOR Method Solution Vector:

Tolerance: 1.0000000000e-12

```
+5.6410256410e-01
+1.7948717949e-01
```

Operations:

```
number of fabs function calls = 0
number of pow function calls = 250
number of sqrt function calls = 125
number of add operations = 4
number of sub operations = 1000
number of div operations = 250
number of mul operations = 3
number of iterations = 125
```

Data Input Dialog

n:

Max Iterations:

Tolerance:

OK Cancel

Original Matrix (and Perhaps Right-Hand Side):

```
+5.0000000000e+00    +1.0000000000e+00    +3.0000000000e+00
+1.0000000000e+00    +8.0000000000e+00    +2.0000000000e+00
```

Gaussian Elimination Solution Vector:

Tolerance: 1.0000000000e-12

```
+5.6410256410e-01
+1.7948717949e-01
```

Operations:

```
number of fabs function calls = 6
number of pow function calls = 0
number of sqrt function calls = 0
number of add operations = 2
number of sub operations = 2
number of div operations = 3
number of mul operations = 3
number of iterations = 0
```

Original Matrix (and Perhaps Right-Hand Side):

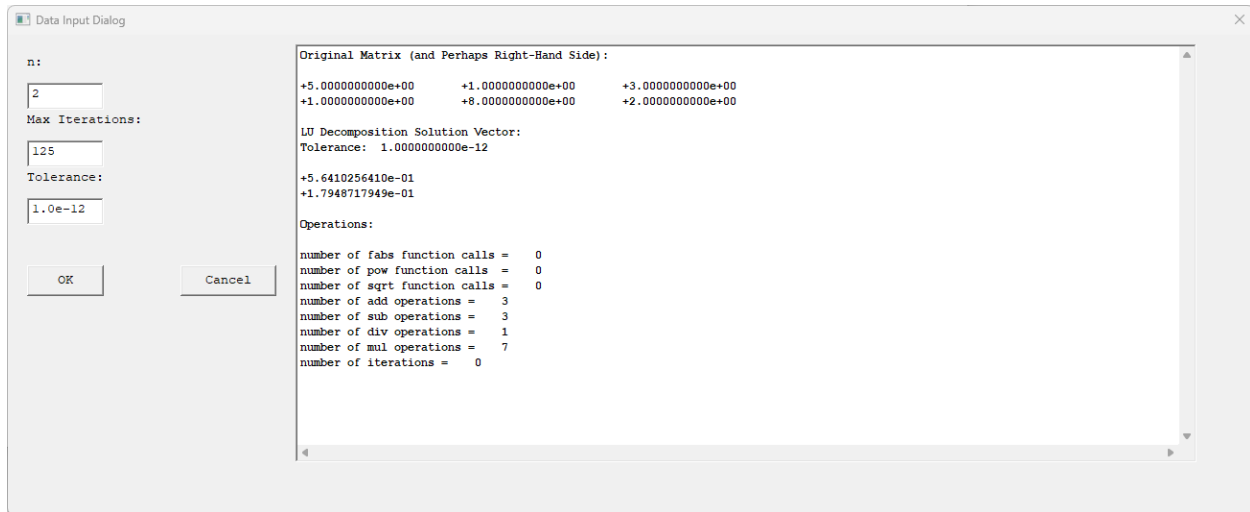
```
+5.0000000000e+00    +1.0000000000e+00    +3.0000000000e+00
+1.0000000000e+00    +8.0000000000e+00    +2.0000000000e+00
```

Gaussian Elimination Solution Vector:
Tolerance: 1.0000000000e-12

+5.6410256410e-01
+1.7948717949e-01

Operations:

number of fabs function calls = 6
number of pow function calls = 0
number of sqrt function calls = 0
number of add operations = 2
number of sub operations = 2
number of div operations = 3
number of mul operations = 3
number of iterations = 0



Original Matrix (and Perhaps Right-Hand Side):

+5.0000000000e+00	+1.0000000000e+00	+3.0000000000e+00
+1.0000000000e+00	+8.0000000000e+00	+2.0000000000e+00

LU Decomposition Solution Vector:
Tolerance: 1.0000000000e-12

+5.6410256410e-01
+1.7948717949e-01

Operations:

number of fabs function calls = 0
number of pow function calls = 0
number of sqrt function calls = 0
number of add operations = 3
number of sub operations = 3
number of div operations = 1
number of mul operations = 7
number of iterations = 0

Data Input Dialog

n:

Max Iterations:

Tolerance:

OK Cancel

Original Matrix (and Perhaps Right-Hand Side):

```
+5.000000000e+00      +1.000000000e+00      +3.000000000e+00
+1.000000000e+00      +8.000000000e+00      +2.000000000e+00
```

LU Decomposition Solution Vector:

```
+5.6410256410e-01
+1.7948717949e-01
```

Tolerance: 1.000000000e-12

Operations:

```
number of fabs function calls = 0
number of pow function calls = 0
number of sqrt function calls = 0
number of add operations = 3
number of sub operations = 3
number of div operations = 1
number of mul operations = 7
number of iterations = 0
```

Original Matrix (and Perhaps Right-Hand Side):

```
+1.2000000000e+01      +3.0000000000e+00      -5.0000000000e+00      +1.0000000000e+00
+1.0000000000e+00      +5.0000000000e+00      +3.0000000000e+00      +2.8000000000e+01
+3.0000000000e+00      +7.0000000000e+00      +1.3000000000e+01      +7.6000000000e+01
```

Gauss-Seidel Iteration Solution Vector:

Tolerance: 1.000000000e-12

```
+1.0000000000e+00
+3.0000000000e+00
+4.0000000000e+00
```

Operations:

```
number of fabs function calls = 60
number of pow function calls = 60
number of sqrt function calls = 20
number of add operations = 300
number of sub operations = 60
number of div operations = 72
number of mul operations = 120
number of iterations = 20
```

Data Input Dialog

n:

Max Iterations:

Tolerance:

OK Cancel

Original Matrix (and Perhaps Right-Hand Side):

```
+1.2000000000e+01      +3.0000000000e+00      -5.0000000000e+00      +1.0000000000e+00
+1.0000000000e+00      +5.0000000000e+00      +3.0000000000e+00      +2.8000000000e+01
+3.0000000000e+00      +7.0000000000e+00      +1.3000000000e+01      +7.6000000000e+01
```

Jacobi Iteration Solution Vector:

```
+1.0000000000e+00
+3.0000000000e+00
+4.0000000000e+00
```

Tolerance: 1.000000000e-12

Operations:

```
number of fabs function calls = 375
number of pow function calls = 375
number of sqrt function calls = 125
number of add operations = 1125
number of sub operations = 750
number of div operations = 375
number of mul operations = 750
number of iterations = 125
```


Original Matrix (and Perhaps Right-Hand Side):

+1.2000000000e+01	+3.0000000000e+00	-5.0000000000e+00	+1.0000000000e+00
+1.0000000000e+00	+5.0000000000e+00	+3.0000000000e+00	+2.8000000000e+01
+3.0000000000e+00	+7.0000000000e+00	+1.3000000000e+01	+7.6000000000e+01

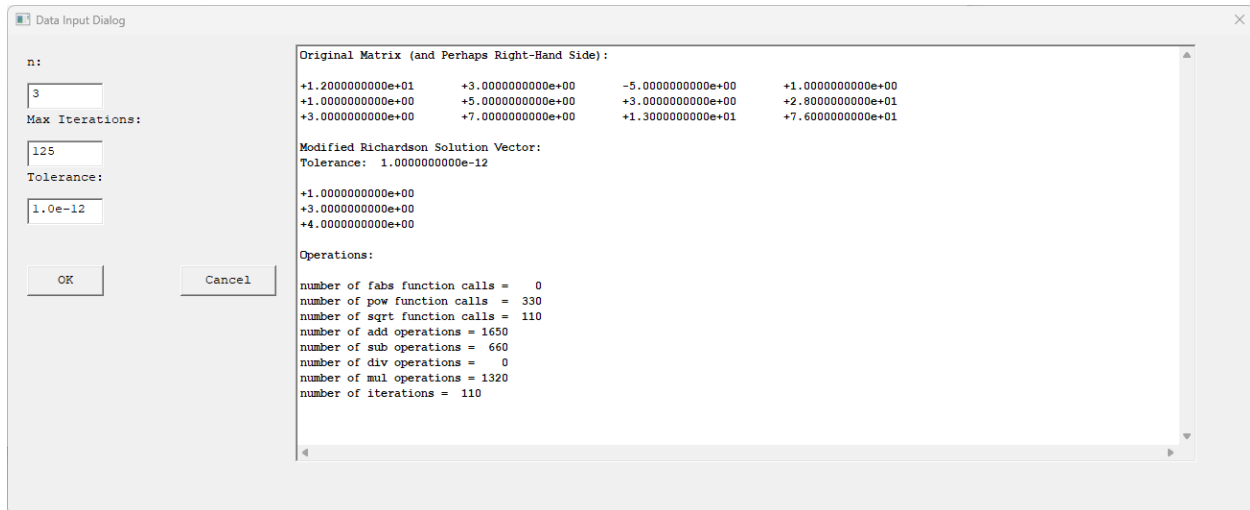
Jacobi Iteration Solution Vector:

Tolerance: 1.0000000000e-12

+1.0000000000e+00
+3.0000000000e+00
+4.0000000000e+00

Operations:

number of fabs function calls = 375
number of pow function calls = 375
number of sqrt function calls = 125
number of add operations = 1125
number of sub operations = 750
number of div operations = 375
number of mul operations = 750
number of iterations = 125



Original Matrix (and Perhaps Right-Hand Side):

+1.2000000000e+01	+3.0000000000e+00	-5.0000000000e+00	+1.0000000000e+00
+1.0000000000e+00	+5.0000000000e+00	+3.0000000000e+00	+2.8000000000e+01
+3.0000000000e+00	+7.0000000000e+00	+1.3000000000e+01	+7.6000000000e+01

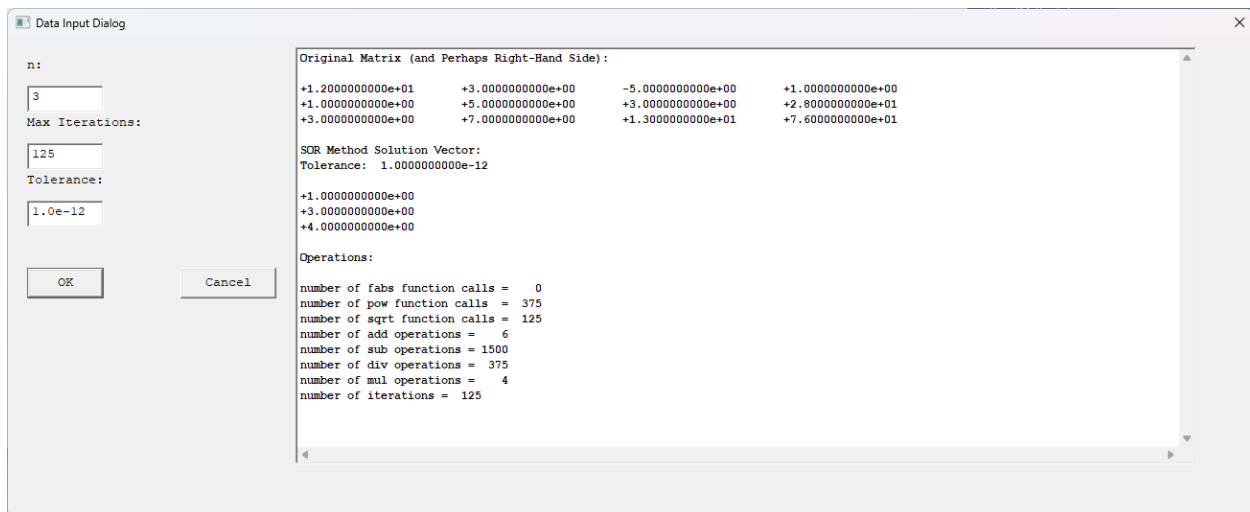
Modified Richardson Solution Vector:

Tolerance: 1.0000000000e-12

+1.0000000000e+00
+3.0000000000e+00
+4.0000000000e+00

Operations:

number of fabs function calls = 0
number of pow function calls = 330
number of sqrt function calls = 110
number of add operations = 1650
number of sub operations = 660
number of div operations = 0
number of mul operations = 1320
number of iterations = 110



Original Matrix (and Perhaps Right-Hand Side):

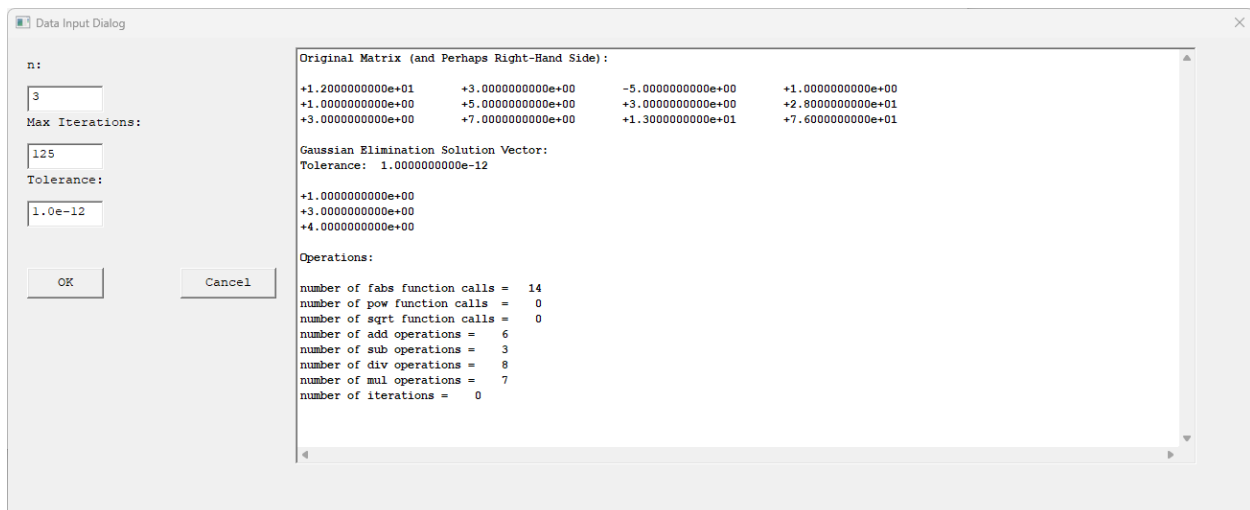
+1.2000000000e+01	+3.0000000000e+00	-5.0000000000e+00	+1.0000000000e+00
+1.0000000000e+00	+5.0000000000e+00	+3.0000000000e+00	+2.8000000000e+01
+3.0000000000e+00	+7.0000000000e+00	+1.3000000000e+01	+7.6000000000e+01

SOR Method Solution Vector:
Tolerance: 1.0000000000e-12

+1.0000000000e+00
+3.0000000000e+00
+4.0000000000e+00

Operations:

number of fabs function calls = 0
number of pow function calls = 375
number of sqrt function calls = 125
number of add operations = 6
number of sub operations = 1500
number of div operations = 375
number of mul operations = 4
number of iterations = 125



Original Matrix (and Perhaps Right-Hand Side):

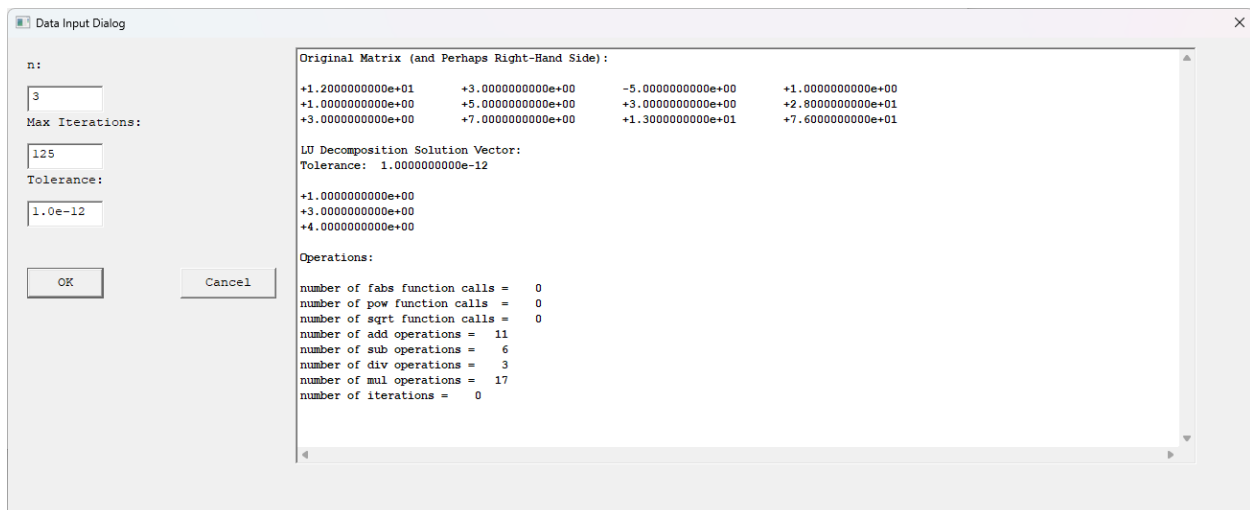
+1.2000000000e+01	+3.0000000000e+00	-5.0000000000e+00	+1.0000000000e+00
+1.0000000000e+00	+5.0000000000e+00	+3.0000000000e+00	+2.8000000000e+01
+3.0000000000e+00	+7.0000000000e+00	+1.3000000000e+01	+7.6000000000e+01

Gaussian Elimination Solution Vector:
Tolerance: 1.0000000000e-12

+1.0000000000e+00
+3.0000000000e+00
+4.0000000000e+00

Operations:

number of fabs function calls = 14
number of pow function calls = 0
number of sqrt function calls = 0
number of add operations = 6
number of sub operations = 3
number of div operations = 8
number of mul operations = 7
number of iterations = 0



Original Matrix (and Perhaps Right-Hand Side):

+1.2000000000e+01	+3.0000000000e+00	-5.0000000000e+00	+1.0000000000e+00
+1.0000000000e+00	+5.0000000000e+00	+3.0000000000e+00	+2.8000000000e+01
+3.0000000000e+00	+7.0000000000e+00	+1.3000000000e+01	+7.6000000000e+01

LU Decomposition Solution Vector:

Tolerance: 1.0000000000e-12

+1.0000000000e+00
+3.0000000000e+00
+4.0000000000e+00

Operations:

number of fabs function calls =	0
number of pow function calls =	0
number of sqrt function calls =	0
number of add operations =	11
number of sub operations =	6
number of div operations =	3
number of mul operations =	17
number of iterations =	0