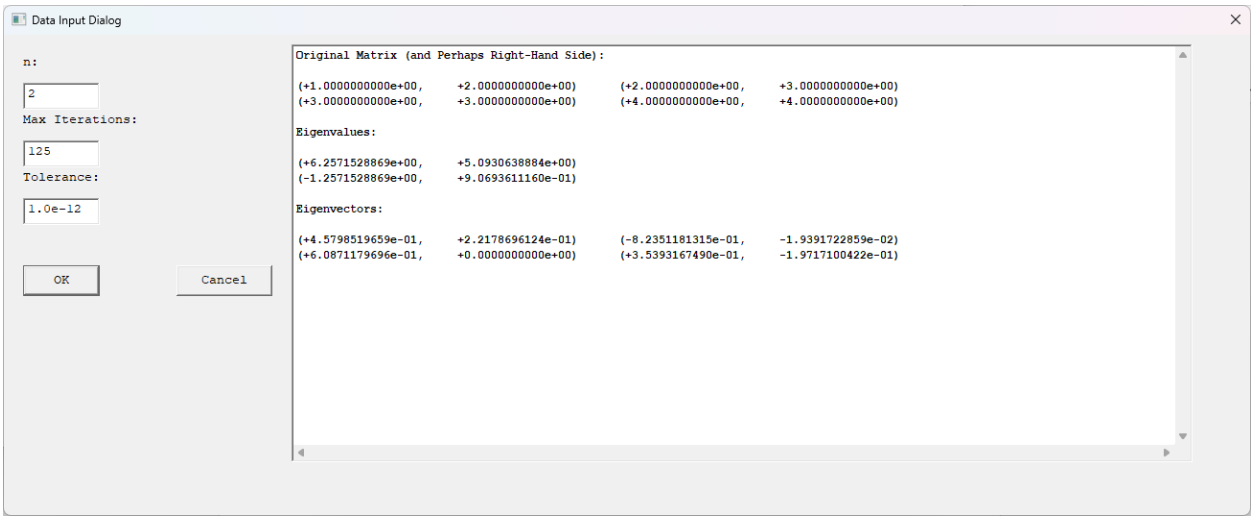
Blog Entry © Friday, August 29, 2025, by James Pate Williams, Jr., Eigenvalue and Eigenvector Calculators and Linear System of Equations Solver

References: 1. ***Elementary Numerical Analysis an Algorithmic Approach Third Edition*** © 1980 by S. D. Conte and Carl de Boor and 2. ***A Numerical Library in C for Scientists and Engineers*** © 1995 by H. T. Lau, Ph.D.

1. Eigenvalue and Eigenvector Calculators
   a. Complex Eigenvalues and Eigenvectors
   b. Real Eigenvalues and Eigenvectors
2. Linear System of Equations Solvers
   a. Classical Iterative Methods
      I. Conjugate Gradient Method
      II. Gauss-Seidel Iteration
      III. Gradient Method
      IV. Jacobi Iteration
      V. Modified Richardson
      VI. Successive Overrelaxation (SOR)
   b. Direct Methods
      I. Gaussian Elimination
      II. LU Decomposition

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Original Matrix (and Perhaps Right-Hand Side):

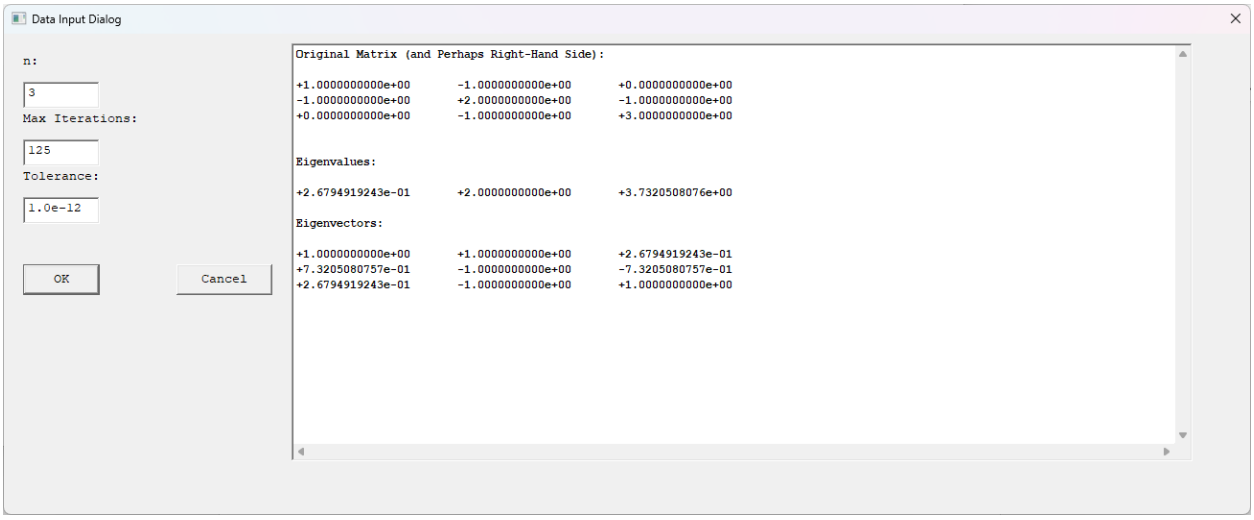
(+1.0000000000e+00,    +2.0000000000e+00)     (+2.0000000000e+00,    +3.0000000000e+00)
(+3.0000000000e+00,    +3.0000000000e+00)     (+4.0000000000e+00,    +4.0000000000e+00)

Eigenvalues:

(+6.2571528869e+00,    +5.0930638884e+00)
(-1.2571528869e+00,    +9.0693611160e-01)
```

Eigenvectors:

```
(+4.5798519659e-01,    +2.2178696124e-01)     (-8.2351181315e-01,    -1.9391722859e-02)
(+6.0871179696e-01,    +0.0000000000e+00)     (+3.5393167490e-01,    -1.9717100422e-01)
```



Original Matrix (and Perhaps Right-Hand Side):

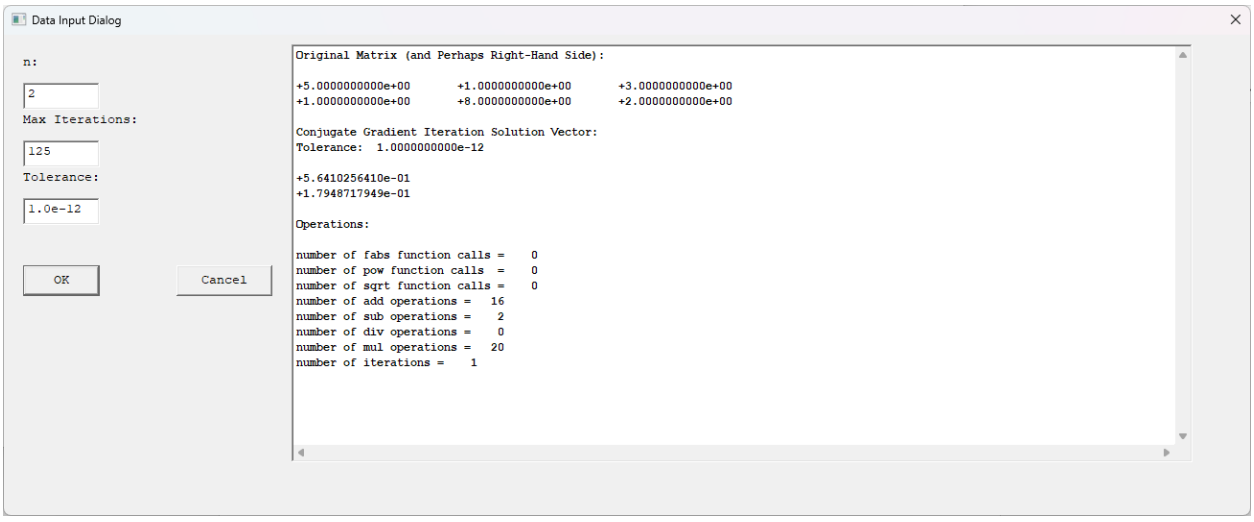
```
+1.0000000000e+00      -1.0000000000e+00      +0.0000000000e+00
-1.0000000000e+00      +2.0000000000e+00      -1.0000000000e+00
+0.0000000000e+00      -1.0000000000e+00      +3.0000000000e+00
```

Eigenvalues:

```
+2.6794919243e-01      +2.0000000000e+00      +3.7320508076e+00
```

Eigenvectors:

```
+1.0000000000e+00      +1.0000000000e+00      +2.6794919243e-01
+7.3205080757e-01      -1.0000000000e+00      -7.3205080757e-01
+2.6794919243e-01      -1.0000000000e+00      +1.0000000000e+00
```



Original Matrix (and Perhaps Right-Hand Side):

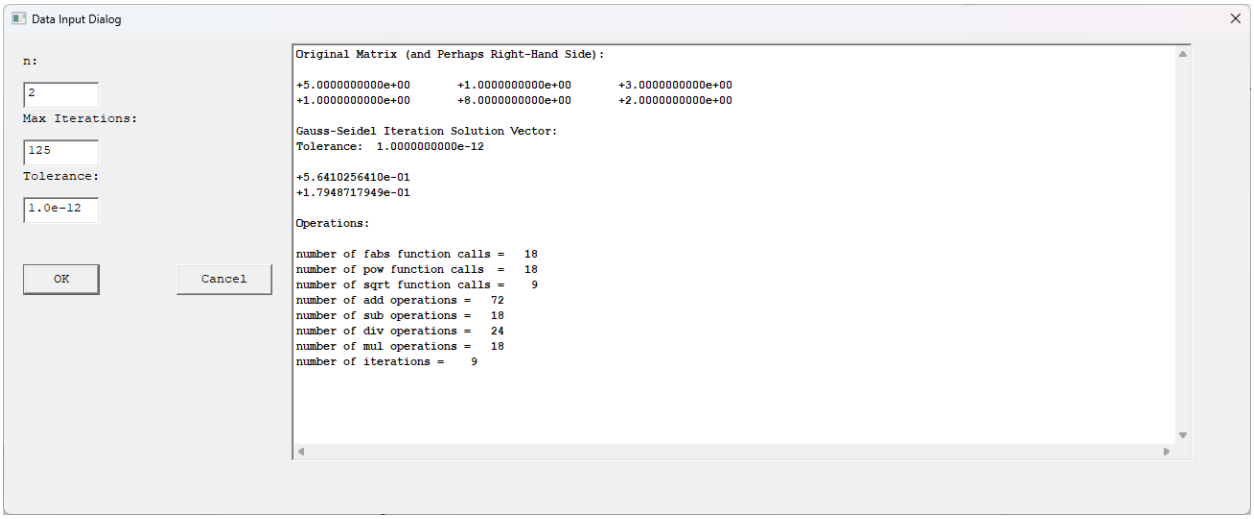
```
+5.0000000000e+00      +1.0000000000e+00      +3.0000000000e+00
+1.0000000000e+00      +8.0000000000e+00      +2.0000000000e+00
```

```
Conjugate Gradient Iteration Solution Vector:
Tolerance:  1.0000000000e-12

+5.6410256410e-01
+1.7948717949e-01

Operations:

number of fabs function calls =    0
number of pow function calls  =    0
number of sqrt function calls =    0
number of add operations =   16
number of sub operations =    2
number of div operations =    0
number of mul operations =   20
number of iterations =    1
```




```
Original Matrix (and Perhaps Right-Hand Side):

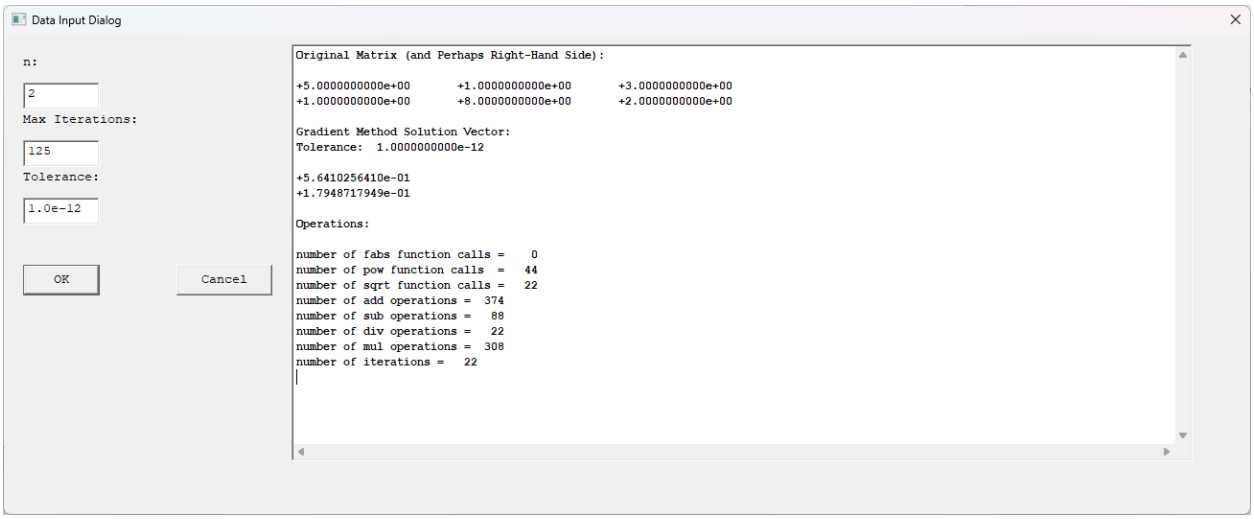
+5.0000000000e+00      +1.0000000000e+00      +3.0000000000e+00
+1.0000000000e+00      +8.0000000000e+00      +2.0000000000e+00

Gauss-Seidel Iteration Solution Vector:
Tolerance:  1.0000000000e-12

+5.6410256410e-01
+1.7948717949e-01

Operations:

number of fabs function calls =   18
number of pow function calls  =   18
number of sqrt function calls =    9
number of add operations =   72
number of sub operations =   18
number of div operations =   24
number of mul operations =   18
number of iterations =    9
```



Original Matrix (and Perhaps Right-Hand Side):

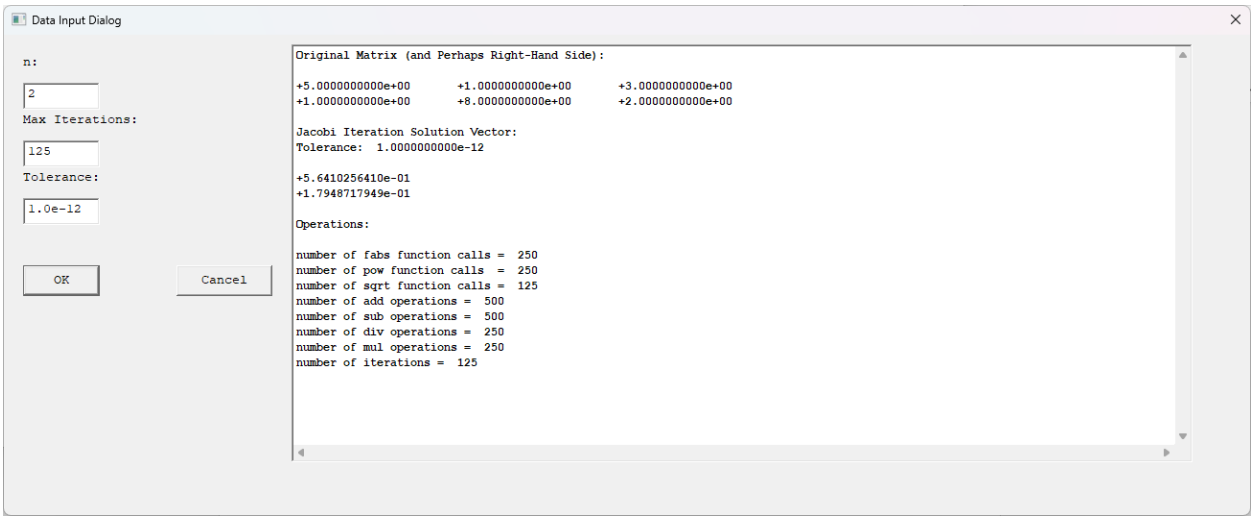
+5.0000000000e+00 +1.0000000000e+00 +3.0000000000e+00
+1.0000000000e+00 +8.0000000000e+00 +2.0000000000e+00

Gradient Method Solution Vector:
Tolerance: 1.0000000000e-12

+5.6410256410e-01
+1.7948717949e-01

Operations:

number of fabs function calls = 0
number of pow function calls = 44
number of sqrt function calls = 22
number of add operations = 374
number of sub operations = 88
number of div operations = 22
number of mul operations = 308
number of iterations = 22



Original Matrix (and Perhaps Right-Hand Side):

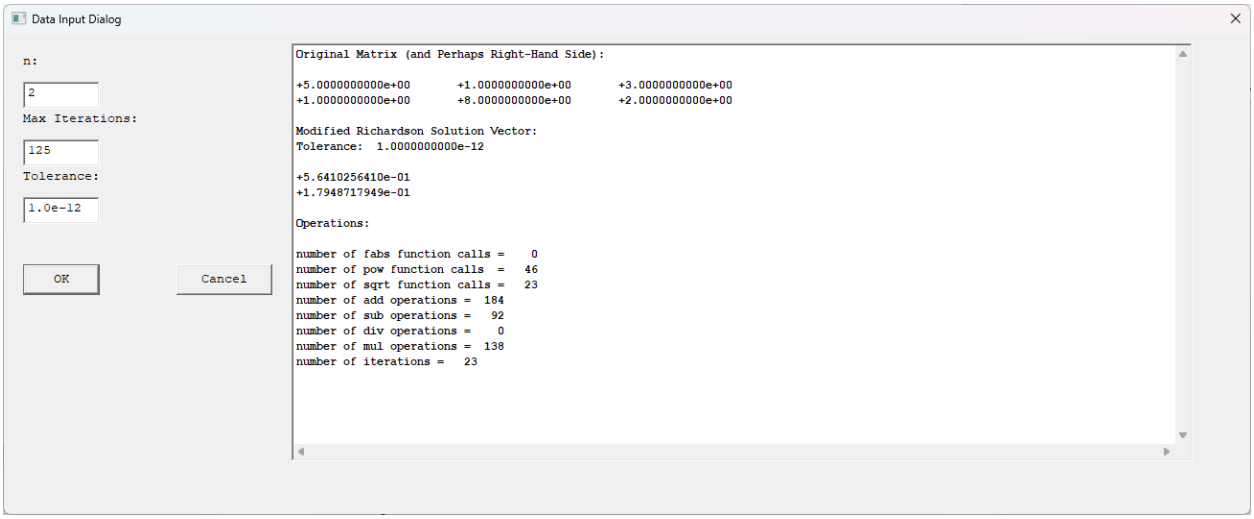
+5.0000000000e+00 +1.0000000000e+00 +3.0000000000e+00
+1.0000000000e+00 +8.0000000000e+00 +2.0000000000e+00

```
Jacobi Iteration Solution Vector:
Tolerance:  1.0000000000e-12

+5.6410256410e-01
+1.7948717949e-01

Operations:

number of fabs function calls =  250
number of pow function calls  =  250
number of sqrt function calls =  125
number of add operations =  500
number of sub operations =  500
number of div operations =  250
number of mul operations =  250
number of iterations =  125
```




```
Original Matrix (and Perhaps Right-Hand Side):

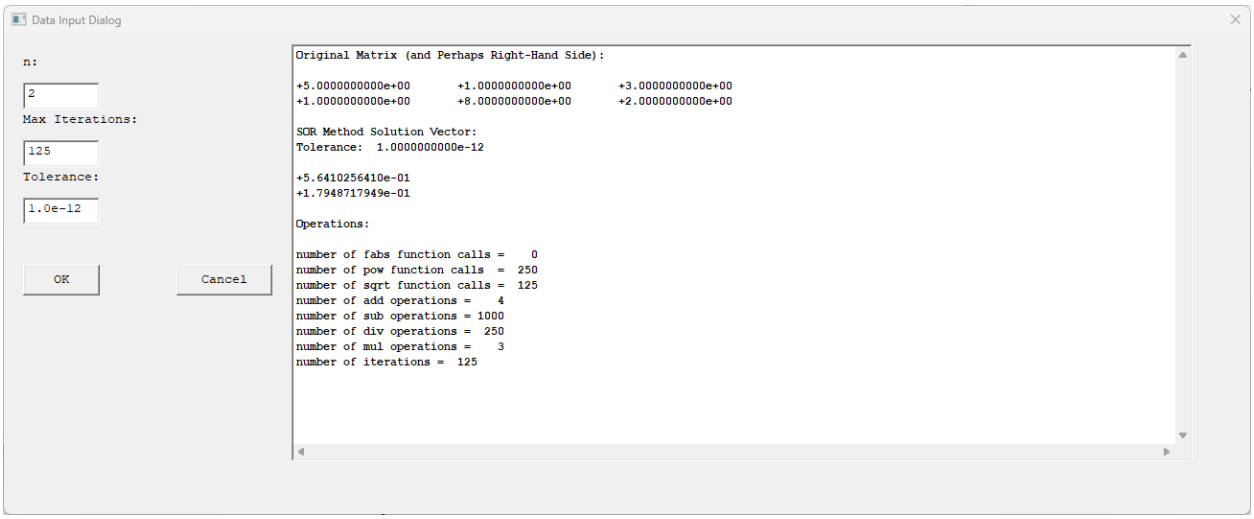
+5.0000000000e+00      +1.0000000000e+00      +3.0000000000e+00
+1.0000000000e+00      +8.0000000000e+00      +2.0000000000e+00

Modified Richardson Solution Vector:
Tolerance:  1.0000000000e-12

+5.6410256410e-01
+1.7948717949e-01

Operations:

number of fabs function calls =    0
number of pow function calls  =   46
number of sqrt function calls =   23
number of add operations =  184
number of sub operations =   92
number of div operations =    0
number of mul operations =  138
number of iterations =   23
```



Original Matrix (and Perhaps Right-Hand Side):

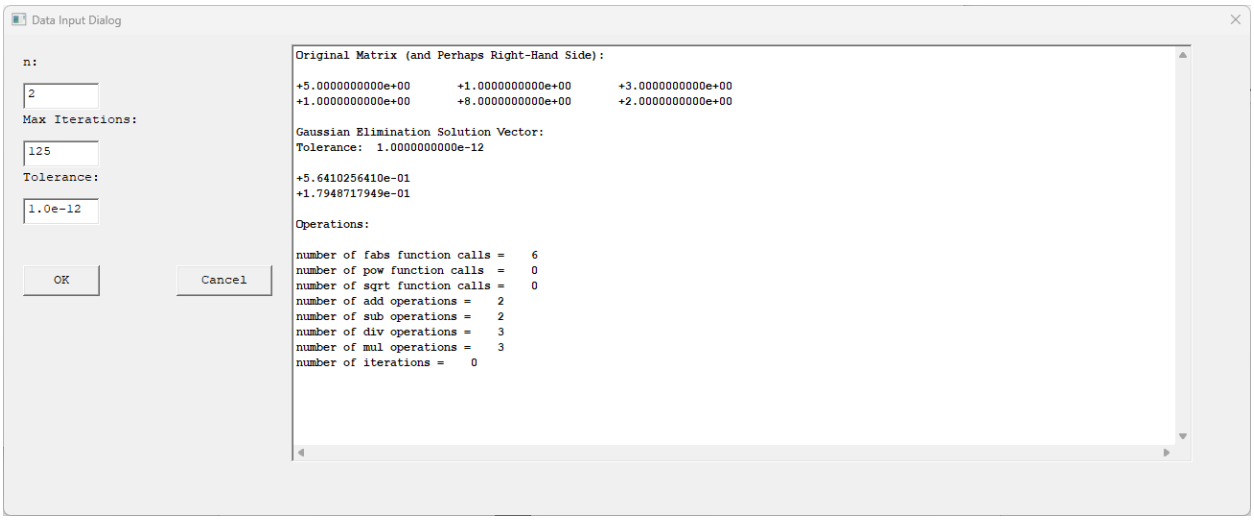
+5.0000000000e+00 +1.0000000000e+00 +3.0000000000e+00
+1.0000000000e+00 +8.0000000000e+00 +2.0000000000e+00

SOR Method Solution Vector:
Tolerance: 1.0000000000e-12

+5.6410256410e-01
+1.7948717949e-01

Operations:

number of fabs function calls = 0
number of pow function calls = 250
number of sqrt function calls = 125
number of add operations = 4
number of sub operations = 1000
number of div operations = 250
number of mul operations = 3
number of iterations = 125



Original Matrix (and Perhaps Right-Hand Side):

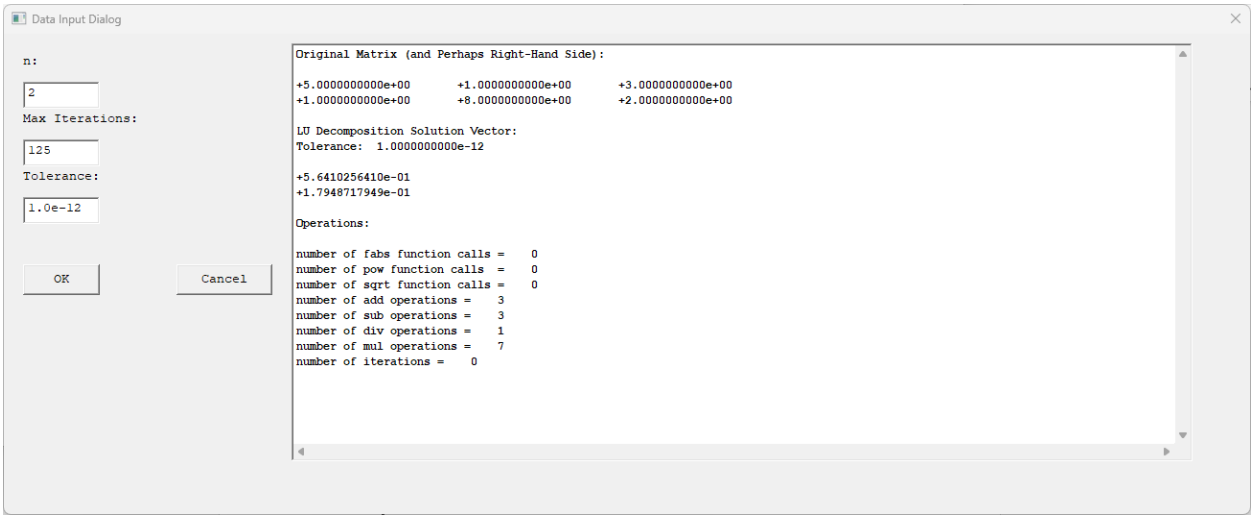
+5.0000000000e+00 +1.0000000000e+00 +3.0000000000e+00
+1.0000000000e+00 +8.0000000000e+00 +2.0000000000e+00

```
Gaussian Elimination Solution Vector:
Tolerance:  1.0000000000e-12

+5.6410256410e-01
+1.7948717949e-01

Operations:

number of fabs function calls =    6
number of pow function calls  =    0
number of sqrt function calls =    0
number of add operations =    2
number of sub operations =    2
number of div operations =    3
number of mul operations =    3
number of iterations =    0
```




```
Original Matrix (and Perhaps Right-Hand Side):

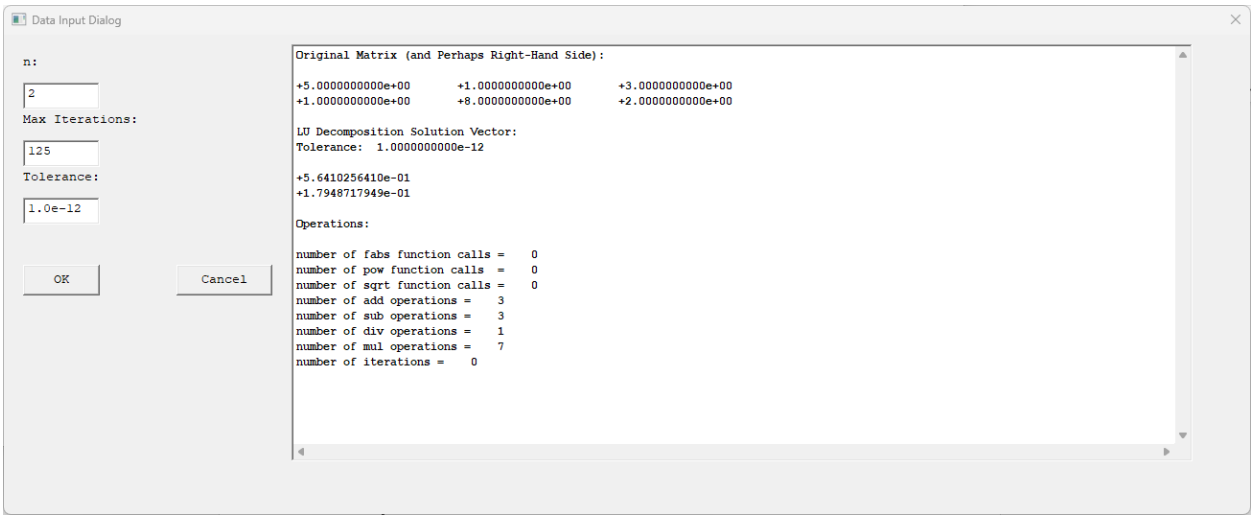
+5.0000000000e+00      +1.0000000000e+00      +3.0000000000e+00
+1.0000000000e+00      +8.0000000000e+00      +2.0000000000e+00

LU Decomposition Solution Vector:
Tolerance:  1.0000000000e-12

+5.6410256410e-01
+1.7948717949e-01

Operations:

number of fabs function calls =    0
number of pow function calls  =    0
number of sqrt function calls =    0
number of add operations =    3
number of sub operations =    3
number of div operations =    1
number of mul operations =    7
number of iterations =    0
```



Original Matrix (and Perhaps Right-Hand Side):

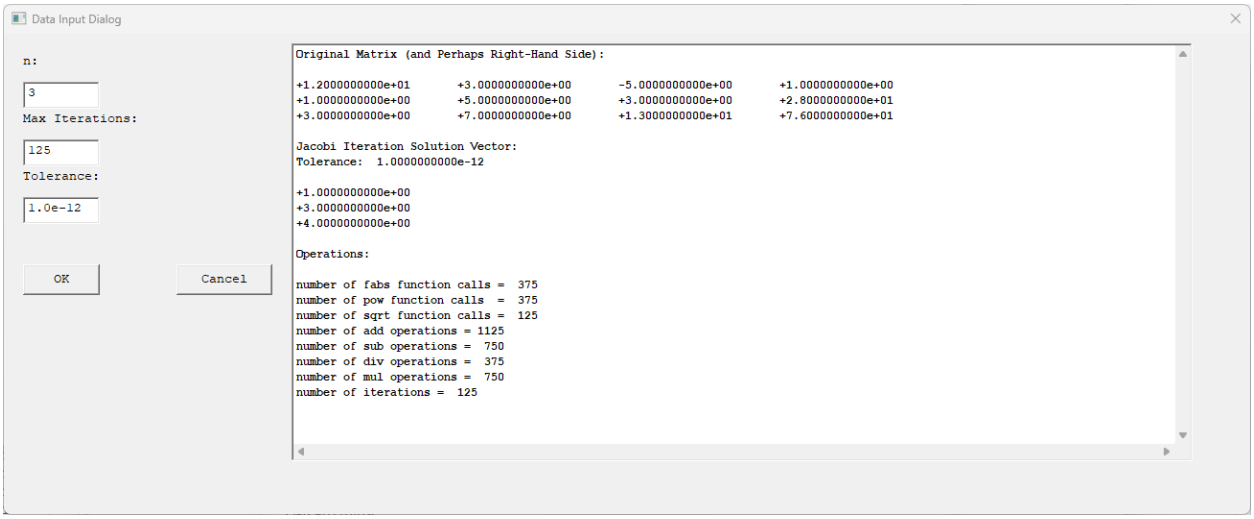
```
+1.2000000000e+01      +3.0000000000e+00      -5.0000000000e+00      +1.0000000000e+00
+1.0000000000e+00      +5.0000000000e+00      +3.0000000000e+00      +2.8000000000e+01
+3.0000000000e+00      +7.0000000000e+00      +1.3000000000e+01      +7.6000000000e+01
```

Gauss-Seidel Iteration Solution Vector:
Tolerance: 1.0000000000e-12

```
+1.0000000000e+00
+3.0000000000e+00
+4.0000000000e+00
```

Operations:

```
number of fabs function calls =   60
number of pow function calls  =   60
number of sqrt function calls =   20
number of add operations =  300
number of sub operations =   60
number of div operations =   72
number of mul operations =  120
number of iterations =   20
```



```
Original Matrix (and Perhaps Right-Hand Side):

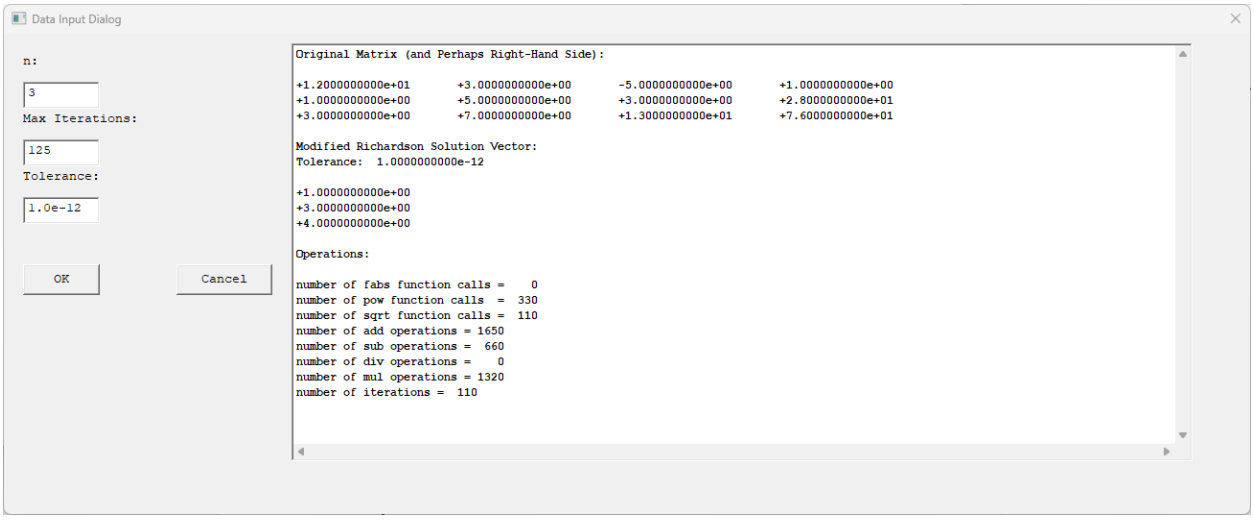
+1.2000000000e+01      +3.0000000000e+00      -5.0000000000e+00      +1.0000000000e+00
+1.0000000000e+00      +5.0000000000e+00      +3.0000000000e+00      +2.8000000000e+01
+3.0000000000e+00      +7.0000000000e+00      +1.3000000000e+01      +7.6000000000e+01

Jacobi Iteration Solution Vector:
Tolerance:  1.0000000000e-12

+1.0000000000e+00
+3.0000000000e+00
+4.0000000000e+00

Operations:

number of fabs function calls =  375
number of pow function calls  =  375
number of sqrt function calls =  125
number of add operations = 1125
number of sub operations =  750
number of div operations =  375
number of mul operations =  750
number of iterations =  125
```



```
Original Matrix (and Perhaps Right-Hand Side):

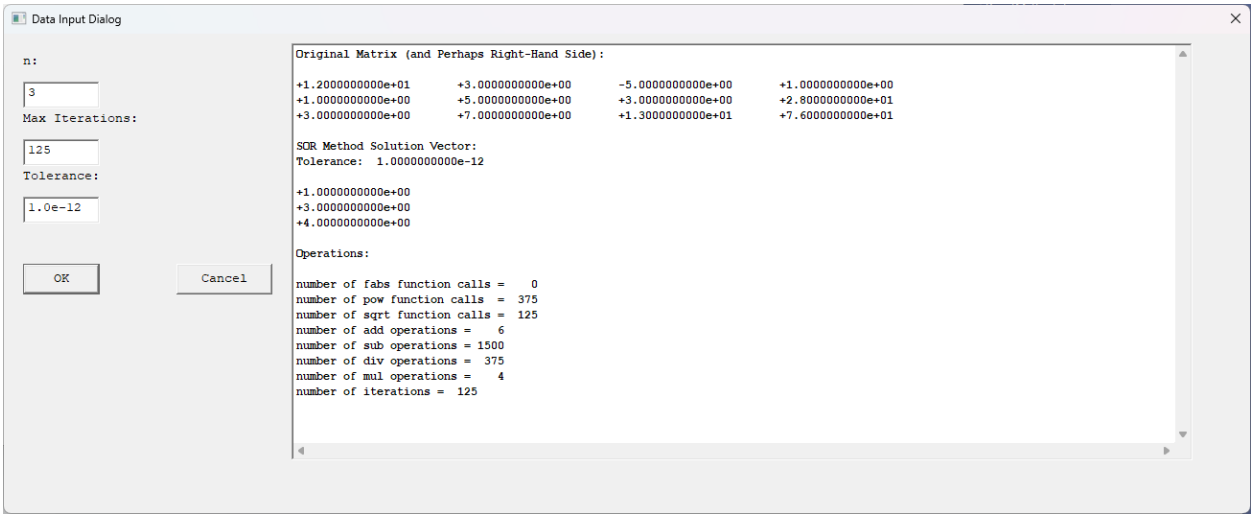
+1.2000000000e+01      +3.0000000000e+00      -5.0000000000e+00      +1.0000000000e+00
+1.0000000000e+00      +5.0000000000e+00      +3.0000000000e+00      +2.8000000000e+01
+3.0000000000e+00      +7.0000000000e+00      +1.3000000000e+01      +7.6000000000e+01

Modified Richardson Solution Vector:
Tolerance:  1.0000000000e-12

+1.0000000000e+00
+3.0000000000e+00
+4.0000000000e+00

Operations:

number of fabs function calls =    0
number of pow function calls  =  330
number of sqrt function calls =  110
number of add operations = 1650
number of sub operations =  660
number of div operations =    0
number of mul operations = 1320
number of iterations =  110
```



```
Original Matrix (and Perhaps Right-Hand Side):

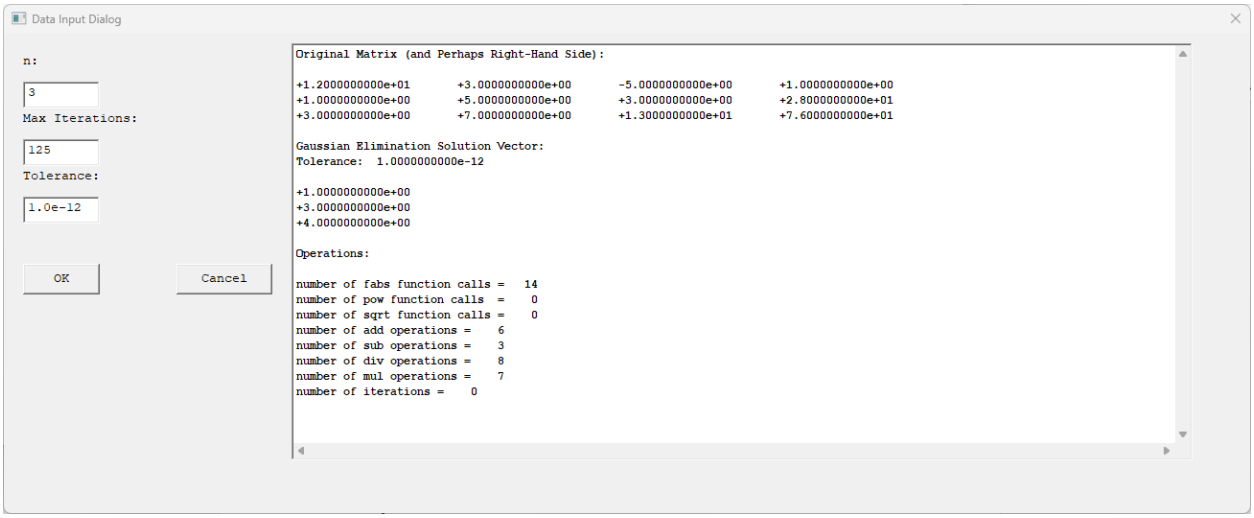
+1.2000000000e+01      +3.0000000000e+00      -5.0000000000e+00      +1.0000000000e+00
+1.0000000000e+00      +5.0000000000e+00      +3.0000000000e+00      +2.8000000000e+01
+3.0000000000e+00      +7.0000000000e+00      +1.3000000000e+01      +7.6000000000e+01

SOR Method Solution Vector:
Tolerance:  1.0000000000e-12

+1.0000000000e+00
+3.0000000000e+00
+4.0000000000e+00

Operations:

number of fabs function calls =    0
number of pow function calls  =  375
number of sqrt function calls =  125
number of add operations =    6
number of sub operations = 1500
number of div operations =  375
number of mul operations =    4
number of iterations =  125
```



```
Original Matrix (and Perhaps Right-Hand Side):

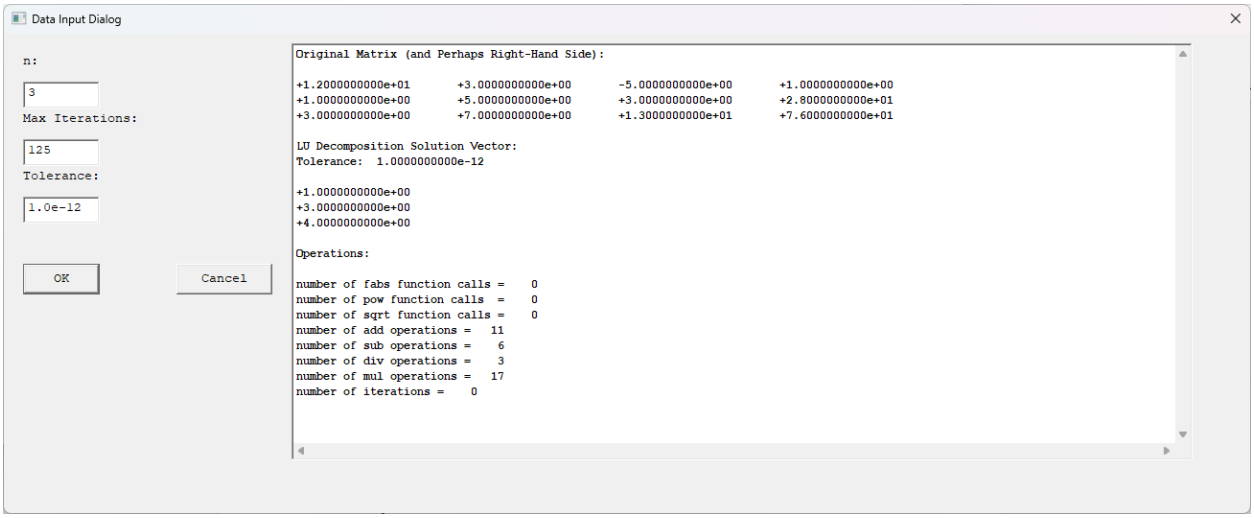
+1.2000000000e+01      +3.0000000000e+00      -5.0000000000e+00      +1.0000000000e+00
+1.0000000000e+00      +5.0000000000e+00      +3.0000000000e+00      +2.8000000000e+01
+3.0000000000e+00      +7.0000000000e+00      +1.3000000000e+01      +7.6000000000e+01

Gaussian Elimination Solution Vector:
Tolerance:  1.0000000000e-12

+1.0000000000e+00
+3.0000000000e+00
+4.0000000000e+00

Operations:

number of fabs function calls =   14
number of pow function calls  =    0
number of sqrt function calls =    0
number of add operations =    6
number of sub operations =    3
number of div operations =    8
number of mul operations =    7
number of iterations =    0
```



```
Original Matrix (and Perhaps Right-Hand Side):

+1.2000000000e+01      +3.0000000000e+00      -5.0000000000e+00      +1.0000000000e+00
+1.0000000000e+00      +5.0000000000e+00      +3.0000000000e+00      +2.8000000000e+01
+3.0000000000e+00      +7.0000000000e+00      +1.3000000000e+01      +7.6000000000e+01

LU Decomposition Solution Vector:
Tolerance:  1.0000000000e-12

+1.0000000000e+00
+3.0000000000e+00
+4.0000000000e+00

Operations:

number of fabs function calls =    0
number of pow function calls  =    0
number of sqrt function calls =    0
number of add operations =   11
number of sub operations =    6
number of div operations =    3
number of mul operations =   17
number of iterations =    0
```