Blog Entry © Tuesday, August 19, 2025, by James Pate Williams, Jr., Continuation of Answers to the Exercises in Chapter 1 of ***Modern Quantum Chemistry an Introduction to Advanced Electronic Structure Theory*** by Atilla Szabo and Neil S. Ostlund

modern-quantum-chemistry.pdf

Exercise 1.5 Properties of 2x2 Determinants

$$\begin{vmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{vmatrix} = A_{11}A_{22} - A_{12}A_{21}$$

$$\begin{vmatrix} 0 & 0 \\ A_{21} & A_{22} \end{vmatrix} = 0$$

$$\begin{vmatrix} A_{11} & 0 \\ A_{21} & 0 \end{vmatrix} = 0$$

$$A^{adj} = \begin{vmatrix} A_{11}^* & A_{21}^* \\ A_{12}^* & A_{22}^* \end{vmatrix}$$

$$\left(A^{adj}\right)^* = \begin{vmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{vmatrix} = |A|$$

$$\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}\begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} = \begin{pmatrix} A_{11}B_{11} + A_{12}B_{21} & A_{11}B_{12} + A_{12}B_{22} \\ A_{21}B_{11} + A_{22}B_{21} & A_{21}B_{12} + A_{22}B_{22} \end{pmatrix}$$

$$\begin{aligned}
&\begin{vmatrix} A_{11}B_{11} + A_{12}B_{21} & A_{11}B_{12} + A_{12}B_{22} \\ A_{21}B_{11} + A_{22}B_{21} & A_{21}B_{12} + A_{22}B_{22} \end{vmatrix} \\
&= (A_{11}B_{11} + A_{12}B_{21})(A_{21}B_{12} + A_{22}B_{22}) \\
&- (A_{11}B_{12} + A_{12}B_{22})(A_{21}B_{11} + A_{22}B_{21}) \\
&= A_{11}B_{11}A_{21}B_{12} + A_{11}B_{11}A_{22}B_{22} + A_{12}B_{21}A_{21}B_{12} + A_{12}B_{21}A_{22}B_{22} \\
&- A_{11}B_{12}A_{21}B_{11} - A_{11}B_{12}A_{22}B_{21} - A_{12}B_{22}A_{21}B_{11} - A_{12}B_{22}A_{22}B_{21} \\
&= A_{11}B_{11}A_{22}B_{22} - A_{11}B_{12}A_{22}B_{21} + A_{12}B_{21}A_{21}B_{12} - A_{12}B_{22}A_{21}B_{11} \\
&= A_{11}A_{22}(B_{11}B_{22} - B_{12}B_{21}) + A_{12}A_{21}(B_{12}B_{21} - B_{11}B_{22}) \\
&= A_{11}A_{22}|B| - A_{12}A_{21}|B| = |A||B|
\end{aligned}$$

$$\begin{aligned}
&\begin{vmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{vmatrix}\begin{vmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{vmatrix} = (A_{11}A_{22} - A_{12}A_{21})(B_{11}B_{22} - B_{12}B_{21}) \\
&= A_{11}A_{22}B_{11}B_{22} - A_{11}A_{22}B_{12}B_{21} - A_{12}A_{21}B_{11}B_{22} + A_{12}A_{21}B_{12}B_{21} \\
&= A_{11}A_{22}(B_{11}B_{22} - B_{12}B_{21}) + A_{12}A_{21}(B_{12}B_{21} - B_{11}B_{22}) \\
&= A_{11}A_{22}|B| - A_{12}A_{21}|B| = |A||B|
\end{aligned}$$