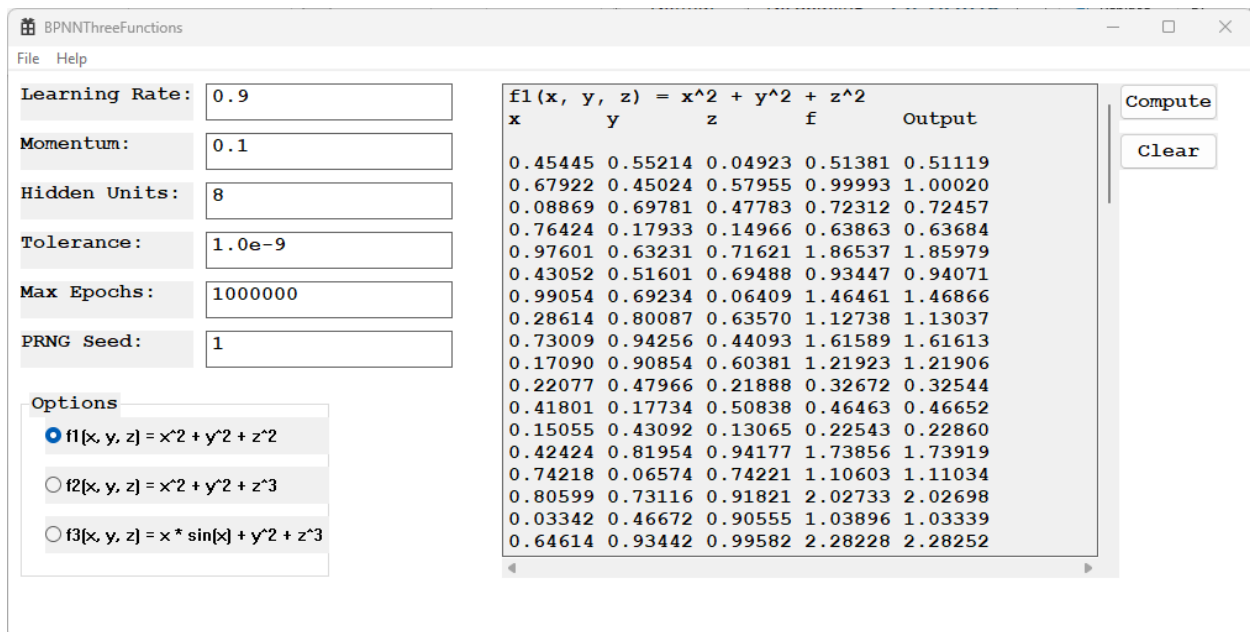
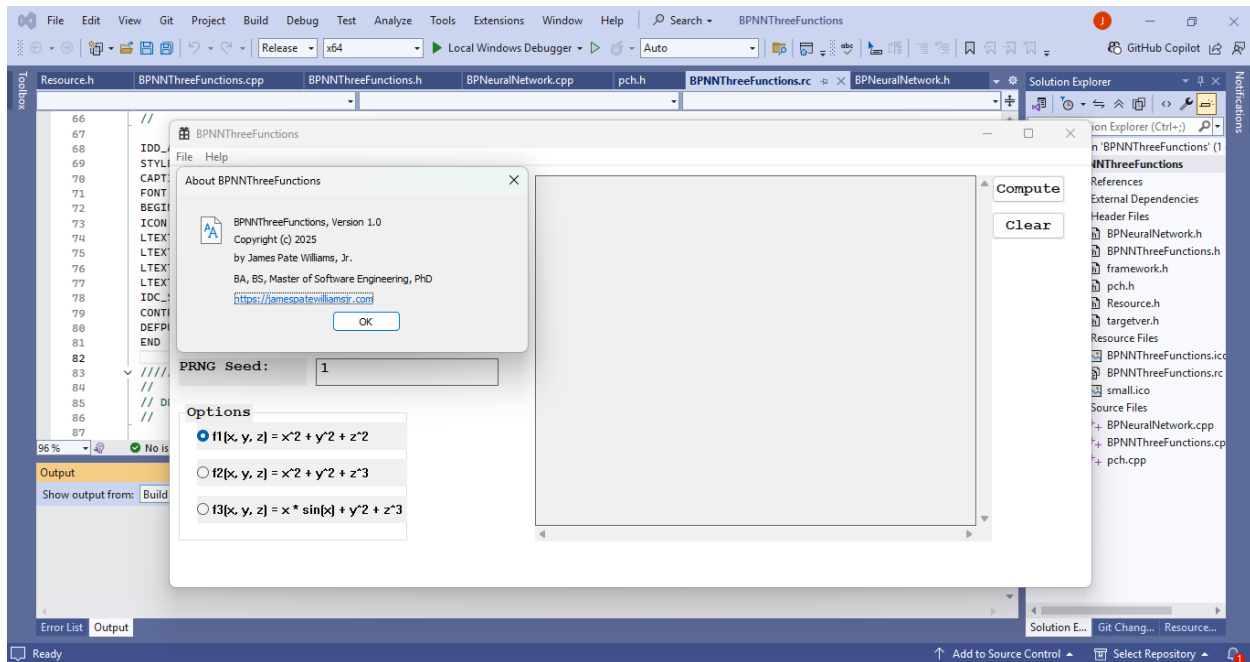


Blog Entry © Wednesday, December 10, 2025, by James Pate Williams, Jr. Backpropagation Neural Network to Learn Three Continuous Functions

References: **Machine Learning** © 1997 by Tom M. Mitchell and **Artificial Intelligence A Modern Approach Second Edition** © 2003 by Stuart Russell and Peter Norvig



$$f1(x, y, z) = x^2 + y^2 + z^2$$

x	y	z	f	Output
0.45445	0.55214	0.04923	0.51381	0.51119
0.67922	0.45024	0.57955	0.99993	1.00020
0.08869	0.69781	0.47783	0.72312	0.72457
0.76424	0.17933	0.14966	0.63863	0.63684
0.97601	0.63231	0.71621	1.86537	1.85979
0.43052	0.51601	0.69488	0.93447	0.94071
0.99054	0.69234	0.06409	1.46461	1.46866
0.28614	0.80087	0.63570	1.12738	1.13037
0.73009	0.94256	0.44093	1.61589	1.61613
0.17090	0.90854	0.60381	1.21923	1.21906
0.22077	0.47966	0.21888	0.32672	0.32544
0.41801	0.17734	0.50838	0.46463	0.46652
0.15055	0.43092	0.13065	0.22543	0.22860
0.42424	0.81954	0.94177	1.73856	1.73919
0.74218	0.06574	0.74221	1.10603	1.11034
0.80599	0.73116	0.91821	2.02733	2.02698
0.03342	0.46672	0.90555	1.03896	1.03339
0.64614	0.93442	0.99582	2.28228	2.28252
0.16349	0.71364	0.07691	0.54193	0.54506
0.92599	0.12210	0.12354	0.88763	0.88898
0.39027	0.15534	0.52898	0.45626	0.45537
0.80740	0.05701	0.70531	1.15261	1.15069
0.11341	0.69677	0.42354	0.67773	0.67326
0.61315	0.03351	0.00751	0.37713	0.38075
0.27235	0.74096	0.57472	0.95350	0.95498
0.76836	0.36167	0.78133	1.33168	1.33401
0.25718	0.12458	0.63887	0.48982	0.48832
0.24216	0.00702	0.44856	0.25990	0.25792
0.03418	0.69076	0.54875	0.77944	0.77390
0.78817	0.24055	0.03900	0.68060	0.67582
0.67421	0.68413	0.38603	1.07162	1.07071
0.15122	0.54512	0.88815	1.10883	1.11118
0.00522	0.55489	0.68490	0.77701	0.78155
0.01221	0.21570	0.53670	0.33472	0.33649
0.81603	0.64125	0.50551	1.33266	1.33842
0.40425	0.19971	0.93945	1.08587	1.07912
0.03867	0.19526	0.63341	0.44083	0.43838
0.28236	0.88433	0.61519	1.24024	1.24110
0.76312	0.94559	0.07566	1.48220	1.48260
0.48451	0.86605	0.38102	1.12998	1.13273
0.90582	0.47725	0.76055	1.62671	1.62783
0.11267	0.36573	0.43153	0.33268	0.33916
0.99902	0.78268	0.13120	1.62785	1.62327
0.01389	0.00037	0.37883	0.14370	0.13792

0.00482	0.01813	0.27827	0.07778	0.07805
0.17524	0.21943	0.08032	0.08531	0.09516
0.42106	0.41707	0.50890	0.61021	0.60222
0.09104	0.98022	0.28043	1.04777	1.04425
0.35301	0.08603	0.92300	0.98395	0.98746
0.13935	0.30219	0.89691	0.91518	0.92533
0.80605	0.67022	0.78878	1.72109	1.72654
0.34245	0.32569	0.82550	0.90479	0.90134
0.33061	0.24262	0.40159	0.32944	0.33292
0.20808	0.53374	0.03568	0.32944	0.32926
0.44978	0.87689	0.79995	1.61116	1.60635
0.28043	0.00153	0.11563	0.09202	0.08881
0.14576	0.82418	0.32398	0.80549	0.81026
0.19565	0.03909	0.91809	0.88270	0.88124
0.19913	0.66399	0.65944	0.91540	0.91649
0.06119	0.51720	0.18836	0.30672	0.30011
0.48848	0.85391	0.84072	1.67459	1.67650
0.48991	0.63677	0.64272	1.05858	1.04909
0.59551	0.39503	0.81320	1.17197	1.16665
0.23707	0.04511	0.53761	0.34726	0.34943

Mean Square Error = 0.000002

Unseen Examples

x	y	z	f	Output
0.47990	0.70425	0.74300	1.27832	1.27898
0.95276	0.34837	0.62481	1.41949	1.41992
0.59691	0.27494	0.42582	0.61322	0.60542
0.53212	0.97311	0.63439	1.63255	1.63691
0.85342	0.85751	0.40727	1.62952	1.64494
0.68780	0.22315	0.33067	0.63220	0.62444
0.29228	0.07706	0.55007	0.39394	0.39812
0.47371	0.00302	0.80081	0.86570	0.86128
0.11237	0.71615	0.36995	0.66235	0.66328
0.07038	0.46254	0.52132	0.49067	0.49565
0.26597	0.92166	0.41450	1.09201	1.10660
0.19514	0.76623	0.04932	0.62762	0.62915
0.68166	0.53691	0.26704	0.82425	0.80659
0.13712	0.98785	0.33915	1.10968	1.11963
0.11176	0.55867	0.17563	0.35545	0.35725
0.41765	0.13013	0.68349	0.65852	0.65721

Mean Square Error = 0.000017

Runtime (s) = 44.248000

BPNNThreeFunctions

File Help

Learning Rate:

0.9

Momentum:

0.1

Hidden Units:

8

Tolerance:

1.0e-9

Max Epochs:

1000000

PRNG Seed:

1

Options

☐ f1[x, y, z] = x^2 + y^2 + z^2

☒ f2[x, y, z] = x^2 + y^2 + z^3

☐ f3[x, y, z] = x * sin[x] + y^2 + z^3

f2(x, y, z) = x^2 + y^2 + z^3

x	y	z	f	Output
0.45445	0.55214	0.04923	0.75943	0.75851
0.67922	0.45024	0.57955	1.07659	1.07371
0.08869	0.69781	0.47783	0.68472	0.68313
0.76424	0.17933	0.14966	0.79975	0.80516
0.97601	0.63231	0.71621	1.74321	1.74326
0.43052	0.51601	0.69488	1.03231	1.03661
0.99054	0.69234	0.06409	1.47014	1.47094
0.28614	0.80087	0.63570	1.18443	1.18043
0.73009	0.94256	0.44093	1.70425	1.69946
0.17090	0.90854	0.60381	1.21648	1.21984
0.22077	0.47966	0.21888	0.46133	0.45623
0.41801	0.17734	0.50838	0.58085	0.57950
0.15055	0.43092	0.13065	0.33847	0.34145
0.42424	0.81954	0.94177	1.93118	1.92995
0.74218	0.06574	0.74221	1.15537	1.15304
0.80599	0.73116	0.91821	2.11475	2.11483
0.03342	0.46672	0.90555	0.99380	0.99116
0.64614	0.93442	0.99582	2.50678	2.50620

Compute

Clear

$$f2(x, y, z) = x^2 + y^2 + z^3$$

x	y	z	f	Output
---	---	---	---	--------

0.45445	0.55214	0.04923	0.75943	0.75851
0.67922	0.45024	0.57955	1.07659	1.07371
0.08869	0.69781	0.47783	0.68472	0.68313
0.76424	0.17933	0.14966	0.79975	0.80516
0.97601	0.63231	0.71621	1.74321	1.74326
0.43052	0.51601	0.69488	1.03231	1.03661
0.99054	0.69234	0.06409	1.47014	1.47094
0.28614	0.80087	0.63570	1.18443	1.18043
0.73009	0.94256	0.44093	1.70425	1.69946
0.17090	0.90854	0.60381	1.21648	1.21984
0.22077	0.47966	0.21888	0.46133	0.45623
0.41801	0.17734	0.50838	0.58085	0.57950
0.15055	0.43092	0.13065	0.33847	0.34145
0.42424	0.81954	0.94177	1.93118	1.92995
0.74218	0.06574	0.74221	1.15537	1.15304
0.80599	0.73116	0.91821	2.11475	2.11483
0.03342	0.46672	0.90555	0.99380	0.99116
0.64614	0.93442	0.99582	2.50678	2.50620
0.16349	0.71364	0.07691	0.67323	0.67500
0.92599	0.12210	0.12354	0.94279	0.94016
0.39027	0.15534	0.52898	0.56242	0.56521
0.80740	0.05701	0.70531	1.16152	1.16337
0.11341	0.69677	0.42354	0.67487	0.67524
0.61315	0.03351	0.00751	0.61427	0.61503
0.27235	0.74096	0.57472	1.01120	1.01443
0.76836	0.36167	0.78133	1.37617	1.37864

0.25718	0.12458	0.63887	0.53346	0.53186
0.24216	0.00702	0.44856	0.33247	0.33651
0.03418	0.69076	0.54875	0.67657	0.67467
0.78817	0.24055	0.03900	0.84609	0.84503
0.67421	0.68413	0.38603	1.19978	1.20199
0.15122	0.54512	0.88815	1.14896	1.14695
0.00522	0.55489	0.68490	0.63439	0.63264
0.01221	0.21570	0.53670	0.21333	0.22025
0.81603	0.64125	0.50551	1.35642	1.35936
0.40425	0.19971	0.93945	1.27326	1.27041
0.03867	0.19526	0.63341	0.33092	0.32978
0.28236	0.88433	0.61519	1.29723	1.29504
0.76312	0.94559	0.07566	1.65768	1.65678
0.48451	0.86605	0.38102	1.28988	1.29507
0.90582	0.47725	0.76055	1.57352	1.57257
0.11267	0.36573	0.43153	0.32680	0.33053
0.99902	0.78268	0.13120	1.61387	1.61335
0.01389	0.00037	0.37883	0.06825	0.06557
0.00482	0.01813	0.27827	0.02670	0.02583
0.17524	0.21943	0.08032	0.22390	0.22882
0.42106	0.41707	0.50890	0.72680	0.71791
0.09104	0.98022	0.28043	1.07393	1.07327
0.35301	0.08603	0.92300	1.14674	1.15276
0.13935	0.30219	0.89691	0.95218	0.95986
0.80605	0.67022	0.78878	1.74601	1.74604
0.34245	0.32569	0.82550	1.01105	1.00944
0.33061	0.24262	0.40159	0.45424	0.45322
0.20808	0.53374	0.03568	0.49300	0.49191
0.44978	0.87689	0.79995	1.73062	1.72819
0.28043	0.00153	0.11563	0.28198	0.27739
0.14576	0.82418	0.32398	0.85904	0.85973
0.19565	0.03909	0.91809	0.97103	0.96612
0.19913	0.66399	0.65944	0.92679	0.92900
0.06119	0.51720	0.18836	0.33537	0.33226
0.48848	0.85391	0.84072	1.81188	1.81757
0.48991	0.63677	0.64272	1.16089	1.16104
0.59551	0.39503	0.81320	1.28931	1.28640
0.23707	0.04511	0.53761	0.39449	0.39326

Mean Square Error = 0.000001

Unseen Examples

x	y	z	f	Output
0.47990	0.70425	0.74300	1.38604	1.38785
0.95276	0.34837	0.62481	1.31803	1.31153
0.59691	0.27494	0.42582	0.74972	0.74386

0.53212	0.97311	0.63439	1.73438	1.72744
0.85342	0.85751	0.40727	1.65629	1.65682
0.68780	0.22315	0.33067	0.77375	0.77377
0.29228	0.07706	0.55007	0.46465	0.46341
0.47371	0.00302	0.80081	0.98727	0.97934
0.11237	0.71615	0.36995	0.67587	0.67164
0.07038	0.46254	0.52132	0.42600	0.42623
0.26597	0.92166	0.41450	1.18664	1.18817
0.19514	0.76623	0.04932	0.78236	0.77345
0.68166	0.53691	0.26704	0.98898	0.98510
0.13712	0.98785	0.33915	1.15199	1.15397
0.11176	0.55867	0.17563	0.42929	0.42481
0.41765	0.13013	0.68349	0.75388	0.74465

Mean Square Error = 0.000007

Runtime (s) = 53.644000

BPNNThreeFunctions

File Help

Learning Rate:

0.9

Momentum:

0.1

Hidden Units:

8

Tolerance:

1.0e-9

Max Epochs:

1000000

PRNG Seed:

1

Options

☐ f1[x, y, z] = x^2 + y^2 + z^2

☐ f2[x, y, z] = x^2 + y^2 + z^3

☒ f3[x, y, z] = x * sin(x) + y^2 + z^3

f3(x, y, z) = x * sin(x) + y^2 + z^3

x	y	z	f	Output
0.45445	0.55214	0.04923	0.50447	0.50300
0.67922	0.45024	0.57955	0.82405	0.82603
0.08869	0.69781	0.47783	0.60389	0.60678
0.76424	0.17933	0.14966	0.56436	0.56248
0.97601	0.63231	0.71621	1.57560	1.57401
0.43052	0.51601	0.69488	0.78146	0.78157
0.99054	0.69234	0.06409	1.30801	1.31006
0.28614	0.80087	0.63570	0.97905	0.97927
0.73009	0.94256	0.44093	1.46108	1.45989
0.17090	0.90854	0.60381	1.07464	1.07440
0.22077	0.47966	0.21888	0.28890	0.28571
0.41801	0.17734	0.50838	0.33253	0.32959
0.15055	0.43092	0.13065	0.21050	0.21112
0.42424	0.81954	0.94177	1.68157	1.67892
0.74218	0.06574	0.74221	0.91482	0.91648
0.80599	0.73116	0.91821	1.89029	1.89188
0.03342	0.46672	0.90555	0.96150	0.96066
0.64614	0.93442	0.99582	2.24969	2.24900

Compute

Clear

f3(x, y, z) = x * sin(x) + y^2 + z^3

x y z f Output

0.45445	0.55214	0.04923	0.50447	0.50300
0.67922	0.45024	0.57955	0.82405	0.82603
0.08869	0.69781	0.47783	0.60389	0.60678
0.76424	0.17933	0.14966	0.56436	0.56248
0.97601	0.63231	0.71621	1.57560	1.57401
0.43052	0.51601	0.69488	0.78146	0.78157
0.99054	0.69234	0.06409	1.30801	1.31006
0.28614	0.80087	0.63570	0.97905	0.97927
0.73009	0.94256	0.44093	1.46108	1.45989
0.17090	0.90854	0.60381	1.07464	1.07440

0.22077	0.47966	0.21888	0.28890	0.28571
0.41801	0.17734	0.50838	0.33253	0.32959
0.15055	0.43092	0.13065	0.21050	0.21112
0.42424	0.81954	0.94177	1.68157	1.67892
0.74218	0.06574	0.74221	0.91482	0.91648
0.80599	0.73116	0.91821	1.89029	1.89188
0.03342	0.46672	0.90555	0.96150	0.96066
0.64614	0.93442	0.99582	2.24969	2.24900
0.16349	0.71364	0.07691	0.53635	0.54047
0.92599	0.12210	0.12354	0.75686	0.75721
0.39027	0.15534	0.52898	0.32062	0.31871
0.80740	0.05701	0.70531	0.93746	0.93583
0.11341	0.69677	0.42354	0.57429	0.57649
0.61315	0.03351	0.00751	0.35396	0.35271
0.27235	0.74096	0.57472	0.81212	0.81299
0.76836	0.36167	0.78133	1.14178	1.14545
0.25718	0.12458	0.63887	0.34170	0.34225
0.24216	0.00702	0.44856	0.14837	0.14967
0.03418	0.69076	0.54875	0.64356	0.64442
0.78817	0.24055	0.03900	0.61679	0.61825
0.67421	0.68413	0.38603	0.94647	0.94716
0.15122	0.54512	0.88815	1.02052	1.02317
0.00522	0.55489	0.68490	0.62920	0.62426
0.01221	0.21570	0.53670	0.20127	0.20224
0.81603	0.64125	0.50551	1.13481	1.13920
0.40425	0.19971	0.93945	1.02802	1.02094
0.03867	0.19526	0.63341	0.29375	0.28998
0.28236	0.88433	0.61519	1.09355	1.09021
0.76312	0.94559	0.07566	1.42201	1.42273
0.48451	0.86605	0.38102	1.03104	1.03204
0.90582	0.47725	0.76055	1.38052	1.37837
0.11267	0.36573	0.43153	0.22679	0.23507
0.99902	0.78268	0.13120	1.45496	1.45206
0.01389	0.00037	0.37883	0.05456	0.05274
0.00482	0.01813	0.27827	0.02190	0.01909
0.17524	0.21943	0.08032	0.07922	0.08909
0.42106	0.41707	0.50890	0.47784	0.47307
0.09104	0.98022	0.28043	0.99117	0.98942
0.35301	0.08603	0.92300	0.91578	0.92002
0.13935	0.30219	0.89691	0.83219	0.83494
0.80605	0.67022	0.78878	1.52157	1.52105
0.34245	0.32569	0.82550	0.78360	0.78618
0.33061	0.24262	0.40159	0.23095	0.23173
0.20808	0.53374	0.03568	0.32791	0.32357
0.44978	0.87689	0.79995	1.47639	1.47821
0.28043	0.00153	0.11563	0.07917	0.07923
0.14576	0.82418	0.32398	0.73445	0.73671

0.19565	0.03909	0.91809	0.81341	0.81228
0.19913	0.66399	0.65944	0.76705	0.76882
0.06119	0.51720	0.18836	0.27792	0.27038
0.48848	0.85391	0.84072	1.55263	1.55546
0.48991	0.63677	0.64272	0.90150	0.89427
0.59551	0.39503	0.81320	1.02784	1.02800
0.23707	0.04511	0.53761	0.21310	0.21650

Mean Square Error = 0.000001

Unseen Examples

x	y	z	f	Output
0.47990	0.70425	0.74300	1.12771	1.12615
0.95276	0.34837	0.62481	1.14179	1.14869
0.59691	0.27494	0.42582	0.48832	0.48016
0.53212	0.97311	0.63439	1.47224	1.46699
0.85342	0.85751	0.40727	1.44595	1.45730
0.68780	0.22315	0.33067	0.52259	0.51534
0.29228	0.07706	0.55007	0.25659	0.25821
0.47371	0.00302	0.80081	0.72966	0.72615
0.11237	0.71615	0.36995	0.57610	0.57790
0.07038	0.46254	0.52132	0.36057	0.36509
0.26597	0.92166	0.41450	0.99058	0.99102
0.19514	0.76623	0.04932	0.62506	0.63034
0.68166	0.53691	0.26704	0.73682	0.74261
0.13712	0.98785	0.33915	1.03361	1.03123
0.11176	0.55867	0.17563	0.33000	0.32689
0.41765	0.13013	0.68349	0.50564	0.50250

Mean Square Error = 0.000007

Runtime (s) = 42.956000