Derivation of the Classical Kinetic Energy Equation from Einstein's Famous 1905 Energy to Mass Relationship

$$E = mc^2 = m_0 c^2 (1 - \beta^2)^{-1/2}$$

$$\beta = \frac{v}{c}$$

Expand the energy equation around 0 using the following expansion:

$$f(\beta) = (1 - \beta^2)^{-1/2}$$

$$f'(\beta) = \beta(1 - \beta^2)^{-3/2}$$

$$f''(\beta) = (1 - \beta^2)^{-3/2} + 3\beta^2(1 - \beta^2)^{-5/2}$$

$$\begin{aligned} f'''(\beta) &= 3\beta(1 - \beta^2)^{-5/2} + 6\beta(1 - \beta^2)^{-5/2} + 15\beta^3(1 - \beta^2)^{-7/2} \\ &= 9\beta(1 - \beta^2)^{-5/2} + 15\beta^3(1 - \beta^2)^{-7/2} \end{aligned}$$

$$\begin{aligned} f^{iv}(\beta) &= 9(1 - \beta^2)^{-5/2} + 45\beta^2(1 - \beta^2)^{-7/2} + 45\beta^2(1 - \beta^2)^{-7/2} + 105\beta^4(1 - \beta^2)^{-9/2} \\ &= 9(1 - \beta^2)^{-5/2} + 90\beta^2(1 - \beta^2)^{-7/2} + 105\beta^4(1 - \beta^2)^{-9/2} \end{aligned}$$

$$\begin{aligned} f^{v}(\beta) &= 45\beta(1 - \beta^2)^{-7/2} + 180\beta(1 - \beta^2)^{-7/2} + 630\beta^3(1 - \beta^2)^{-9/2} + 420\beta^3(1 - \beta^2)^{-9/2} \\ &\quad + 3780\beta^5(1 - \beta^2)^{-11/2} \\ &= 225\beta(1 - \beta^2)^{-7/2} + 1050\beta^3(1 - \beta^2)^{-9/2} + 3780\beta^5(1 - \beta^2)^{-11/2} \end{aligned}$$

$$\begin{aligned} f^{vi}(\beta) &= 225(1 - \beta^2)^{-7/2} + 1575\beta^2(1 - \beta^2)^{-9/2} + 3150\beta^2(1 - \beta^2)^{-9/2} \\ &\quad + 18900\beta^4(1 - \beta^2)^{-11/2} + 41580\beta^6(1 - \beta^2)^{-13/2} \\ &= 225(1 - \beta^2)^{-7/2} + 4725\beta^2(1 - \beta^2)^{-9/2} + 18900\beta^4(1 - \beta^2)^{-11/2} \\ &\quad + 41580\beta^6(1 - \beta^2)^{-13/2} \end{aligned}$$

$$\begin{aligned} E = m_0 c^2 f(\beta) &= m_0 c^2 + \frac{1}{2!} m_0 c^2 \left(\frac{v}{c}\right)^2 + \frac{9}{4!} m_0 c^2 \left(\frac{v}{c}\right)^4 + \frac{225}{6!} m_0 c^2 \left(\frac{v}{c}\right)^6 + \cdots \\ &= m_0 c^2 + \frac{1}{2} m_0 v^2 + \frac{3}{8} m_0 \frac{v^4}{c^2} + \frac{225}{720} m_0 \frac{v^6}{c^4} + \cdots \end{aligned}$$

Suppose the following constraint is valid:

$$v \ll c$$

Then the energy is approximately:

$$E \sim m_0 c^2 + \frac{1}{2} m_0 v^2 = m_0 \left(c^2 + \frac{1}{2} v^2\right)$$

The first term above is a constant, and the second term is the familiar classical kinetic energy term.

Suppose we have a slow neutron slowly moving then

$$m_0 = 1.67492750056 \times 10^{-27} kg$$

$$c = 2.99792458 \times 10^8 m/s$$

$$c^2 = 8.9875517873681764 \times 10^{16}\, m^2/s^2$$

$$v = 1.0 \times 10^3 m/s$$

$$v^2 = 1.0 \times 10^6\, m^2/s^2$$

I wrote a little C++ program to calculate the energy using the preceding variables:

```
Enter v = 1000
# Terms = 3
KE = 1.50535e-10 joules
KE = 3.59787e-23 kilotons TNT
KE[0] = 1.505350e-10 joules
KE[0] = 3.597872e-23 kilotons TNT
KE[1] = 8.374638e-22 joules
KE[1] = 2.001586e-34 kilotons TNT
KE[2] = 6.988531e-33 joules
KE[2] = 1.670299e-45 kilotons TNT
KE[3] = 6.479824e-44 joules
KE[3] = 1.548715e-56 kilotons TNT
Enter v = 10000
# Terms = 3
KE = 1.505350e-10 joules
KE = 3.597872e-23 kilotons TNT
KE[0] = 1.505350e-10 joules
KE[0] = 3.597872e-23 kilotons TNT
KE[1] = 8.374638e-20 joules
KE[1] = 2.001586e-32 kilotons TNT
KE[2] = 6.988531e-29 joules
KE[2] = 1.670299e-41 kilotons TNT
KE[3] = 6.479824e-38 joules
KE[3] = 1.548715e-50 kilotons TNT
Enter v = 0

D:\KineticEnergy\x64\Debug\KineticEnergy.exe (process 28828)
exited with code 0 (0x0).
Press any key to close this window . . .
```