Blog Entry © Monday, March 23, 2026, by James Pate Williams, Jr. Gauss-Legendre Quadrature

Reference: DLMF: §3.5 Quadrature ‣ Areas ‣ Chapter 3 Numerical Methods

Test Integral (evaluation due to wolfram.com online definite integral calculator definite integral calculator - Wolfram|Alpha):

$$\int_{-1}^{1} x^3 e^{-x}\, dx =$$

$$= -0.4495074018249866748078053798780488836184839091083662021 91453571$$

Gauss-Legendre using n = 20 yields:

$$\int_{-1}^{1} x^3 e^{-x}\, dx = \quad -0.449507401824987$$

Simpson's Rule yields after 100000 steps:

$$\int_{-1}^{1} x^3 e^{-x}\, dx = \quad -0.449507401824237$$

Detailed results are displayed below:

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Abscissas (x) and Weights (w)
for Gauss-Legendre Quadrature
x                           w
-0.973906528517172          0.066671344308688
-0.865063366688985          0.149451349150581
-0.679409568299024          0.219086362515982
-0.433395394129247          0.269266719309996
-0.148874338981631          0.295524224714753
 0.148874338981631          0.295524224714753
 0.433395394129247          0.269266719309996
 0.679409568299024          0.219086362515982
 0.865063366688985          0.149451349150581
 0.973906528517172          0.066671344308688
                   10      -0.44950740182498 6
Abscissas (x) and Weights (w)
for Gauss-Legendre Quadrature
x                           w
-0.993128599185095          0.017614007139152
-0.963971927277914          0.040601429800387
-0.912234428251326          0.062672048334109
-0.839116971822219          0.083276741576705
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-0.746331906460151	0.101930119817240
-0.636053680726515	0.118194531961518
-0.510867001950827	0.131688638449177
-0.373706088715420	0.142096109318382
-0.227785851141645	0.149172986472604
-0.076526521133497	0.152753387130726
0.076526521133497	0.152753387130726
0.227785851141645	0.149172986472604
0.373706088715420	0.142096109318382
0.510867001950827	0.131688638449177
0.636053680726515	0.118194531961518
0.746331906460151	0.101930119817240
0.839116971822219	0.083276741576705
0.912234428251326	0.062672048334109
0.963971927277914	0.040601429800387
0.993128599185095	0.017614007139152
20	-0.449507401824987

Abscissas (x) and Weights (w)
for Gauss-Legendre Quadrature

x	w
-0.996893484074650	0.007968192496167
-0.985258674130786	0.017999216408973
-0.960021864968308	0.028784707883323
-0.926200047429274	0.038799192569627
-0.882560535792053	0.048402672830594
-0.829565762382768	0.057493156217619
-0.767777432104826	0.065974229882180
-0.697850494793316	0.073755974737705
-0.620526182989243	0.080755895229420
-0.536624148142020	0.086899787201083
-0.447033769538089	0.092122522237786
-0.352704725530878	0.096368737174644
-0.254636926167890	0.099593420586795
-0.153869913608584	0.101762389748405
-0.051471842555318	0.102852652893559
0.051471842555318	0.102852652893559
0.153869913608584	0.101762389748405
0.254636926167890	0.099593420586795
0.352704725530878	0.096368737174644
0.447033769538089	0.092122522237786
0.536624148142020	0.086899787201083
0.620526182989243	0.080755895229420
0.697850494793316	0.073755974737705
0.767777432104826	0.065974229882180
0.829565762382768	0.057493156217619
0.882560535792053	0.048402672830594
0.926200047429274	0.038799192569627

```
     0.960021864968308         0.028784707883323
     0.983668123279747         0.018466468311091
     0.996893484074650         0.007968192496167
                     30        -0.448613963738790
Abscissas (x) and Weights (w)
for Gauss-Legendre Quadrature
x                          w
-0.998237709710559          0.004521277098533
-0.990726238699457          0.010498284531153
-0.977259949983774          0.016421058381908
-0.957916819213792          0.022245849194167
-0.932812808278677          0.027937006980023
-0.902098806968874          0.033460195282548
-0.865959503212260          0.038782167974472
-0.824612230833312          0.043870908185673
-0.778305651426519          0.048695807635072
-0.727318255189927          0.053227846983937
-0.671956684614180          0.057439769099392
-0.612553889667980          0.061306242492929
-0.549467125095128          0.064804013456601
-0.483075801686179          0.067912045815234
-0.413779204371605          0.070611647391287
-0.341994090825758          0.072886582395804
-0.268152185007254          0.074723169057968
-0.192697580701371          0.076110361900626
-0.116084070675255          0.077039818164248
-0.038772417506051          0.077505947978425
 0.038772417506051          0.077505947978425
 0.116084070675255          0.077039818164248
 0.192697580701371          0.076110361900626
 0.268152185007254          0.074723169057968
 0.341994090825758          0.072886582395804
 0.413779204371605          0.070611647391287
 0.483075801686179          0.067912045815234
 0.549467125095128          0.064804013456601
 0.612553889667980          0.061306242492929
 0.671956684614180          0.057439769099392
 0.727318255189927          0.053227846983937
 0.778305651426519          0.048695807635072
 0.824612230833312          0.043870908185673
 0.865959503212259          0.038782167974472
 0.902098806968874          0.033460195282548
 0.932812808278677          0.027937006980023
 0.957916819213792          0.022245849194167
 0.977259949983774          0.016421058381908
 0.990726238699457          0.010498284531153
 0.998237709710559          0.004521277098533
```

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                40          -0.449507401824986
Simpson 50000               -0.449507401825246
Simpson 60000               -0.449507401825104
Simpson 70000               -0.449507401824803
Simpson 80000               -0.449507401825114
Simpson 90000               -0.449507401825659
Simpson 100000              -0.449507401824237


D:\Quadrature1d\x64\Debug\Quadrature1d.exe (process 11104)
exited with code 0 (0x0).
Press any key to close this window . . .
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Hats off to Gauss and Legendre.