

Blog Entry © Sunday, March 29, 2026, James Pate Williams, Jr. Properties of Determinants

Reference: [DLMF: §1.3 Determinants, Linear Operators, and Spectral Expansions](#) ▶ [Topics of Discussion](#) ▶ [Chapter 1 Algebraic and Analytic Methods](#)

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \det(A) = ad - bc$$

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \det(A) = 4 - 6 = -2$$

$$A^{-1} = -\frac{1}{2} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 1.5 & -0.5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} -2 & 1 \\ 1.5 & -0.5 \end{bmatrix} = \begin{bmatrix} -2 + 3 & 1 - 1 \\ -6 + 6 & 3 - 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\det(A^{-1}) = 1 - 1.5 = \frac{2}{2} - \frac{3}{2} = \frac{2}{2} - \frac{3}{2} = -\frac{1}{2} = \frac{1}{\det(A)}$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$\det(A) = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} \begin{vmatrix} 1 & 2 \\ 4 & 5 \\ 7 & 8 \end{vmatrix} = 45 + 84 + 96 - 72 - 48 - 105 = 225 - 225 = 0$$

The 3-by-3 matrix, A, is singular, i.e. it does not have an inverse.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 6 \\ 0 & 0 & 9 \end{bmatrix}$$

$$\det(A) = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 5 & 6 \\ 0 & 0 & 9 \end{vmatrix} \begin{vmatrix} 1 & 2 \\ 0 & 5 \\ 0 & 0 \end{vmatrix} = 45 + 0 + 0 - 0 - 0 = 45 = \prod_{i=0}^3 a_{ii}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 5 & 0 \\ 7 & 8 & 9 \end{bmatrix}$$

$$\det(A) = \begin{vmatrix} 1 & 0 & 0 \\ 4 & 5 & 0 \\ 7 & 8 & 9 \end{vmatrix} \begin{vmatrix} 1 & 0 \\ 4 & 0 \\ 7 & 8 \end{vmatrix} = 45 + 0 + 0 - 0 - 0 - 0 = 45 = \prod_{i=0}^3 a_{ii}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$\det(A) = 1 * 5 * 9 = 45 = \prod_{i=0}^3 a_{ii}$$

The image shows a software dialog box titled "Data Input Dialog" with a close button (X) in the top right corner. Inside the dialog, there is a label "n:" followed by a dropdown menu showing the number "2". Below this is the label "input matrix:" followed by a large text input area. The first line of the input area contains the text "1 2 3 4 |". At the bottom of the dialog, there is a label "determinant:" followed by an empty text input field. Two buttons, "Compute" and "Cancel", are located at the bottom of the dialog.

Figure 1 2x2 Determinant Input



Figure 2 2x2 Determinant Output

Data Input Dialog

n:

input matrix:

1	2	3	4	5	6	7	8	9

determinant:

Compute Cancel

Figure 3 3x3 Determinant Input



Figure 4 3x3 Determinant Output



Figure 5 3x3 Determinant Input

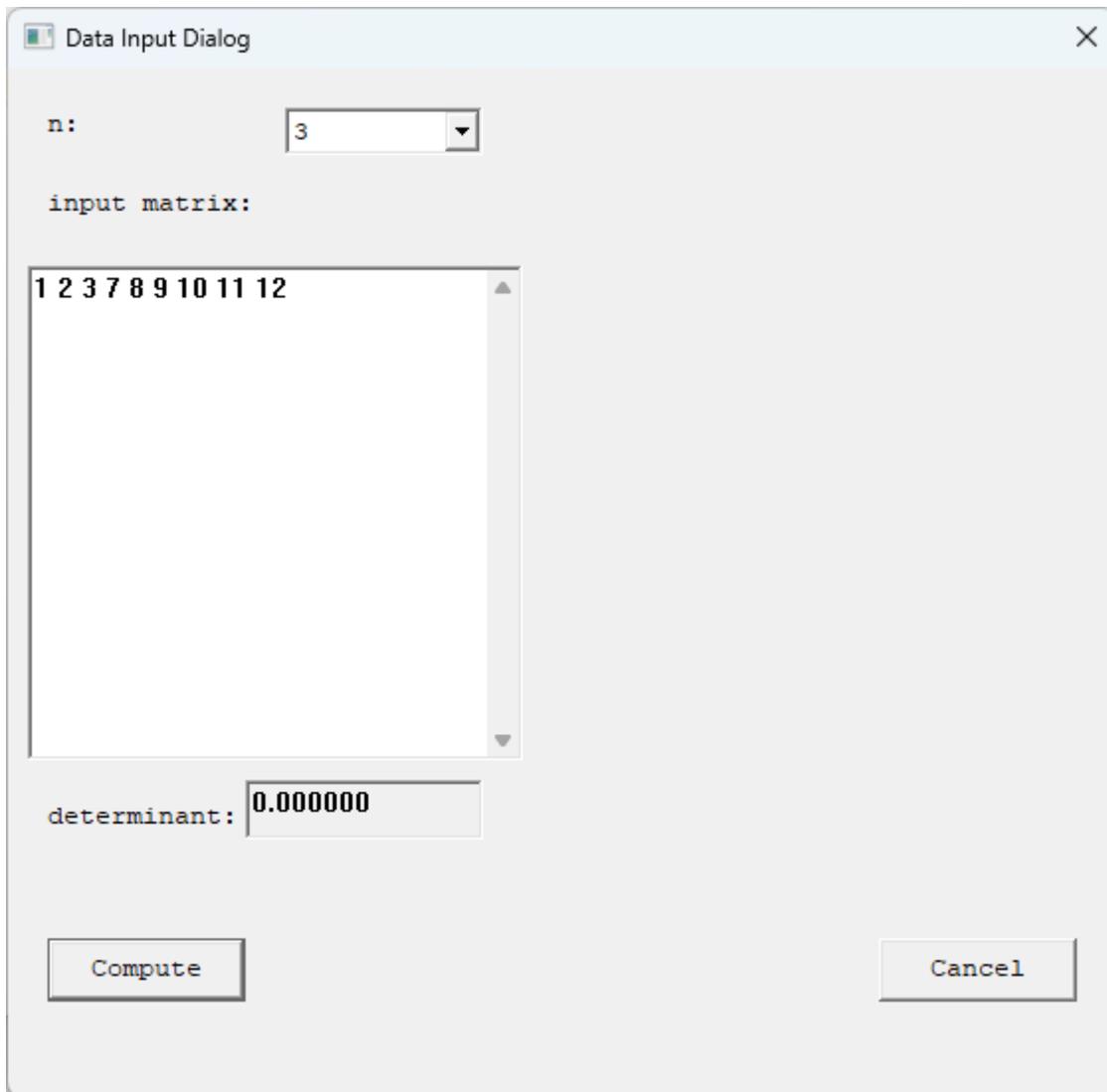


Figure 6 3x3 Determinant Output

Data Input Dialog

n:

input matrix:

```
10 6 7 12 |
```

determinant:

Compute Cancel

Figure 7 2x2 Determinant Input

Data Input Dialog



n:

2

input matrix:

10 6 7 12

determinant:

78.000000

Compute

Cancel

Data Input Dialog

n:

input matrix:

```
1 2 3 0 5 6 0 0 9 |
```

determinant:

Compute Cancel

Figure 8 3x3 Upper Triangular Determinant Input

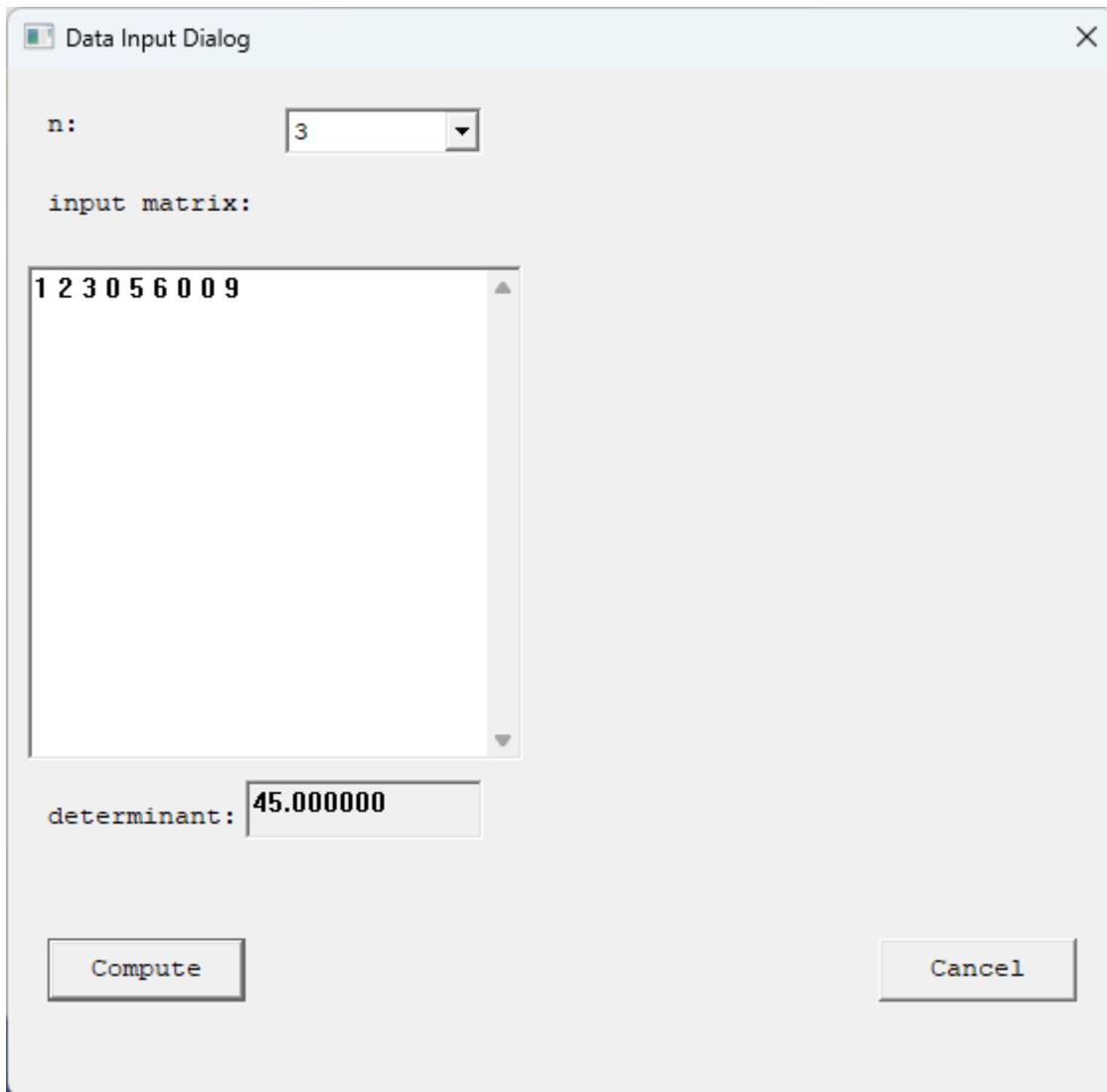


Figure 9 3x3 Upper Triangular Determinant Output

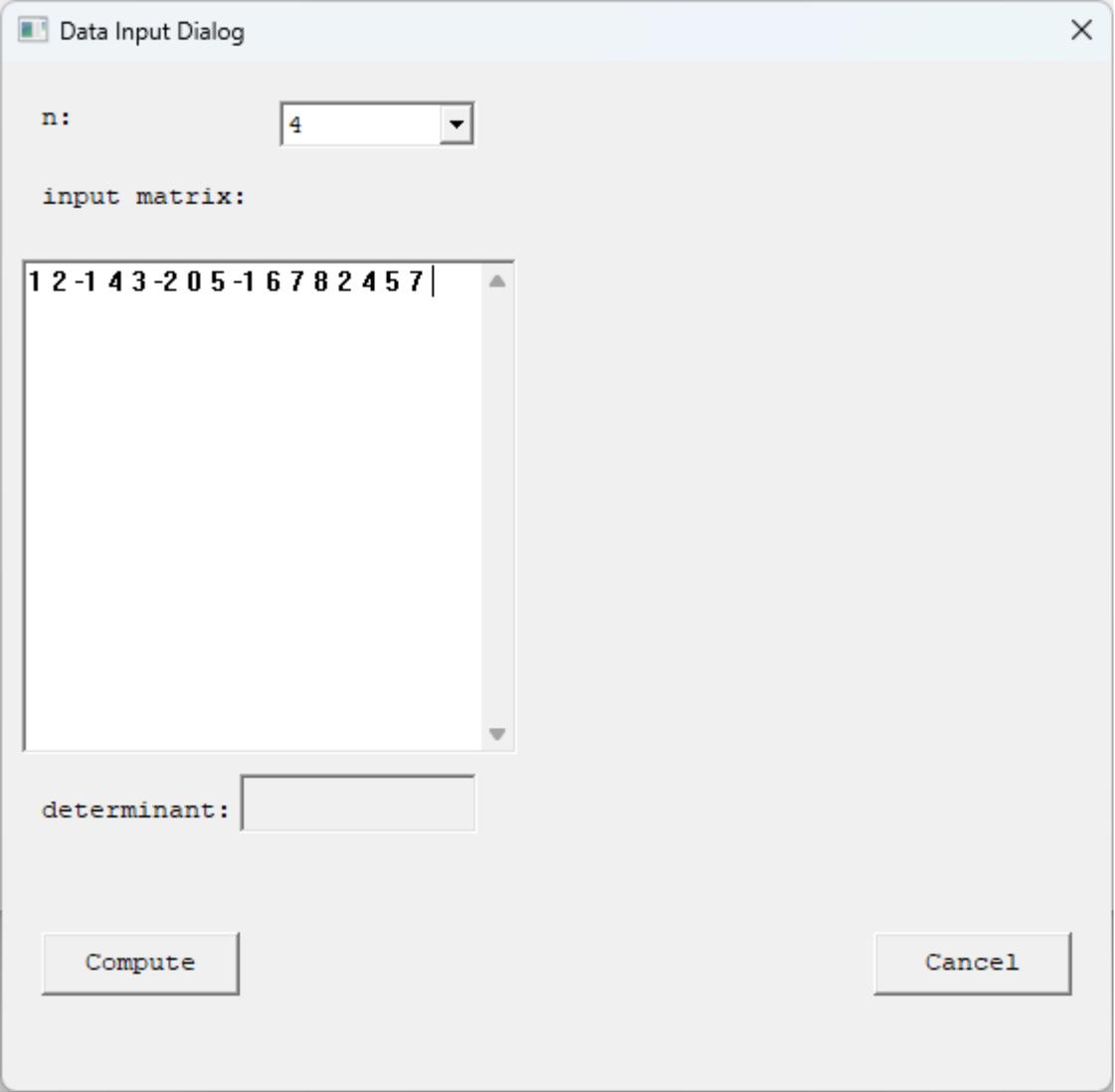


Figure 10 4x4 Determinant Input

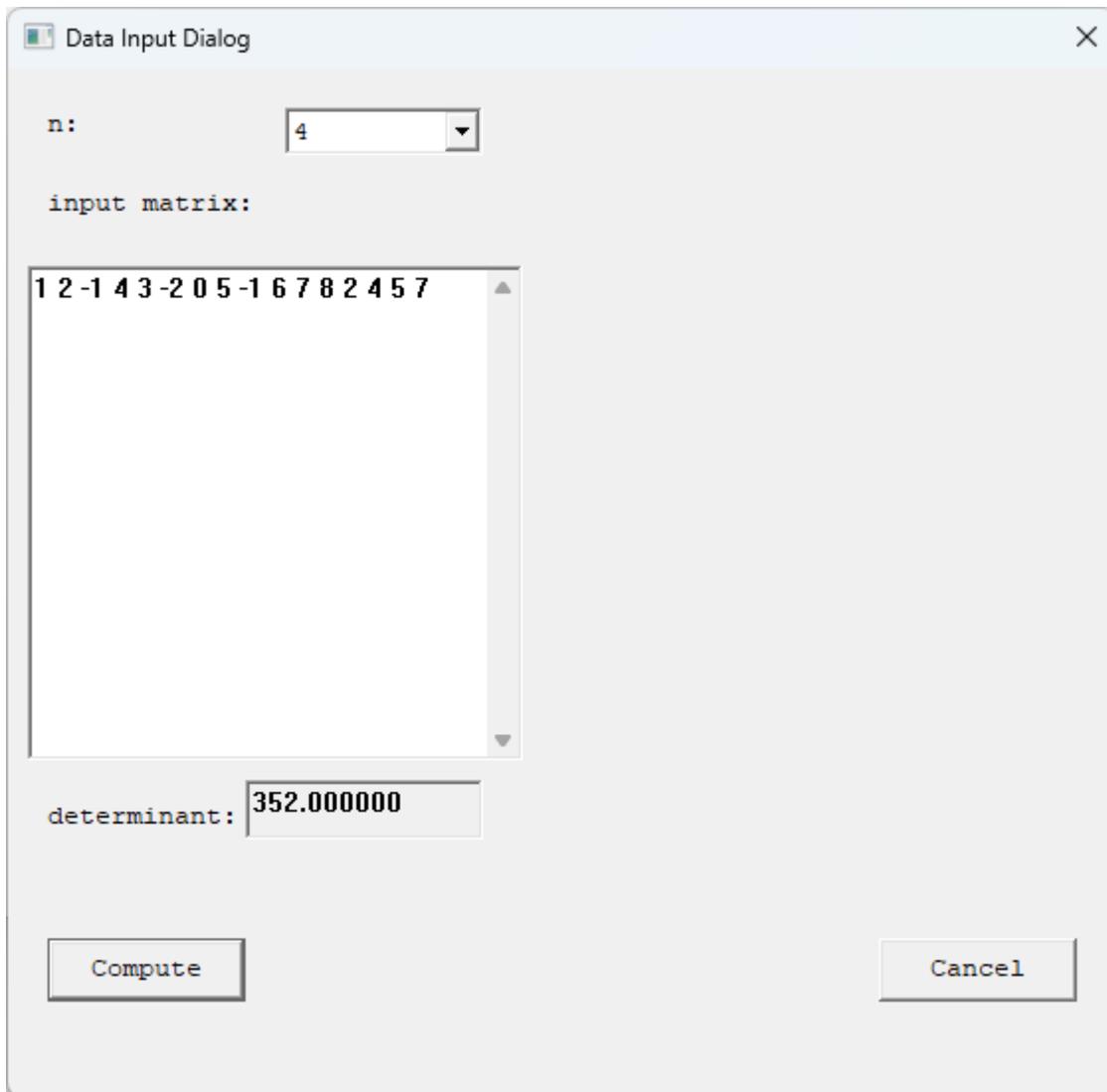


Figure 11 4x4 Determinant Output

[Determinant Calculator: Step-by-Step Solutions - Wolfram|Alpha](#)

The 4x4 determinant has the same value as the Wolfram.com calculator result.