Blog Entry © Sunday, March 29, 2026, James Pate Williams, Jr. Properties of Determinants

Reference: DLMF: §1.3 Determinants, Linear Operators, and Spectral Expansions ▸ Topics of Discussion ▸ Chapter 1 Algebraic and Analytic Methods

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \det(A) = ad - bc$$

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \det(A) = 4 - 6 = -2$$

$$A^{-1} = -\frac{1}{2} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 1.5 & -0.5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} -2 & 1 \\ 1.5 & -0.5 \end{bmatrix} = \begin{bmatrix} -2 + 3 & 1 - 1 \\ -6 + 6 & 3 - 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\det(A^{-1}) = 1 - 1.5 = \frac{2}{2} - \frac{3}{2} = \frac{2}{2} - \frac{3}{2} = -\frac{1}{2} = \frac{1}{\det(A)}$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$\det(A) = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} \begin{matrix} 1 & 2 \\ 4 & 5 \\ 7 & 8 \end{matrix} = 45 + 84 + 96 - 72 - 48 - 105 = 225 - 225 = 0$$

The 3-by-3 matrix, A, is singular, i.e. it does not have an inverse.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 6 \\ 0 & 0 & 9 \end{bmatrix}$$

$$\det(A) = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 5 & 6 \\ 0 & 0 & 9 \end{vmatrix} \begin{matrix} 1 & 2 \\ 0 & 5 \\ 0 & 0 \end{matrix} = 45 + 0 + 0 - 0 - 0 = 45 = \prod_{i=0}^{3} a_{ii}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 5 & 0 \\ 7 & 8 & 9 \end{bmatrix}$$

$$\det(A) = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 5 & 0 \\ 7 & 8 & 9 \end{bmatrix} \begin{matrix} 1 & 0 \\ 4 & 0 \\ 7 & 8 \end{matrix} = 45 + 0 + 0 - 0 - 0 - 0 = 45 = \prod_{i=0}^{3} a_{ii}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$\det(A) = 1 * 5 * 9 = 45 = \prod_{i=0}^{3} a_{ii}$$

Figure 1 2x2 Determinant Input

Figure 2 2x2 Determinant Output

Figure 3 3x3 Determinant Input

Figure 4 3x3 Determinant Output

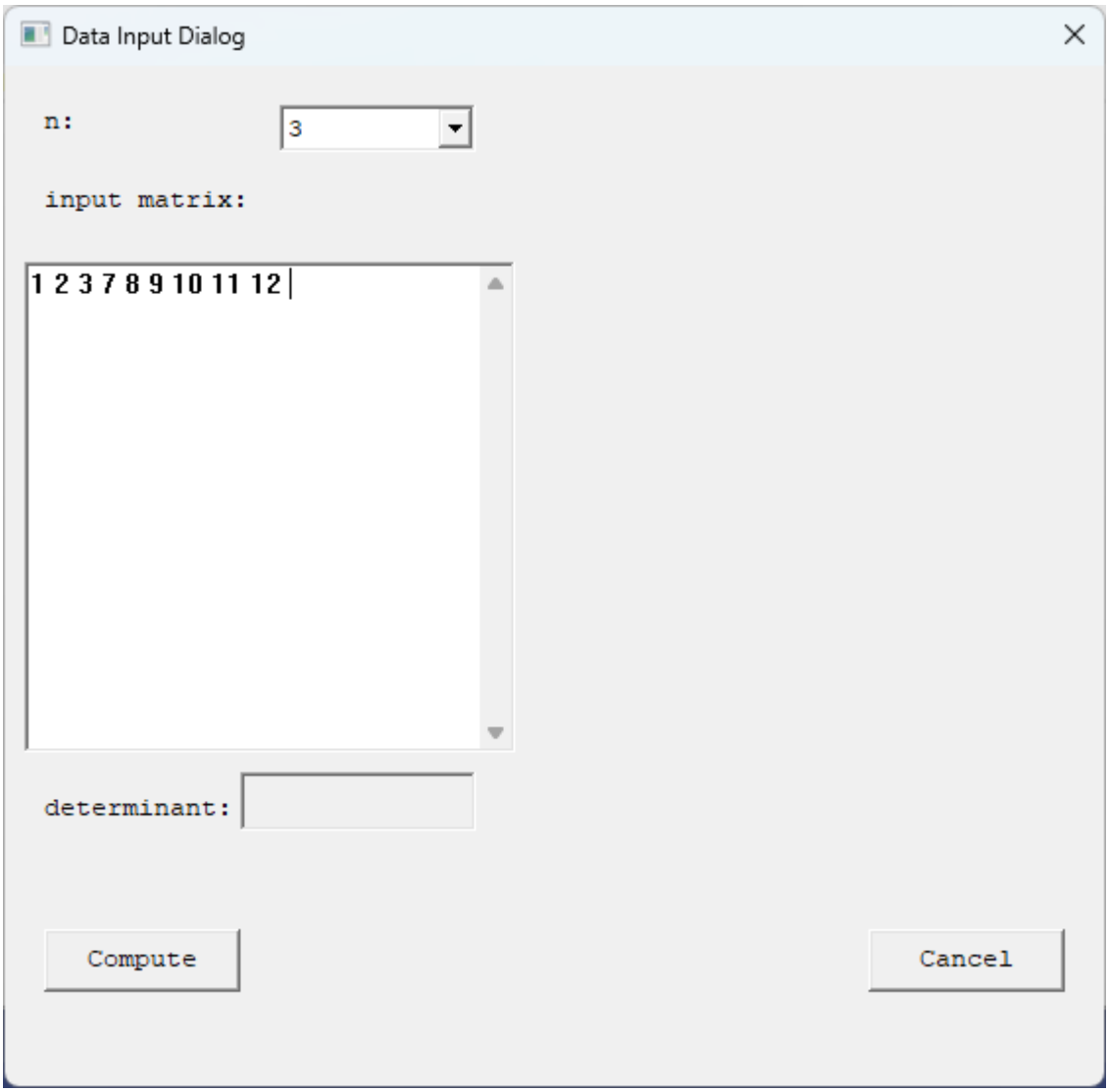

Figure 5 3x3 Determinant Input

Figure 6 3x3 Determinant Output

Figure 7 2x2 Determinant Input


Data Input Dialog
n:
2
input matrix:
10 6 7 12
determinant:
78.000000
Compute
Cancel

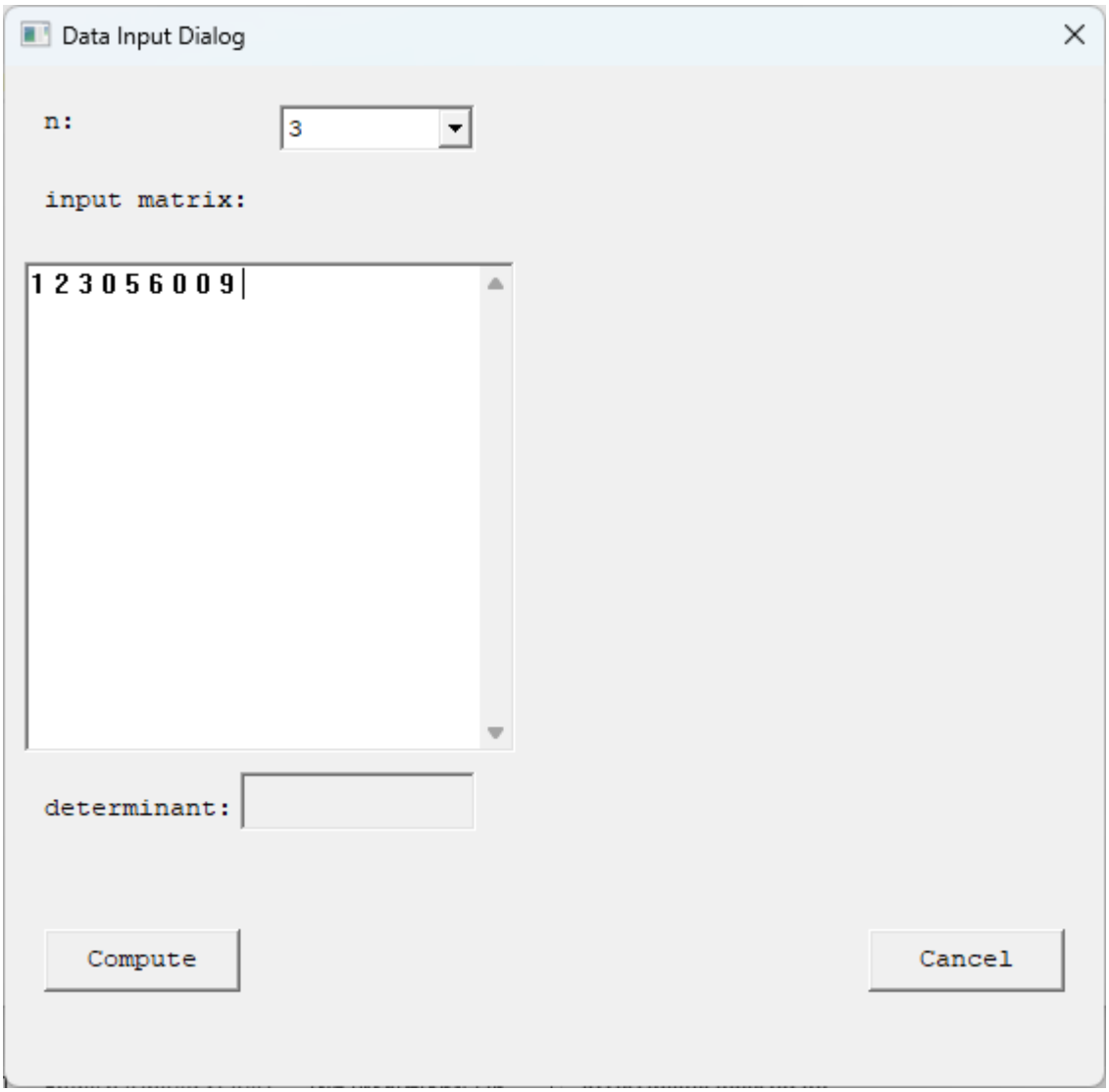

Figure 8 3x3 Upper Triangular Determinant Input

Figure 9 3x3 Upper Triangular Determinant Output

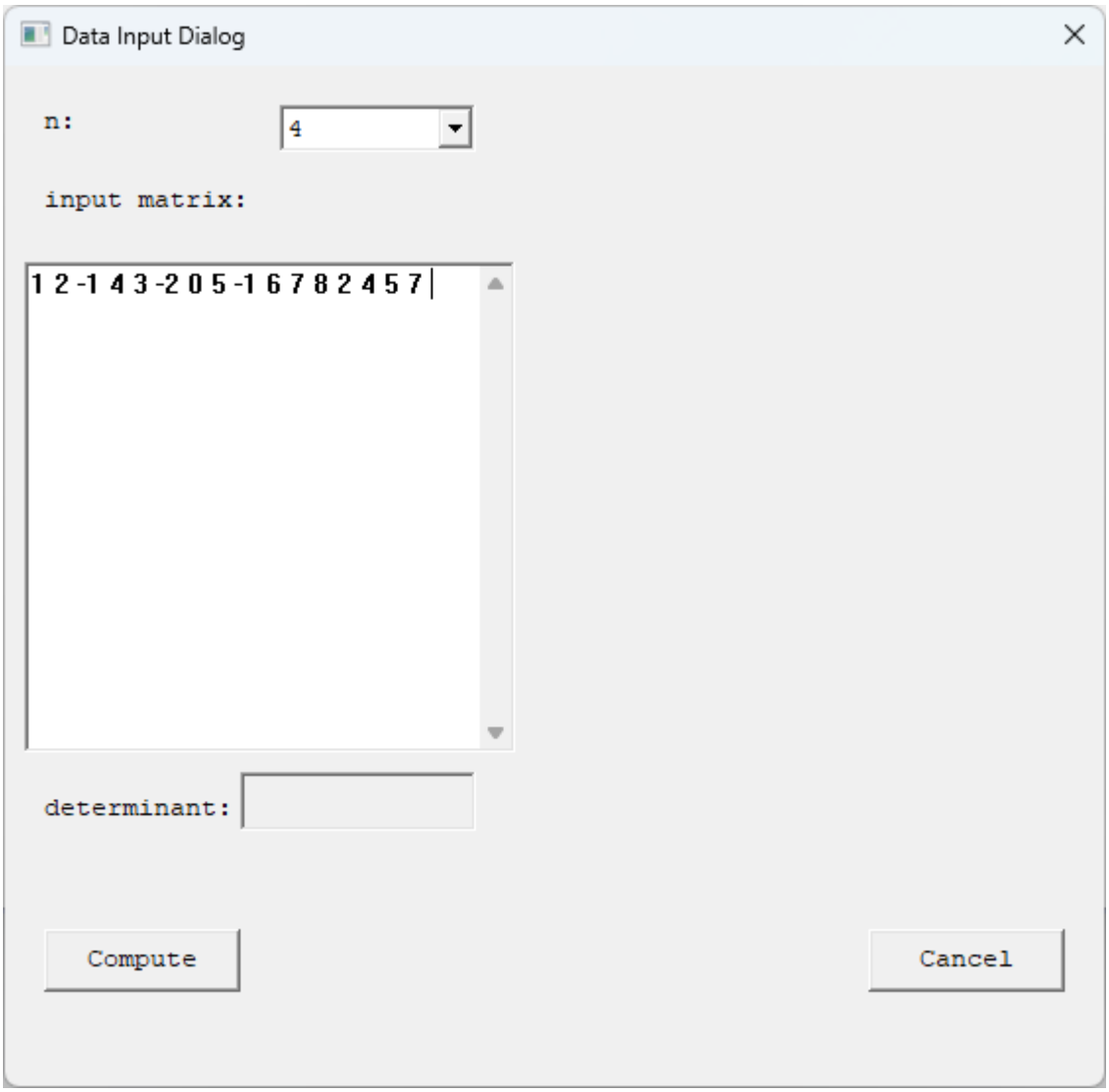

Figure 10 4x4 Determinant Input

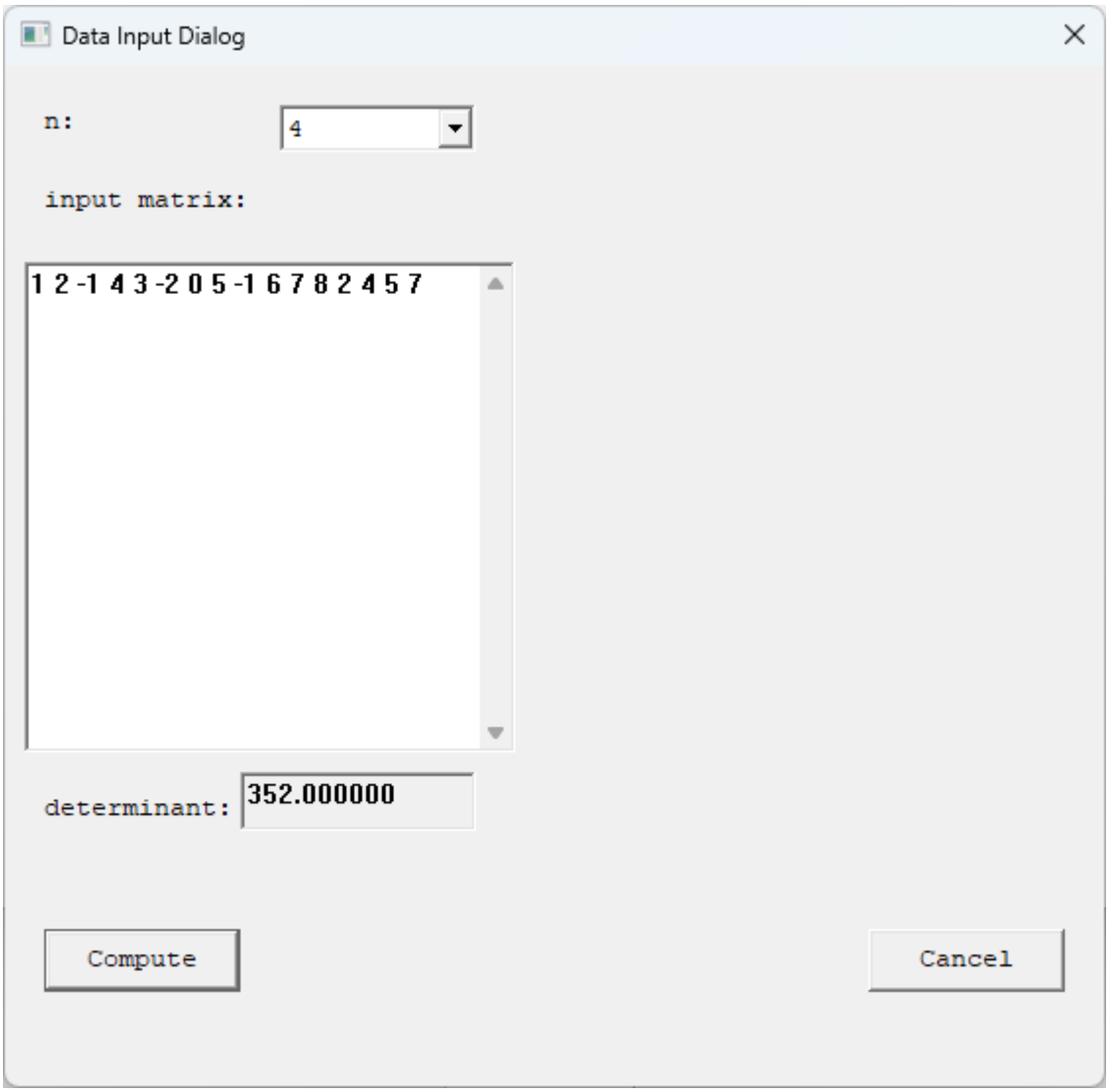

Figure 11 4x4 Determinant Output

[Determinant Calculator: Step-by-Step Solutions - Wolfram|Alpha]

The 4x4 determinant has the same value as the Wolfram.com calculator result.