Blog Entry © Sunday March 15, 2026, by James Pate Williams, Jr. Graphing Integer Order Bessel Functions of the First Kind and the real Gamma Function

References:

1. Bessel function - Wikipedia
2. Gamma function - Wikipedia

The Hansen-Bessel Formula is:

$$J_n(x) = \frac{1}{\pi}\int_0^{\infty} \cos(n\tau - x\sin(\tau))d\tau$$

The Bessel function infinite series is:

$$J_\alpha(x) = \sum_{m=0}^{\infty} \frac{(-1)^m}{m!\,\Gamma(m+\alpha+1)}\left(\frac{x}{2}\right)^{2m+\alpha}$$

The Gamma function is defined as follows:

$$\Gamma(z) = \int_0^{\infty} t^{z-1}e^{-t}dt$$

If z is a positive integer, n, then:

$$\Gamma(n) = (n-1)!$$

$$\Gamma(1) = \int_0^{\infty} e^{-t}dt = -e^{-t}\Big|_0^{\infty} = -0-(-1) = 1 = (1-1)!$$

$$\Gamma(2) = \int_0^{\infty} te^{-t}dt = -te^{-t}\Big|_0^{\infty} - e^{-t}\Big|_0^{\infty} = 0-0+1 = 1 = (2-1)!$$

$$\Gamma(3) = \int_0^{\infty} t^2e^{-t}dt = -t^2e^{-t}\Big|_0^{\infty} + 2te^{-t}\Big|_0^{\infty} = -2e^{-t}\Big|_0^{\infty} = 0-(-2) = 2! = (3-1)!$$

I leave it to the reader to prove the integer gamma function's formula by mathematical induction.

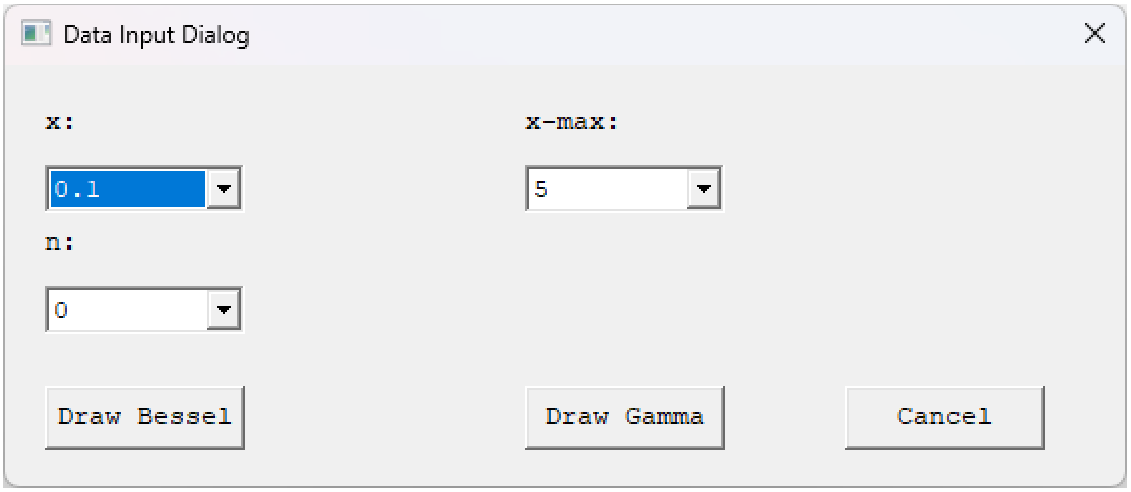

Figure 1 Data Input Dialog

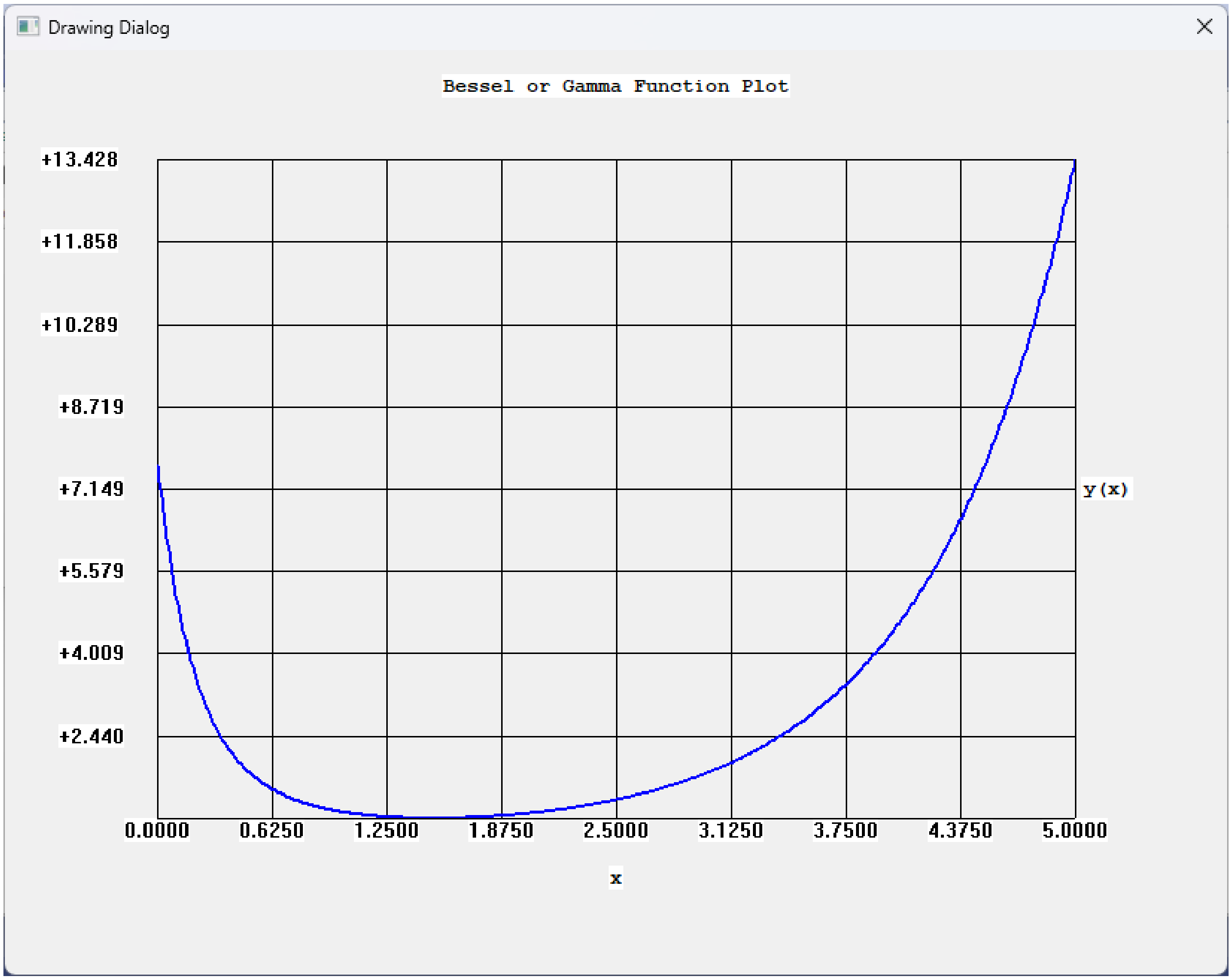

Figure 2 Gamma Function

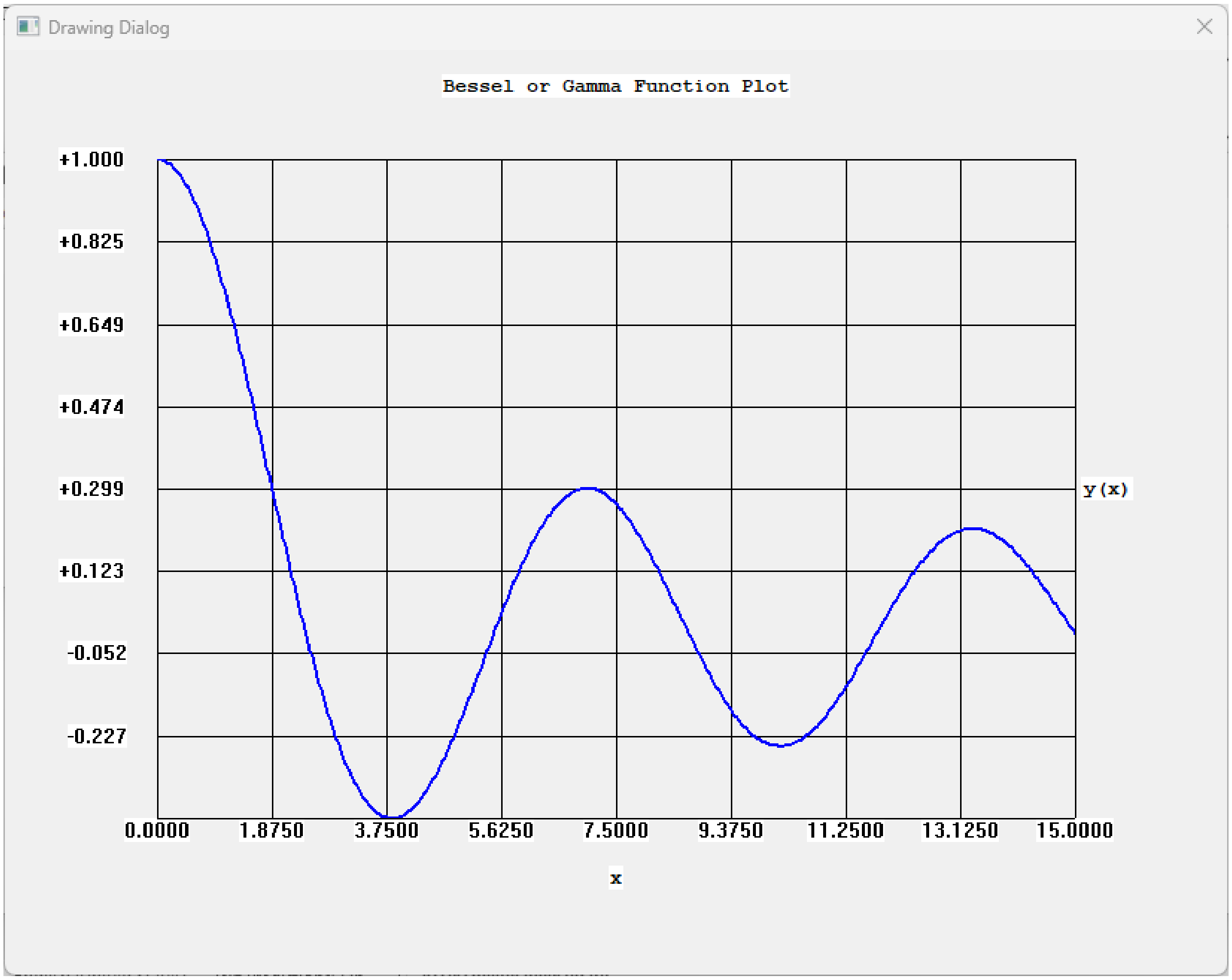

Figure 3 Bessel Functions of the First Kind and Order 0

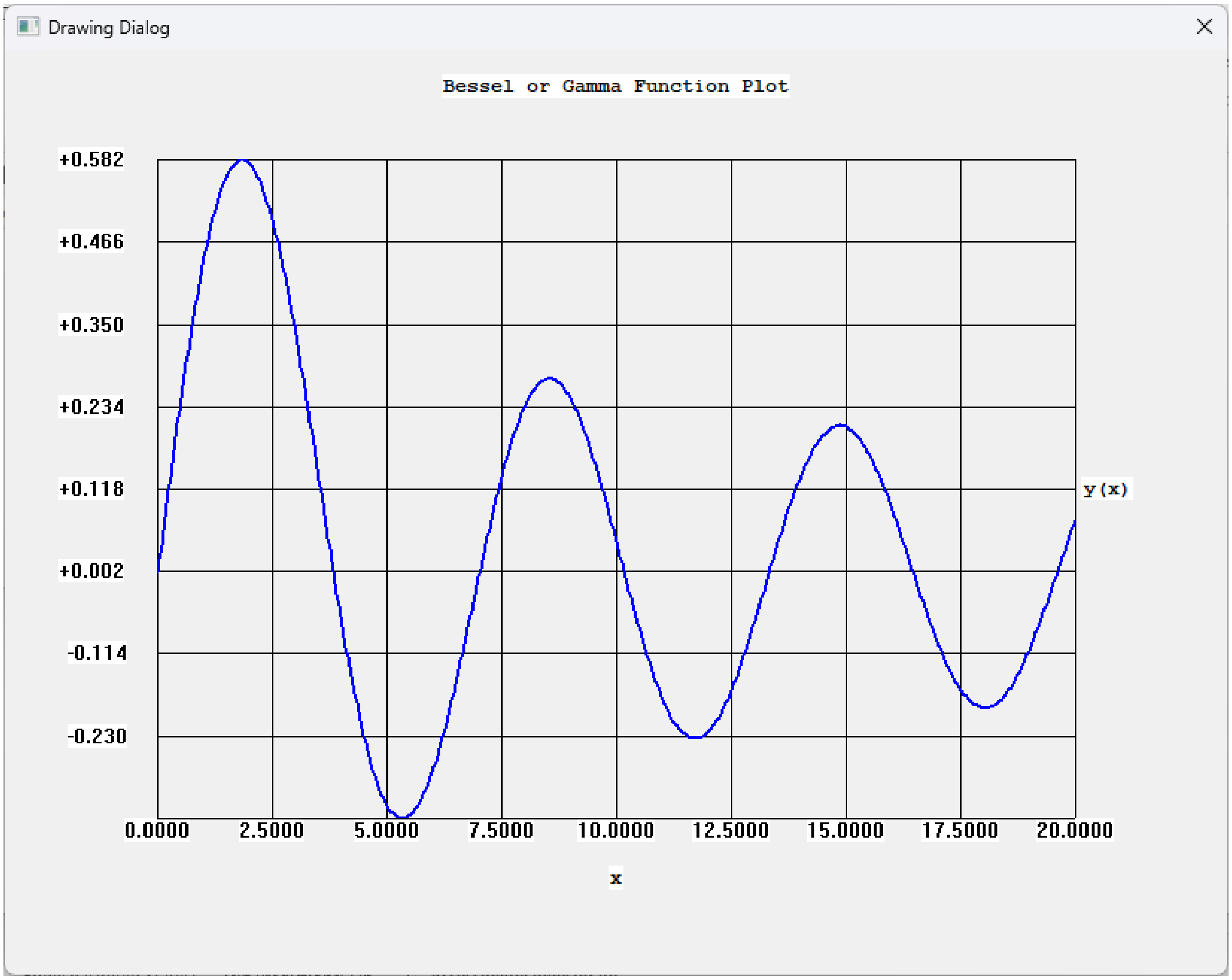

Figure 4 Bessel Function of First Kind and Order 1

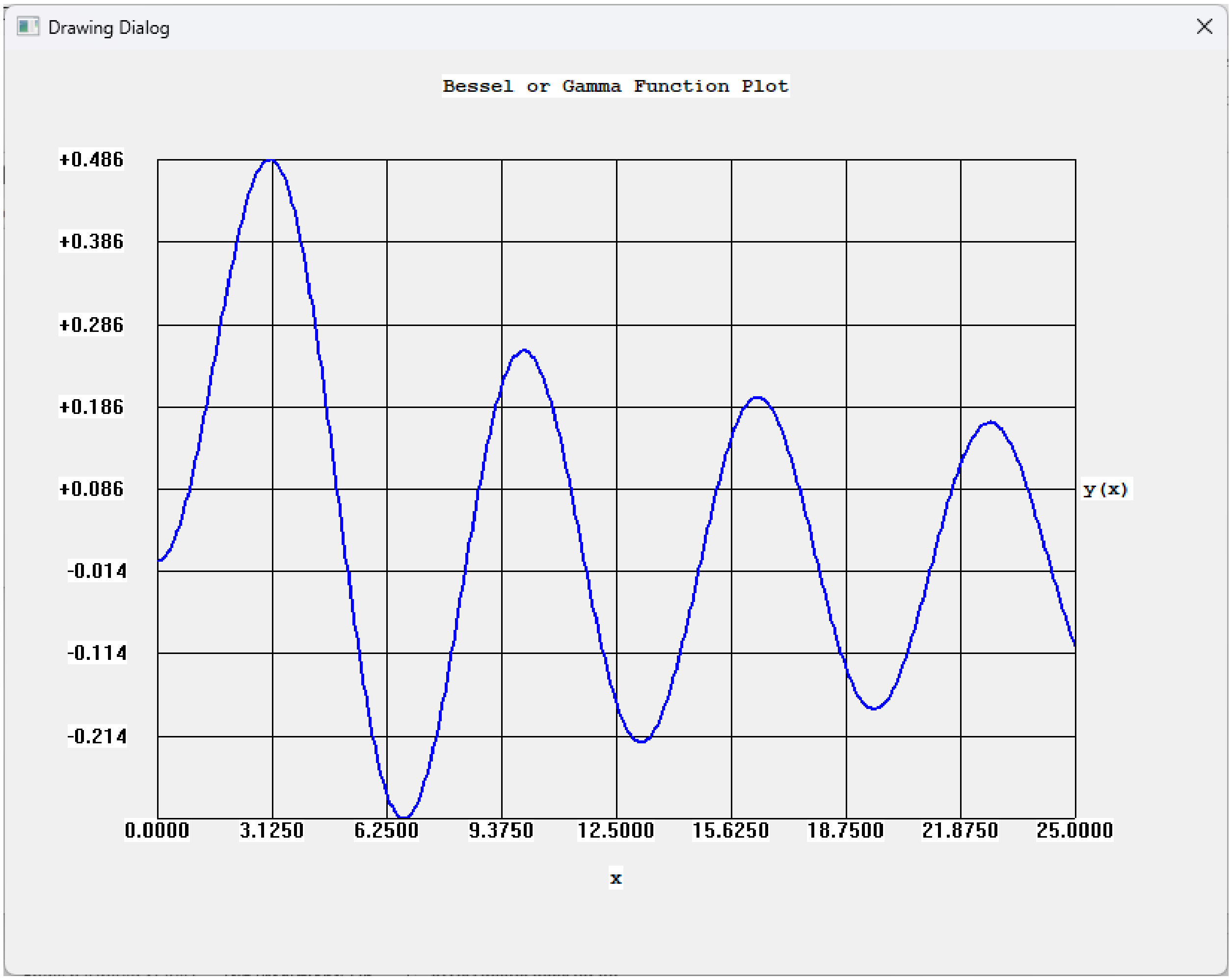

Figure 5 Bessel Function of the First Kind and Order 2