Blog Entry © Sunday, March 22, 2026, by James Pate Williams, Jr. Mueller's Method

Mueller's Method is a technique for finding all the real and/or complex roots of a real and/or complex polynomial. I originally coded the method in FORTRAN IV while I was in a student in the Scientific Computing I course in the Summer Quarter of 1982 at the Georgia Institute of Technology. My professor was Doctor Gunter Meyer. Our textbook was "Elementary Numerical Analysis: An Algorithmic Approach" © 1980 by S. D. Conte and Carl de Boor. In March 2015 I translated the textbook's FORTRAN77 algorithm into C#. On Tuesday, July 23, 2024, I translated my C# code into the computer language C. The complex division function is from "A Numerical Library in C for Scientists and Engineers" © 1994 by H. T. Lau Chapter 1.

Below is the output from two monic polynomial examples:

$$p(x) = x^2 + 1 \ and \ q(x) = (x - 1)(x - 2)(x + 3) = x^3 - 7x + 6$$

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Degree (0 to quit) = 2
coefficient[2].real = 1
coefficient[2].imag = 0
coefficient[1].real = 0
coefficient[1].imag = 0
coefficient[0].real = 1
coefficient[0].imag = 0

zero[0].real =  0.0000000000e+00        zero[0].imag =  1.0000000000e+00
zero[1].real =  0.0000000000e+00        zero[1].imag = -1.0000000000e+00

func[0].real =  0.0000000000e+00        func[0].imag =  0.0000000000e+00
func[1].real =  0.0000000000e+00        func[1].imag =  0.0000000000e+00

Degree (0 to quit) = 3
coefficient[3].real = 1
coefficient[3].imag = 0
coefficient[2].real = 0
coefficient[2].imag = 0
coefficient[1].real = -7
coefficient[1].imag = 0
coefficient[0].real = 6
coefficient[0].imag = 0

zero[0].real =  1.0000000000e+00        zero[0].imag =  0.0000000000e+00
zero[1].real =  2.0000000000e+00        zero[1].imag =  0.0000000000e+00
zero[2].real = -3.0000000000e+00        zero[2].imag =  0.0000000000e+00

func[0].real =  0.0000000000e+00        func[0].imag =  0.0000000000e+00
func[1].real =  0.0000000000e+00        func[1].imag =  0.0000000000e+00
func[2].real =  0.0000000000e+00        func[2].imag =  0.0000000000e+00

Degree (0 to quit) = 0

D:\MuellersMethod\x64\Debug\MuellersMethod.exe (process 19920) exited with code 0
(0x0).
Press any key to close this window . . .
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Now find the roots of the complex polynomial:

$$p(z) = (1 + i)z^4 + (2 + 2i)z^3 + (3 + 3i)z^2 + (4 + 4i)z + 5 + 5i$$

```
Degree (0 to quit) = 4
coefficient[4].real = 1
coefficient[4].imag = 1
coefficient[3].real = 2
coefficient[3].imag = 2
coefficient[2].real = 3
coefficient[2].imag = 3
coefficient[1].real = 4
coefficient[1].imag = 4
coefficient[0].real = 5
coefficient[0].imag = 5

zero[0].real = -1.2878154796e+00          zero[0].imag = -8.5789675833e-01
zero[1].real = -1.2878154796e+00          zero[1].imag =  8.5789675833e-01
zero[2].real =  2.8781547956e-01          zero[2].imag =  1.4160930802e+00
zero[3].real =  2.8781547956e-01          zero[3].imag = -1.4160930802e+00

func[0].real =  8.8817841970e-16          func[0].imag =  0.0000000000e+00
func[1].real =  0.0000000000e+00          func[1].imag =  8.8817841970e-16
func[2].real =  1.7763568394e-15          func[2].imag = -2.6645352591e-15
func[3].real = -2.6645352591e-15          func[3].imag =  2.6645352591e-15

Degree (0 to quit) = 0

D:\MuellersMethod\x64\Debug\MuellersMethod.exe (process 25820) exited with code 0
(0x0).
Press any key to close this window . . .
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Let the real monic polynomial be:

$$p(x) = x^5 + 2x^4 + 3x^3 + 4x^2 + 5x + 6$$

```
Degree (0 to quit) = 5
coefficient[5].real = 1
coefficient[5].imag = 0
coefficient[4].real = 2
coefficient[4].imag = 0
coefficient[3].real = 3
coefficient[3].imag = 0
coefficient[2].real = 4
coefficient[2].imag = 0
coefficient[1].real = 5
coefficient[1].imag = 0
coefficient[0].real = 6
coefficient[0].imag = 0

zero[0].real = -8.0578646939e-01          zero[0].imag =  1.2229047134e+00
zero[1].real = -8.0578646939e-01          zero[1].imag = -1.2229047134e+00
zero[2].real =  5.5168546346e-01          zero[2].imag =  1.2533488603e+00
zero[3].real =  5.5168546346e-01          zero[3].imag = -1.2533488603e+00
zero[4].real = -1.4917979881e+00          zero[4].imag = -5.9164567892e-31

func[0].real =  8.8817841970e-16          func[0].imag =  4.4408920985e-16
func[1].real =  8.8817841970e-16          func[1].imag = -4.4408920985e-16
func[2].real =  8.8817841970e-16          func[2].imag = -2.6645352591e-15
func[3].real =  8.8817841970e-16          func[3].imag = -1.7763568394e-15
func[4].real = -2.6645352591e-15          func[4].imag = -6.6847957074e-30
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Degree (0 to quit) = 0

D:\MuellersMethod\x64\Debug\MuellersMethod.exe (process 13944) exited with code 0
(0x0).
Press any key to close this window . . .
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Next let the real monic polynomial be:

$$p(x) = x^6 - 5x^5 + 4x^4 - 3x^3 + 2x^2 - x + 1$$

```
Degree (0 to quit) = 6
coefficient[6].real = 1
coefficient[6].imag = 0
coefficient[5].real = -5
coefficient[5].imag = 0
coefficient[4].real = 4
coefficient[4].imag = 0
coefficient[3].real = -3
coefficient[3].imag = 0
coefficient[2].real = 2
coefficient[2].imag = 0
coefficient[1].real = -1
coefficient[1].imag = 0
coefficient[0].real = 1
coefficient[0].imag = 0

zero[0].real =   3.1453925746e-01        zero[0].imag = -7.1300382143e-01
zero[1].real =   3.1453925746e-01        zero[1].imag =  7.1300382143e-01
zero[2].real =   8.4984407105e-01        zero[2].imag =  3.5293108401e-18
zero[3].real = -3.3536285937e-01         zero[3].imag =  5.9139916052e-01
zero[4].real = -3.3536285937e-01         zero[4].imag = -5.9139916052e-01
zero[5].real =   4.1918031328e+00        zero[5].imag = -1.1359597035e-28

func[0].real =   1.1102230246e-16        func[0].imag = -1.6653345369e-16
func[1].real =   0.0000000000e+00        func[1].imag =  1.6653345369e-16
func[2].real =   5.5511151231e-16        func[2].imag = -1.6449894948e-17
func[3].real = -2.2204460493e-16         func[3].imag =  3.8857805862e-16
func[4].real =   7.7715611724e-16        func[4].imag = -2.7755575616e-16
func[5].real =   4.2188474936e-14        func[5].imag = -1.2298721152e-25

Degree (0 to quit) = 0

D:\MuellersMethod\x64\Debug\MuellersMethod.exe (process 4000) exited with code 0
(0x0).
Press any key to close this window . . .
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The next four examples are from the numerical analysis reference textbook. The first finds two real roots of the Bessel function of the First Kind of Zeroth Order:

$$J_0(x) = 1 - \frac{x^2}{4} + \frac{x^4}{64} - \frac{x^6}{2304} + \cdots$$

```
Degree (0 to quit) = 6
coefficient[6].real = -4.3402777778e-4
coefficient[6].imag = 0
coefficient[5].real = 0
coefficient[5].imag = 0
coefficient[4].real = 0.015625
coefficient[4].imag = 0
coefficient[3].real = 0
coefficient[3].imag = 0
coefficient[2].real = -0.25
coefficient[2].imag = 0
coefficient[1].real = 0
coefficient[1].imag = 0
coefficient[0].real = 1
coefficient[0].imag = 0

zero[0].real = -2.3916466909e+00        zero[0].imag =  0.0000000000e+00
zero[1].real =  2.3916466909e+00        zero[1].imag =  0.0000000000e+00
zero[2].real =  4.1958233454e+00        zero[2].imag = -1.5700065265e+00
zero[3].real = -4.1958233454e+00        zero[3].imag = -1.5700065265e+00
zero[4].real = -4.1958233454e+00        zero[4].imag =  1.5700065265e+00
zero[5].real =  4.1958233454e+00        zero[5].imag =  1.5700065265e+00

func[0].real = -2.2204460493e-16        func[0].imag =  0.0000000000e+00
func[1].real = -2.2204460493e-16        func[1].imag =  0.0000000000e+00
func[2].real =  0.0000000000e+00        func[2].imag = -3.8857805862e-16
func[3].real = -4.4408920985e-16        func[3].imag =  5.5511151231e-16
func[4].real =  0.0000000000e+00        func[4].imag = -3.8857805862e-16
func[5].real =  5.5511151231e-16        func[5].imag =  5.5511151231e-16

Degree (0 to quit) = 0

D:\MuellersMethod\x64\Release\MuellersMethod.exe (process 18060) exited with code 0
(0x0).
Press any key to close this window . . .
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The next polynomial from the textbook is:

$$p(x) = x^3 + x - 3$$

```
Degree (0 to quit) = 3
coefficient[3].real = 1
coefficient[3].imag = 0
coefficient[2].real = 0
coefficient[2].imag = 0
coefficient[1].real = 1
coefficient[1].imag = 0
coefficient[0].real = -3
coefficient[0].imag = 0

zero[0].real =  1.2134116628e+00        zero[0].imag =  0.0000000000e+00
zero[1].real = -6.0670583138e-01        zero[1].imag =  1.4506122492e+00
zero[2].real = -6.0670583138e-01        zero[2].imag = -1.4506122492e+00

func[0].real = -4.4408920985e-16        func[0].imag =  0.0000000000e+00
func[1].real = -8.8817841970e-16        func[1].imag =  2.2204460493e-16
func[2].real =  8.8817841970e-16        func[2].imag =  8.8817841970e-16

Degree (0 to quit) = 0
```

$$p(x) = x^5 - 3.7x^4 + 7.4x^3 - 10.8x^2 + 10.8x - 6.8$$

```
Degree (0 to quit) = 5
coefficient[5].real = 1
coefficient[5].imag = 0
coefficient[4].real = -3.7
coefficient[4].imag = 0
coefficient[3].real = 7.4
coefficient[3].imag = 0
coefficient[2].real = -10.8
coefficient[2].imag = 0
coefficient[1].real = 10.8
coefficient[1].imag = 0
coefficient[0].real = -6.8
coefficient[0].imag = 0

zero[0].real =  1.0000000000e+00        zero[0].imag = -1.0000000000e+00
zero[1].real =  1.0000000000e+00        zero[1].imag =  1.0000000000e+00
zero[2].real =  3.3732656708e-17        zero[2].imag = -1.4142135624e+00
zero[3].real =  1.6845142627e-16        zero[3].imag =  1.4142135624e+00
zero[4].real =  1.7000000000e+00        zero[4].imag =  8.8746851837e-31

func[0].real =  8.8817841970e-16        func[0].imag =  0.0000000000e+00
func[1].real =  8.8817841970e-16        func[1].imag =  0.0000000000e+00
func[2].real =  0.0000000000e+00        func[2].imag =  2.6743455480e-15
func[3].real = -2.6645352591e-15        func[3].imag =  1.7021785424e-15
func[4].real =  8.8817841970e-16        func[4].imag =  6.4661843717e-30

Degree (0 to quit) = 0
```

$$p(x) = x^8 - 170x^6 + 7392x^4 - 39172x^2 + 51200$$

```
Degree (0 to quit) = 8
coefficient[8].real = 1
coefficient[8].imag = 0
coefficient[7].real = 0
coefficient[7].imag = 0
coefficient[6].real = -170
coefficient[6].imag = 0
coefficient[5].real = 0
coefficient[5].imag = 0
coefficient[4].real = 7392
coefficient[4].imag = 0
coefficient[3].real = 0
coefficient[3].imag = 0
coefficient[2].real = -39712
coefficient[2].imag = 0
coefficient[1].real = 0
coefficient[1].imag = 0
coefficient[0].real = 51200
coefficient[0].imag = 0

zero[0].real = -1.4142135624e+00          zero[0].imag =  0.0000000000e+00
zero[1].real =  1.4142135624e+00          zero[1].imag =  0.0000000000e+00
zero[2].real = -2.0000000000e+00          zero[2].imag =  0.0000000000e+00
zero[3].real =  2.0000000000e+00          zero[3].imag =  0.0000000000e+00
zero[4].real = -8.0000000000e+00          zero[4].imag =  0.0000000000e+00
zero[5].real =  8.0000000000e+00          zero[5].imag =  0.0000000000e+00
zero[6].real = -1.0000000000e+01          zero[6].imag =  0.0000000000e+00
zero[7].real =  1.0000000000e+01          zero[7].imag =  0.0000000000e+00

func[0].real =  0.0000000000e+00          func[0].imag =  0.0000000000e+00
func[1].real =  0.0000000000e+00          func[1].imag =  0.0000000000e+00
func[2].real =  0.0000000000e+00          func[2].imag =  0.0000000000e+00
func[3].real = -1.4551915228e-11          func[3].imag =  0.0000000000e+00
func[4].real =  0.0000000000e+00          func[4].imag =  0.0000000000e+00
func[5].real =  0.0000000000e+00          func[5].imag =  0.0000000000e+00
func[6].real = -1.4333636500e-09          func[6].imag =  0.0000000000e+00
func[7].real =  0.0000000000e+00          func[7].imag =  0.0000000000e+00

Degree (0 to quit) = 0

D:\MuellersMethod\x64\Release\MuellersMethod.exe (process 26360) exited with code 0
(0x0).
Press any key to close this window . . .
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