

Blog Entry © Tuesday, March 24, 2026, by James Pate Williams, Jr. Cubature on a Disk

Reference: [Cubature formulas for the unit disk](#)

First integral:

$$I(x^i y^j) = \iint_{x^2+y^2 \leq 1} x^i y^j dx dy = \int_{-1}^1 |\rho| \rho^{i+j} d\rho \int_{-1}^1 (1-t^2)^{-1/2} (1-t^2)^{i/2} t^j dt$$

Second integral:

$$I(\vec{x}) = \int_0^1 \prod_{k=1}^n \left| x_k - \frac{1}{3} \right|^{1/2} d\vec{x}$$

Actual value of second integral:

$$I(\vec{x}) = \left(\frac{2}{3}\right)^n \left[\left(\frac{2}{3}\right)^{3/2} + \left(\frac{1}{3}\right)^{3/2} \right]^n$$

Integral # 1

5	0.014382687766782
10	0.014721183667169
15	0.014725571825593
20	0.014726062543354
25	0.014726165085081
Simpson 5000000	0.014726215548205
Simpson 6000000	0.014726215548922
Simpson 7000000	0.014726215553673
Simpson 8000000	0.014726215573049
Simpson 9000000	0.014726215593847
Monte Carlo 10000000	0.014714961075823
Integral # 2	
Monte Carlo 10000000	0.014043552082956
actual	0.014043761994629
% error	0.001494696886834
Integral # 3	
Monte Carlo 10000000	3.141754800000000
actual	3.141592653589793
% error	0.005161280537812

D:\CubatureDisk\x64\Debug\CubatureDisk.exe (process 1580) exited with code 0 (0x0).

Press any key to close this window . . .