Blog Entry © Monday, April 20, 2026, by James Pate Williams, Jr., Vector Analysis Continued and Perhaps Corrected

My first formal course in Vector Analysis was Math 4583 Vector Analysis at the Georgia Institute of Technology in the Fall Quarter of 1981. I made a B in the class. I also studied CHEM 7121 Ligand Theory in which I earned an A and MATH 4347 Partial Differential Equations in which I garnered a B. I seem to recall that Professor Sloan taught MATH 4347. My future research advisor Professor Donald J. Royer taught me ligand field theory which is the quantum chemical theory of organometallic compounds. I took CHEM 6141 Group Theory Applications in the Spring Quarter 1981 under Professor Royer, and I scored an A in the course. In the Spring Quarter of 1981, I also studied CHEM 6422 Chemical Thermodynamics (B grade) and MATH 4582 Advanced Engineering Math (B grade). Professor Pierotti taught CHEM 6422 which was Chemical Statistical Thermodynamics. In that class I learned about chemical ensembles Bose-Einstein statistics that govern the behavior of bosons (integer spin particles) and Fermi-Dirac statistics that govern the behavior of fermions (particles with fractional spins).

$$\hat{e}_1 \times \hat{e}_2 = \begin{vmatrix} \hat{e}_1 & \hat{e}_2 & \hat{e}_3 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} \begin{matrix} \hat{e}_1 & \hat{e}_2 \\ 1 & 0 \\ 0 & 1 \end{matrix} = \hat{e}_3$$

$$\hat{e}_1 \times \hat{e}_3 = \begin{vmatrix} \hat{e}_1 & \hat{e}_2 & \hat{e}_3 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{matrix} \hat{e}_1 & \hat{e}_2 \\ 1 & 0 \\ 0 & 0 \end{matrix} = -\hat{e}_2$$

$$\hat{e}_3 \times \hat{e}_2 = \begin{vmatrix} \hat{e}_1 & \hat{e}_2 & \hat{e}_3 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} \begin{matrix} \hat{e}_1 & \hat{e}_2 \\ 0 & 0 \\ 0 & 1 \end{matrix} = -\hat{e}_1$$

$$\vec{A} \cdot \left(\vec{B} \times \vec{C}\right) = \begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix} \begin{matrix} A_1 & A_2 \\ B_1 & B_2 \\ C_1 & C_2 \end{matrix}$$
$$= A_1(B_2C_3 - B_3C_2) + A_2(B_3C_1 - B_1C_3) + A_3(B_1C_2 - B_2C_1)$$

$$\vec{A} \cdot \left(\vec{B} \times \vec{C}\right) = \varepsilon_{ijk} A_i B_j C_k$$
$$= \varepsilon_{123} A_1 B_2 C_3 + \varepsilon_{132} A_1 B_3 C_2 + \varepsilon_{231} A_2 B_3 C_1 + \varepsilon_{213} A_2 B_1 C_3 + \varepsilon_{312} A_3 B_1 C_2$$
$$+ \varepsilon_{321} A_3 B_2 C_1 = A_1 B_2 C_3 - A_1 B_3 C_2 + A_2 B_3 C_1 - A_2 B_1 C_3 + A_3 B_1 C_2 - A_3 B_2 C_1$$

$$\varepsilon_{ijk} = \begin{matrix} +1 \ (ijk) = (123), (231), (312) \\ -1 \ (ijk) = \ (321), (213), (132) \\ 0 \ (ijk) = otherwise \end{matrix}$$

The code uses 0 for, 1 for 2, and 2 for 3, i.e. C/C++ indexing instead of mathematical (FORTRAN) indexing. We use FORTRAN indexing in this document.

$$\vec{A}\times\left(\vec{B}\times\vec{C}\right)=\begin{vmatrix}\hat{e}_1 & \hat{e}_2 & \hat{e}_3 \\ A_1 & A_2 & A_3 \\ B_2C_3-B_3C_2 & B_3C_1-B_1C_3 & B_1C_2-B_2C_1\end{vmatrix}\begin{matrix}\hat{e}_1 & \hat{e}_2 \\ A_1 & A_2 \\ B_2C_3-B_3C_2 & B_3C_1-B_1C_3\end{matrix}$$

$$= A_2(B_1C_2-B_2C_1)\hat{e}_1 - A_3(B_3C_1-B_1C_3)\hat{e}_1 + A_3(B_2C_3-B_3C_2)\hat{e}_2$$
$$- A_1(B_1C_2-B_2C_1)\hat{e}_2 + A_1(B_3C_1-B_1C_3)\hat{e}_3 - A_2(B_2C_3-B_3C_2)\hat{e}_3$$

$$\vec{A}\times\left(\vec{B}\times\vec{C}\right)=\varepsilon_{ijk}A_i\left(\vec{B}\times\vec{C}\right)_i=\varepsilon_{ijk}\varepsilon_{klm}A_jB_lC_m\hat{e}_i=\left(\delta_{il}\delta_{jm}-\delta_{im}\delta_{jl}\right)A_jB_lC_m\hat{e}_i$$
$$=A_jB_iC_j\hat{e}_i-A_jB_jC_i\hat{e}_i=A_jC_jB_i\hat{e}_i-A_jB_jC_i\hat{e}_i=\left(\vec{A}\cdot\vec{C}\right)\vec{B}-\left(\vec{A}\cdot\vec{B}\right)\vec{C}$$