Blog Entry © Monday, April 20, 2026, by James Pate Williams, Jr., Vector Analysis Continued and Perhaps Corrected

My first formal course in Vector Analysis was Math 4583 Vector Analysis at the Georgia Institute of Technology in the Fall Quarter of 1981. I made a B in the class. I also studied CHEM 7121 Ligand Theory in which I earned an A and MATH 4347 Partial Differential Equations in which I garnered a B. I seem to recall that Professor Sloan taught MATH 4347.

$$\hat{e}_1 \times \hat{e}_2 = \begin{vmatrix} \hat{e}_1 & \hat{e}_2 & \hat{e}_3 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} \begin{matrix} \hat{e}_1 & \hat{e}_2 \\ 1 & 0 \\ 0 & 1 \end{matrix} = \hat{e}_3$$

$$\hat{e}_1 \times \hat{e}_3 = \begin{vmatrix} \hat{e}_1 & \hat{e}_2 & \hat{e}_3 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{matrix} \hat{e}_1 & \hat{e}_2 \\ 1 & 0 \\ 0 & 0 \end{matrix} = -\hat{e}_2$$

$$\hat{e}_3 \times \hat{e}_2 = \begin{vmatrix} \hat{e}_1 & \hat{e}_2 & \hat{e}_3 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} \begin{matrix} \hat{e}_1 & \hat{e}_2 \\ 0 & 0 \\ 0 & 1 \end{matrix} = -\hat{e}_1$$

$$\vec{A} \cdot \left(\vec{B} \times \vec{C}\right) = \begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix} \begin{matrix} A_1 & A_2 \\ B_1 & B_2 \\ C_1 & C_2 \end{matrix}$$
$$= A_1(B_2C_3 - B_3C_2) + A_2(B_3C_1 - B_1C_3) + A_3(B_1C_2 - B_2C_1)$$

$$\vec{A} \cdot \left(\vec{B} \times \vec{C}\right) = \varepsilon_{ijk}A_iB_jC_k = \varepsilon_{ijk}A_iB_jC_k$$

$$\varepsilon_{ijk} = \begin{matrix} +1\ (ijk) = (123), (231), (312) \\ -1\ (ijk) = \ (321), (213), (132) \\ 0\ (ijk) = otherwise \end{matrix}$$

The code uses 0 for, 1 for 2, and 2 for 3, i.e. C/C++ indexing instead of mathematical (FORTRAN) indexing. We use FORTRAN indexing in this document.

$$\vec{A} \times \left(\vec{B} \times \vec{C}\right) = \begin{vmatrix} \hat{e}_1 & \hat{e}_2 & \hat{e}_3 \\ A_1 & A_2 & A_3 \\ B_2C_3 - B_3C_2 & B_3C_1 - B_1C_3 & B_1C_2 - B_2C_1 \end{vmatrix} \begin{matrix} \hat{e}_1 & \hat{e}_2 \\ A_1 & A_2 \\ B_2C_3 - B_3C_2 & B_3C_1 - B_1C_3 \end{matrix}$$
$$= A_2(B_1C_2 - B_2C_1)\hat{e}_1 - A_3(B_3C_1 - B_1C_3)\hat{e}_1 + A_3(B_2C_3 - B_3C_2)\hat{e}_2$$
$$- A_1(B_1C_2 - B_2C_1)\hat{e}_2 + A_1(B_3C_1 - B_1C_3)\hat{e}_3 - A_2(B_2C_3 - B_3C_2)\hat{e}_3$$

$$\vec{A} \times \left(\vec{B} \times \vec{C}\right) = \varepsilon_{ijk}A_i\left(\vec{B} \times \vec{C}\right)_i = \varepsilon_{ijk}\varepsilon_{klm}A_jB_lC_m\hat{e}_i = \left(\delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}\right)A_jB_lC_m\hat{e}_i$$
$$= A_jB_iC_j\hat{e}_i - A_jB_jC_i\hat{e}_i = A_jC_jB_i\hat{e}_i - A_jB_jC_i\hat{e}_i = \left(\vec{A} \cdot \vec{C}\right)\vec{B} - \left(\vec{A} \cdot \vec{B}\right)\vec{C}$$