Blog Entry © Sunday, April 19, 2026, by James Pate Williams, Jr., Scattering from a Spherically Symmetric Potential

This C/C++ Win32 program was translated from a C# application created on June 25, 2015, and shared on the Microsoft TechNet.

References: ***Handbook of Mathematical Functions*** *edited by Milton Abramowitz and Irene A. Stegun* and *Chapter 19 of* ***Quantum Mechanics Third Edition*** *by Leonard I. Schiff namely Equations (19.12) and (19.13) on page 120. We also use Equation (19.15) on page 121 which is not in the following picture.* Spherical Bessel Function of the First Kind -- from Wolfram MathWorld phys516_2021_lec_040121.pdf

$$\left(\frac{1}{z}\frac{d}{dz}\right)^m [z^{n+1} f_n(z)] = z^{n-m+1} f_{n-m}(z)$$

$$z^n f_n'(z) + (n+1) z^{n-1} f_n(z) = z^n f_{n-1}(z)$$

$$f_n'(z) = f_{n-1}(z) - (n+1) z^{-1} f_n(z)$$

$$j_0(z) = \frac{\sin z}{z}$$

$$j_0'(z) = \frac{\cos z}{z} - \frac{\sin z}{z^2}$$

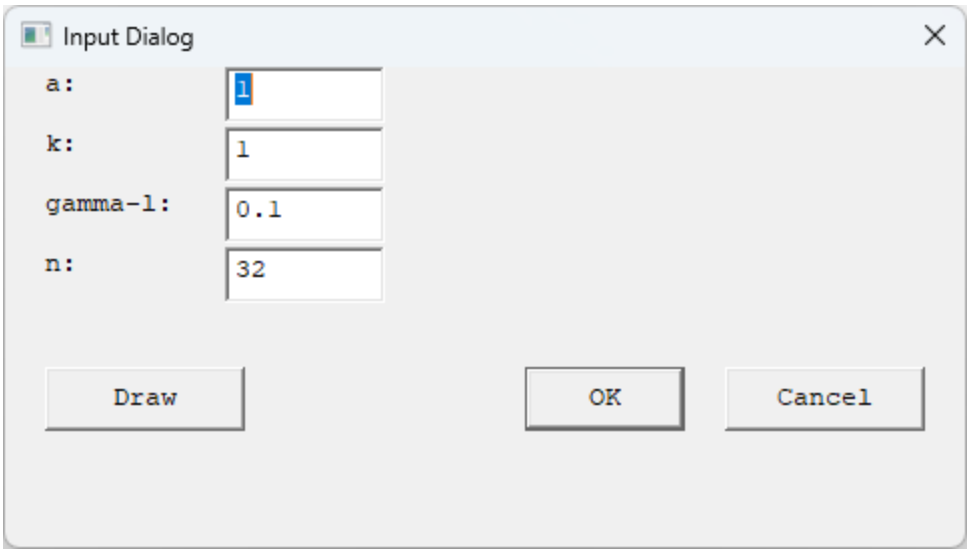
$$y_0(z) = -\frac{\cos z}{z}$$

$$y_0'(z) = \frac{\sin z}{z} + \frac{\cos z}{z^2}$$

$$\sigma(\theta) = \frac{1}{k^2}\left|\sum_{l=0}^{\infty} (2l+1) e^{i\delta_l} \sin\delta_l P_l(\cos\theta)\right|^2$$

$$\sigma = 2\pi \int_0^{\pi} \sigma(\theta) \sin\theta d\theta = \frac{4\pi}{k^2} \sum_{l=0}^{\infty} (2l+1) (\sin\delta_l)^2$$

The P function above is a Legendre polynomial. The sigma(theta) function is the differential cross section, and the final equation is the total elastic cross section. In the computer output we omit the adjective “elastic”.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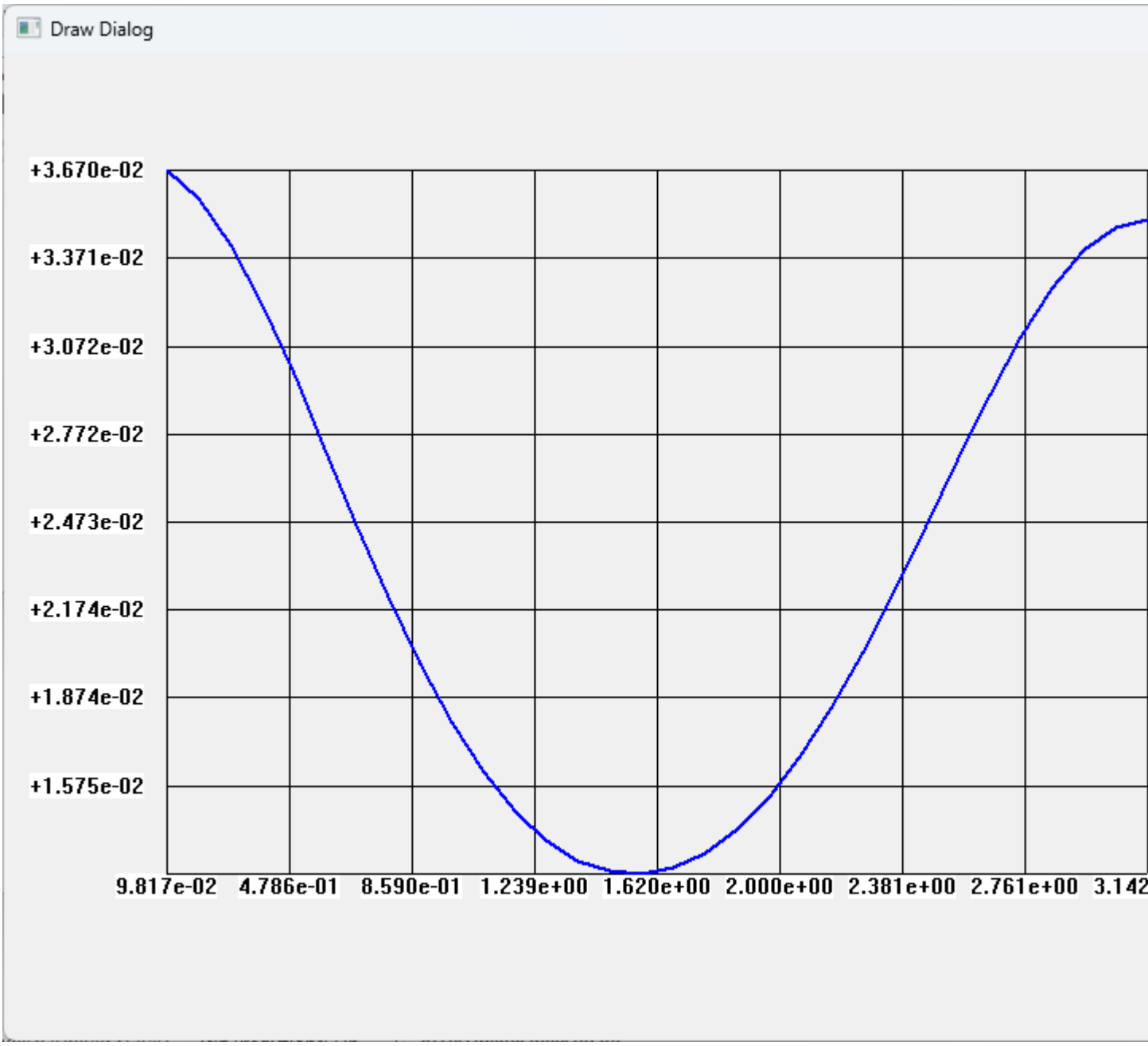
n:
32
Draw
OK
Cancel


Draw Dialog
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9.817e-02
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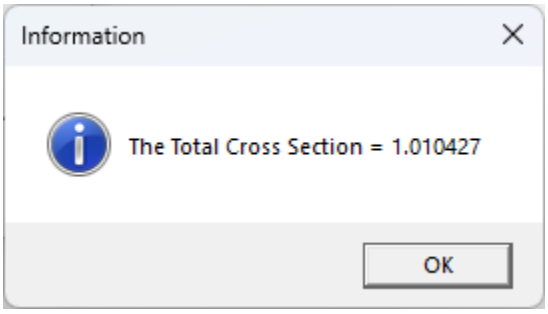
3.142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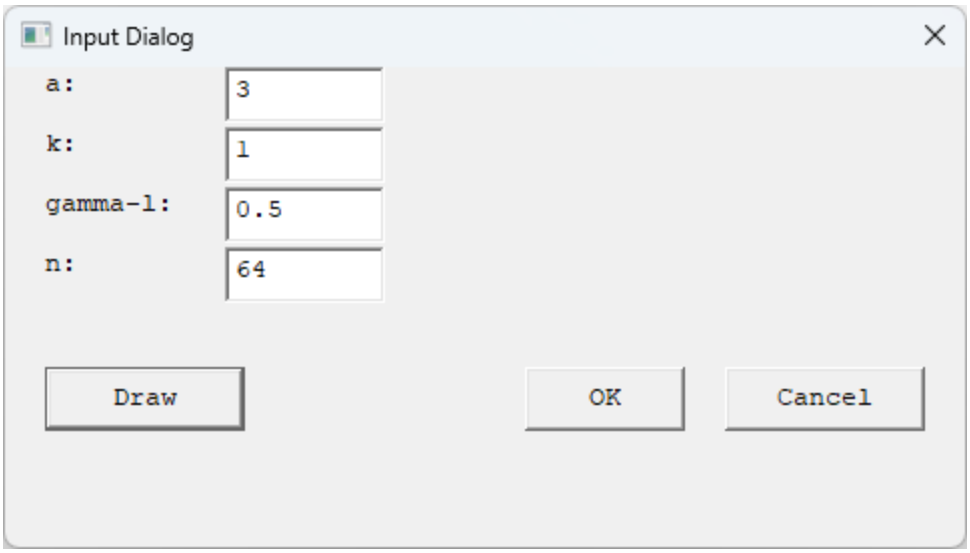
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gamma-1:
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n:
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OK
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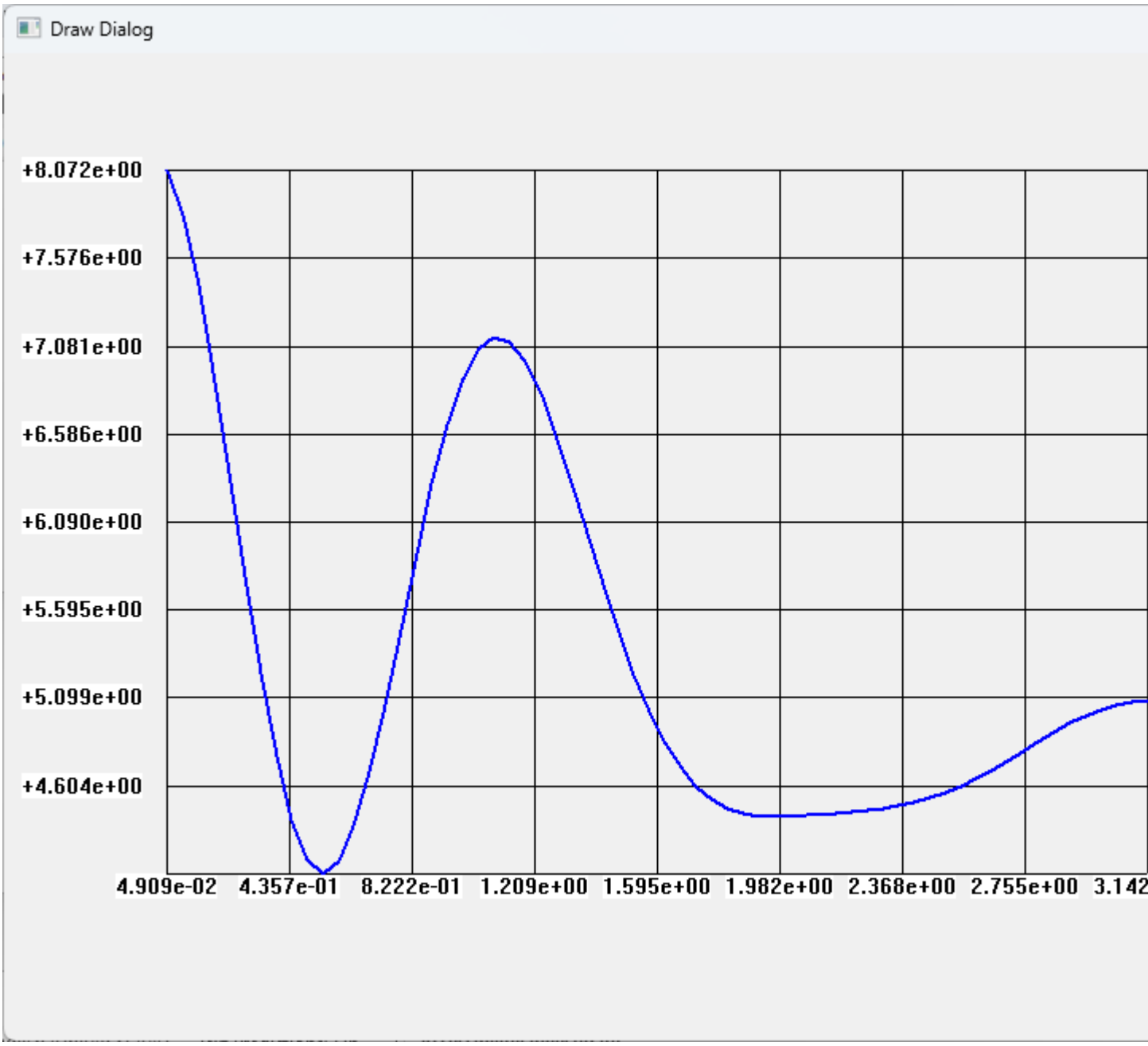
Draw Dialog
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+7.081e+00
+6.586e+00
+6.090e+00
+5.595e+00
+5.099e+00
+4.604e+00
4.909e-02
4.357e-01
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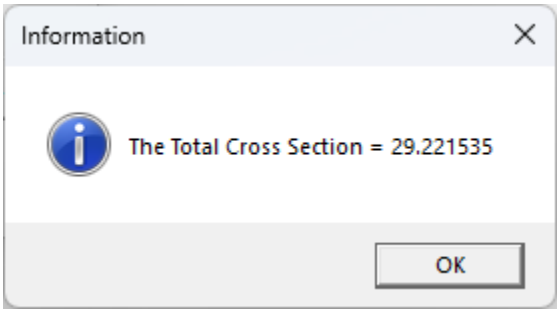
2.755e+00
3.142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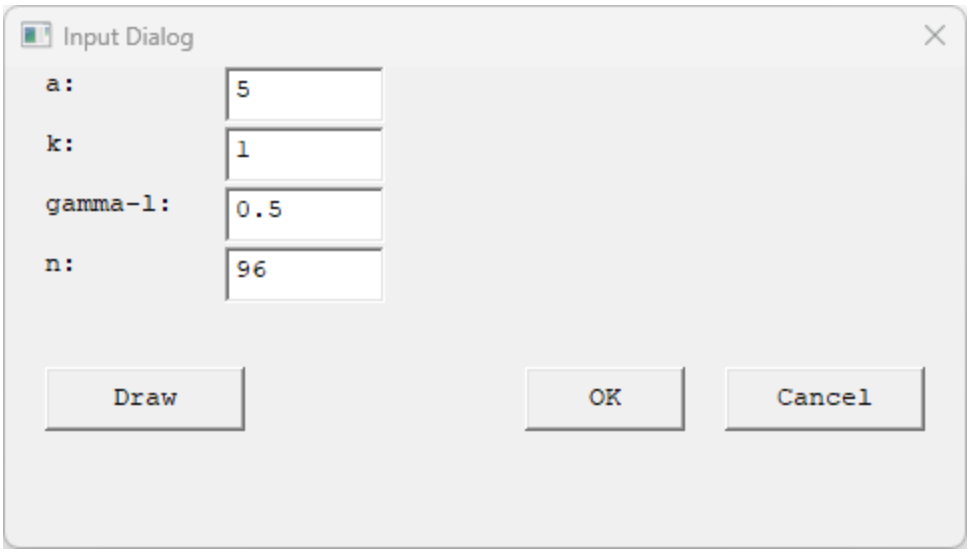
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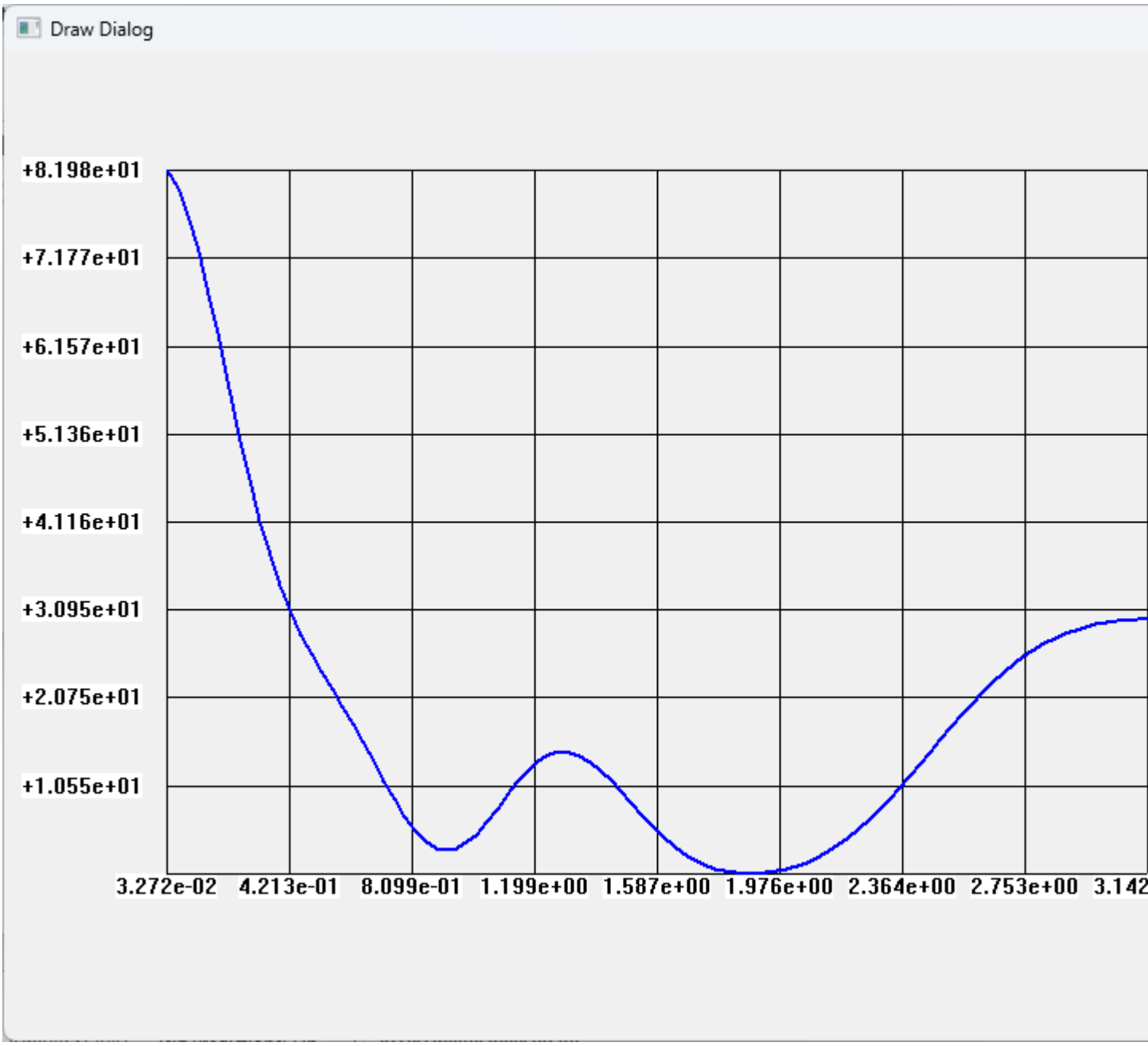
Draw Dialog
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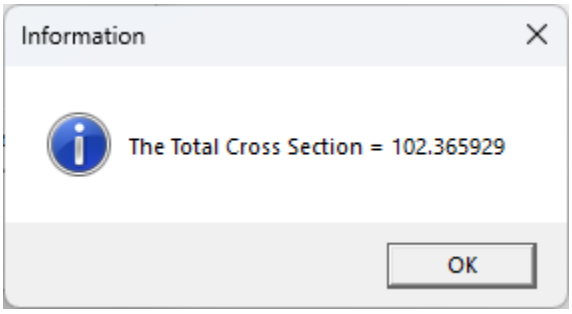
1.976e+00
2.364e+00
2.753e+00
3.142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The Total Cross Section = 102.365929
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