Blog Entry © Thursday, April 2, 2026, by James Pate Williams, Jr., Matrix Inverses that Exist and Their Characteristic Polynomials

Reference: 5.2: The Characteristic Polynomial - Mathematics LibreTexts

Inverse of the matrix:

$$A = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}, A^{-1} = \frac{1}{5-4}\begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix}$$

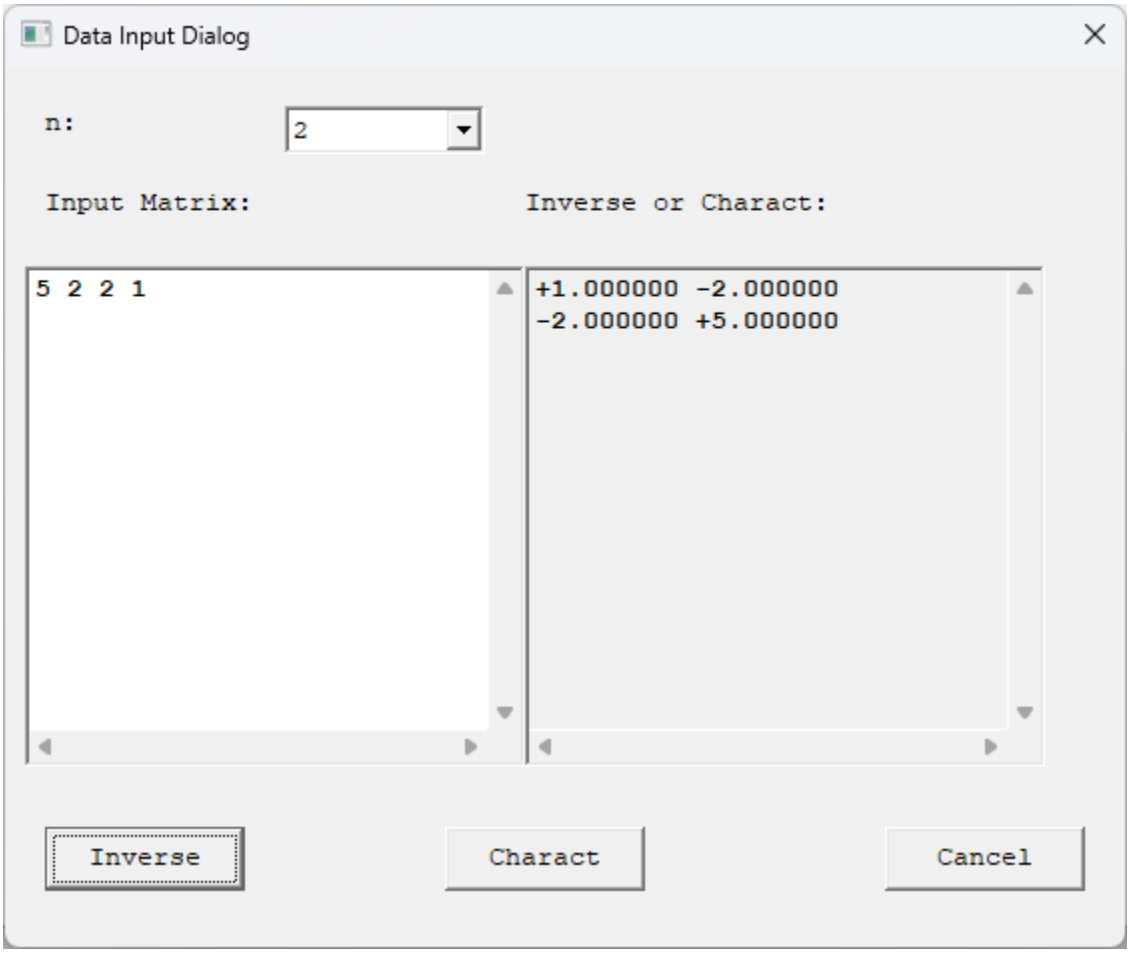

Figure 1 Inverse of a 2x2 Matrix

The characteristic polynomial is computed from the secular determinant:

$$\begin{vmatrix} 5-\lambda & 2 \\ 2 & 1-\lambda \end{vmatrix} = (5-\lambda)(1-\lambda) - 4 = 5 - 4 - \lambda - 5\lambda + \lambda^2 = 1 - 6\lambda + \lambda^2 = 0$$

The eigenvalues are:

$$\lambda = \frac{6 \pm \sqrt{36-4}}{2} = 3 \pm \frac{\sqrt{32}}{2} = 3 \pm \frac{\sqrt{16 \cdot 2}}{2} = 3 \pm \frac{4\sqrt{2}}{2} = 3 \pm 2\sqrt{2}$$

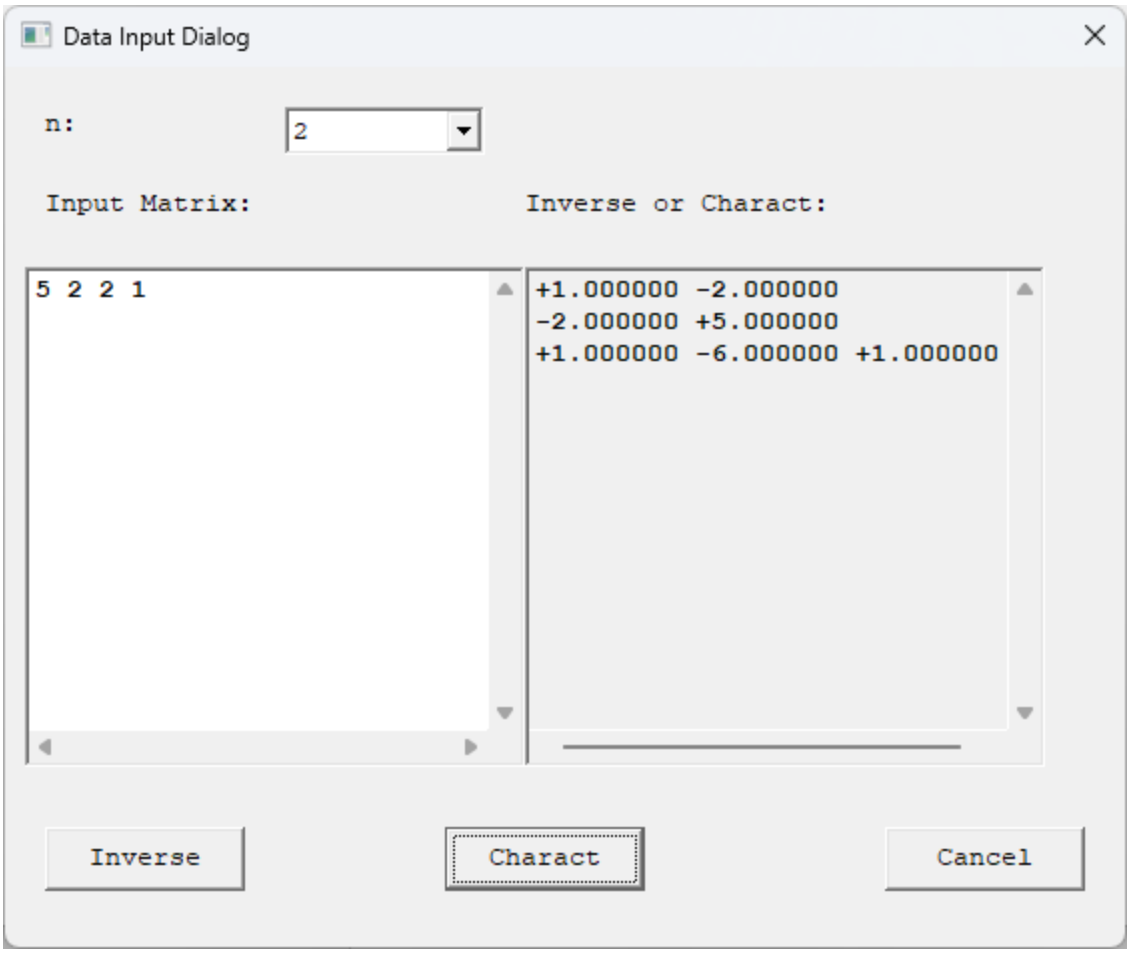

Figure 2 Characteristic Polynomial of a 2x2 Matrix

A 3x3 matrix:

$$A = \begin{bmatrix} 0 & 6 & 8 \\ 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \end{bmatrix}$$

The inverse of the 3x3 matrix is numerically calculated:

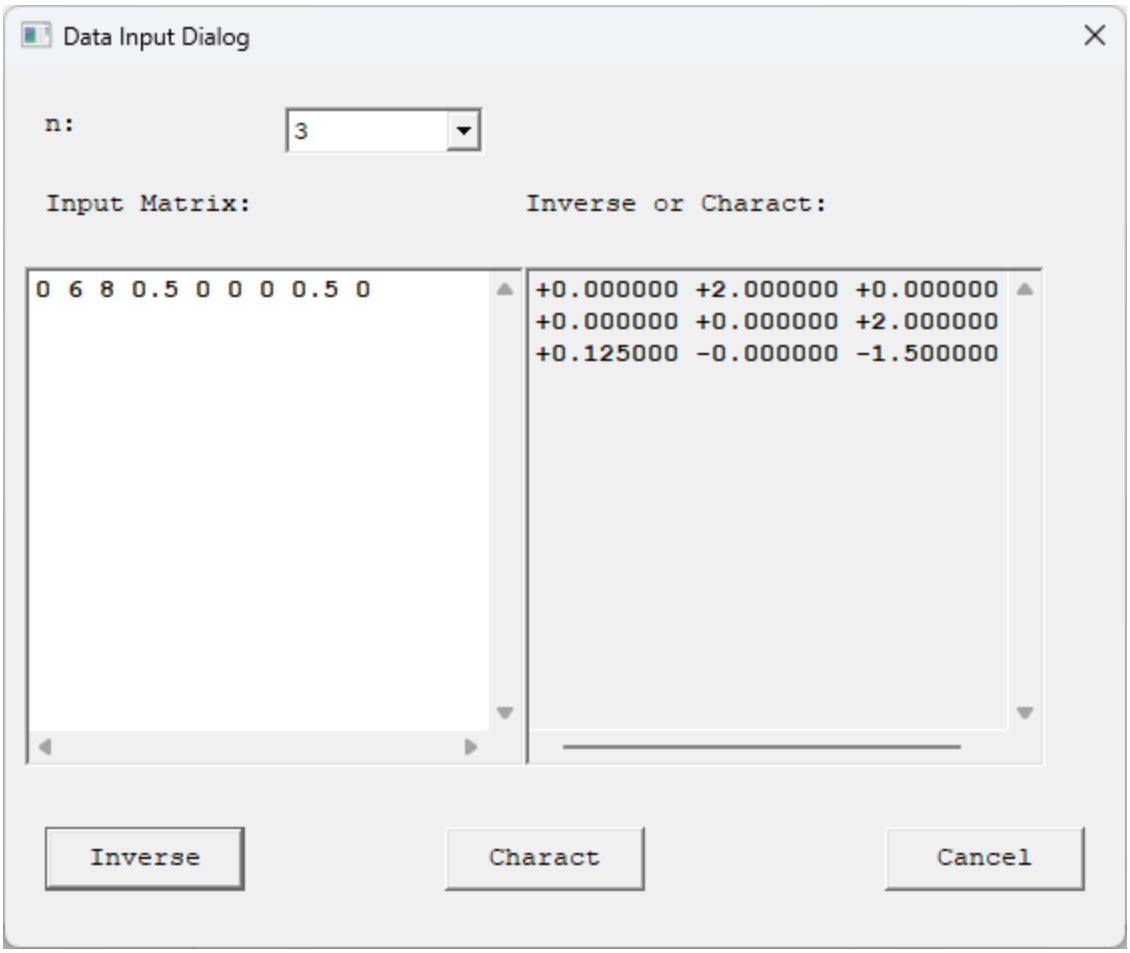

Figure 3 Inverse of a 3x3 Matrix

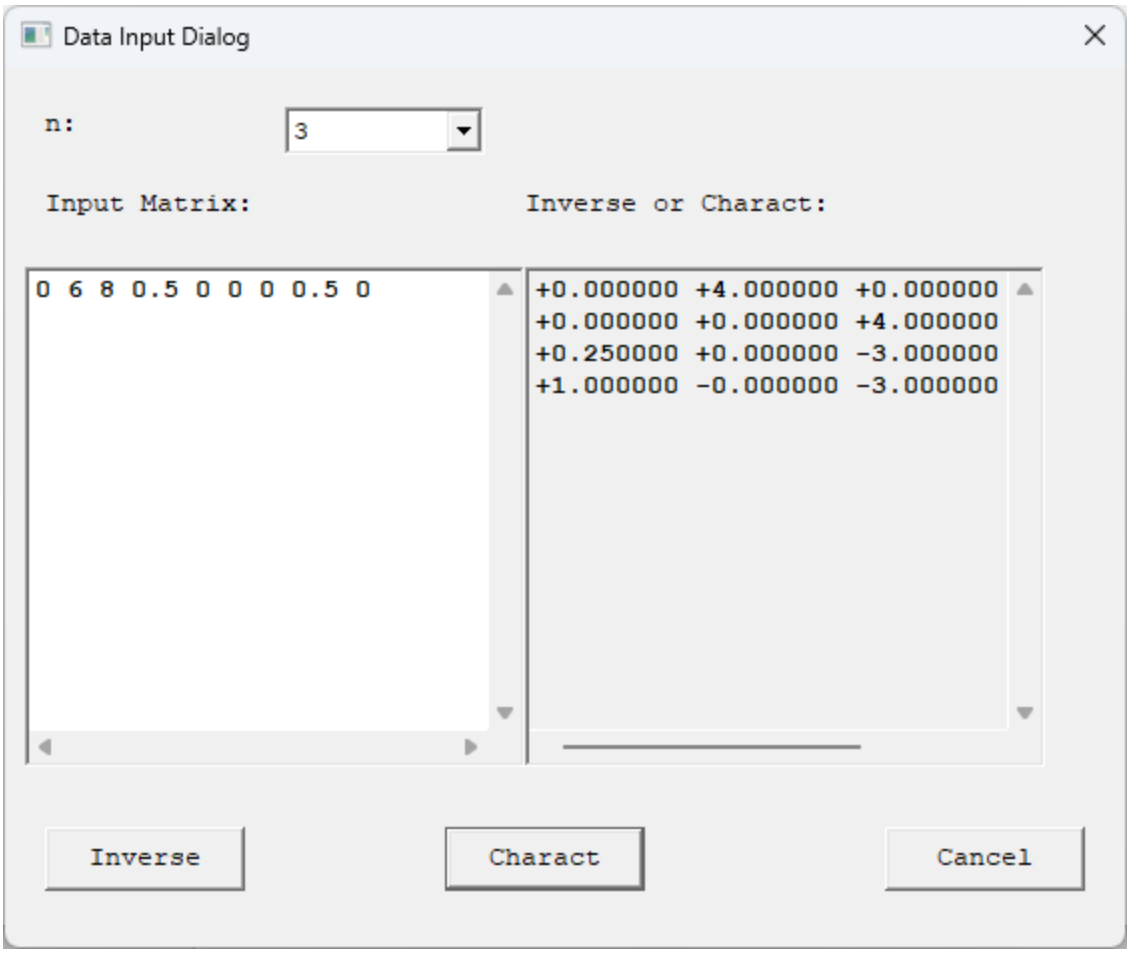

Figure 4 Characteristic Polynomial of a 3x3 Matrix

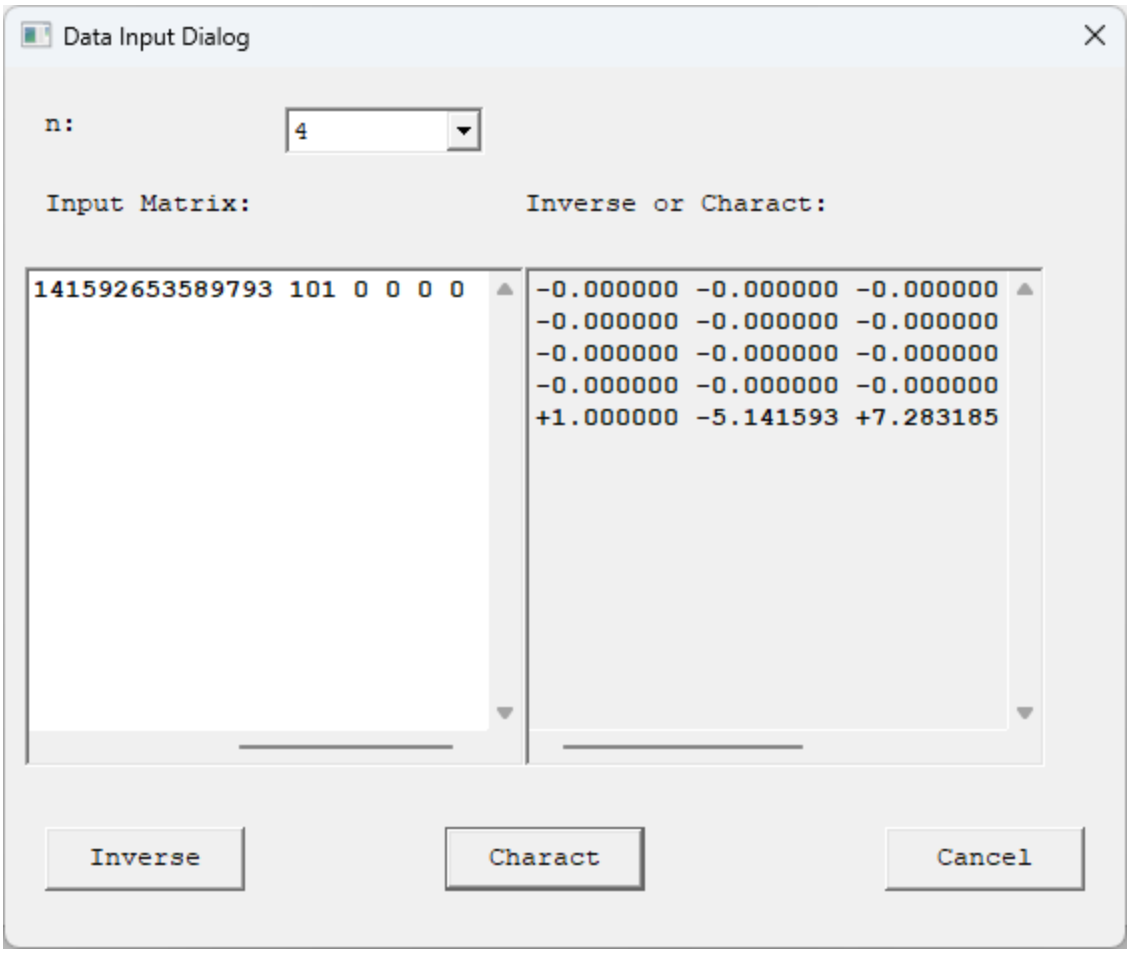

Figure 5 Characteristic Polynomial of a 4x4 Matrix

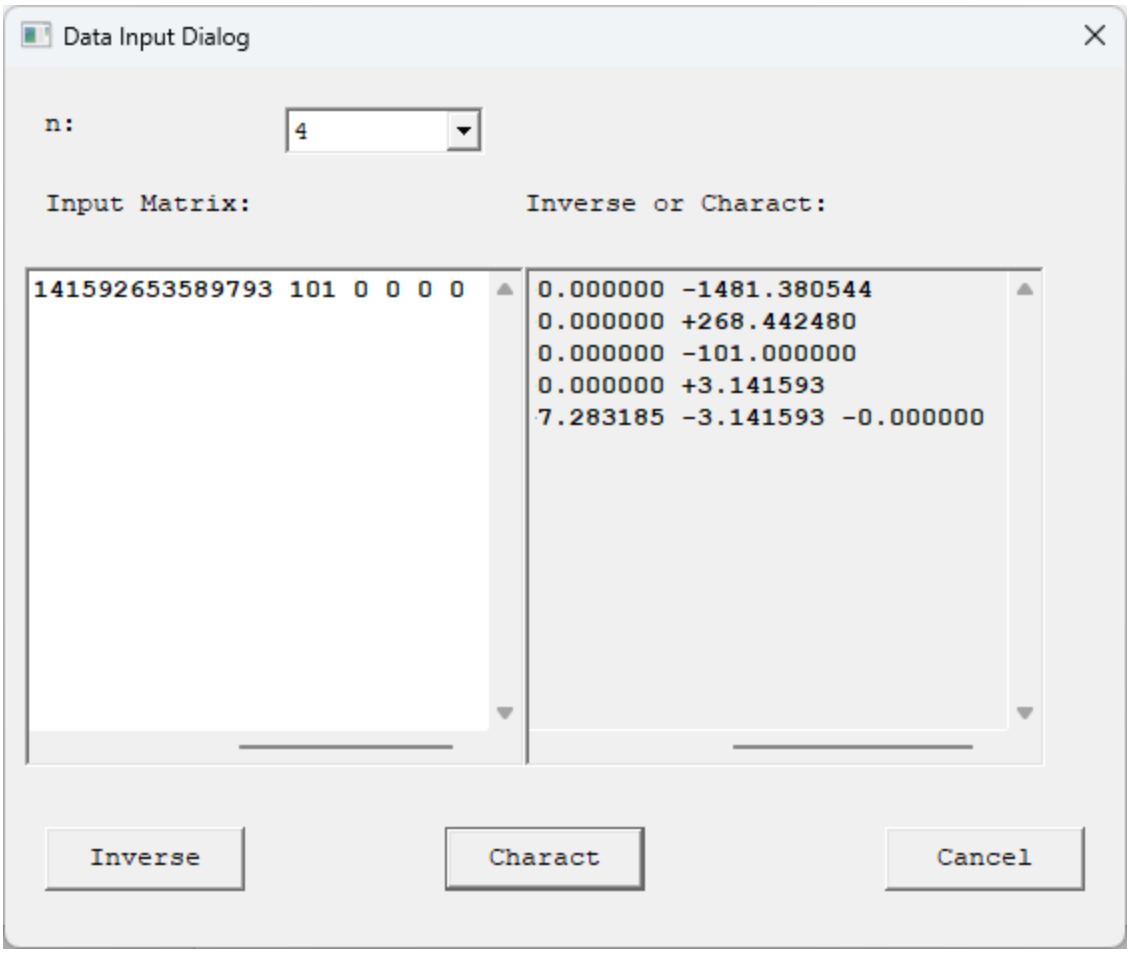

Figure 6 Characteristic Polynomial of a 4x4 Matrix Continued

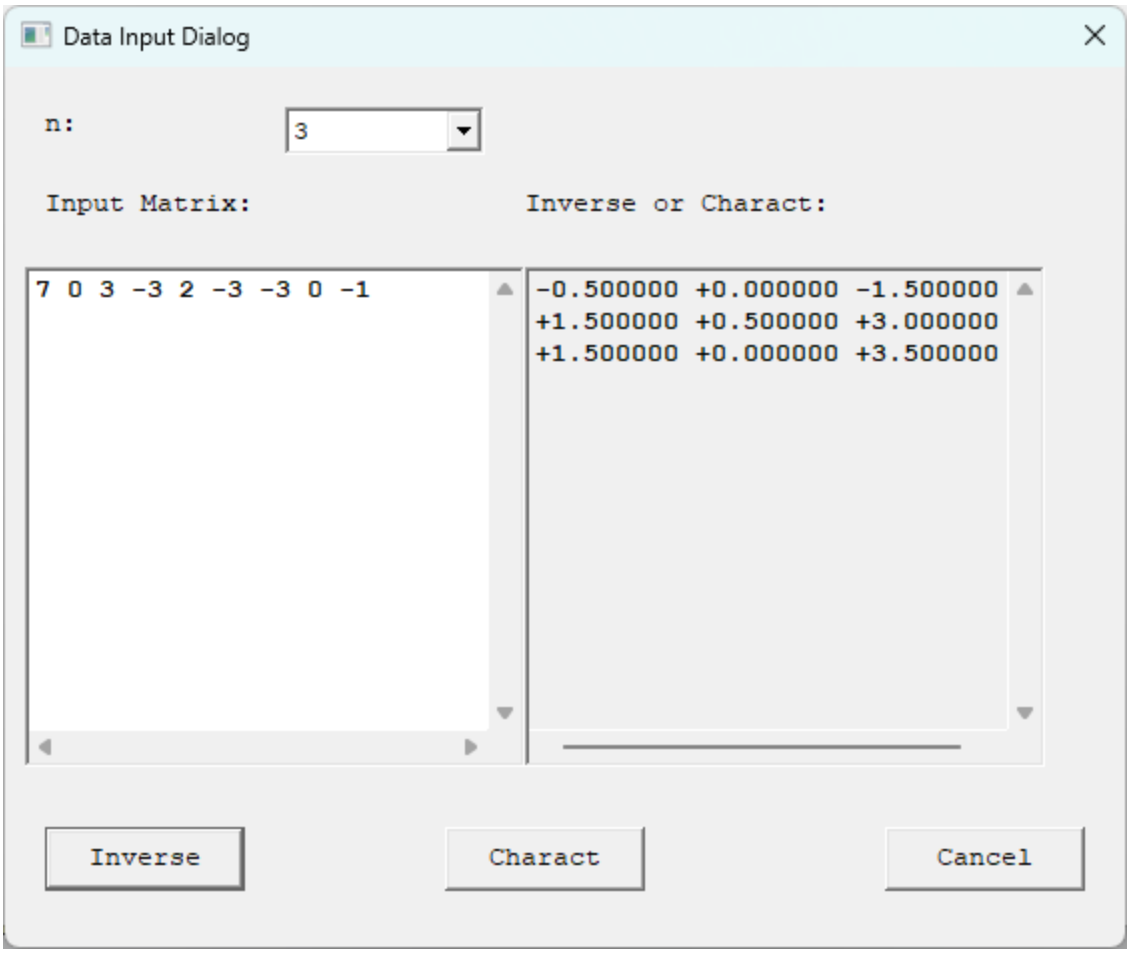

Figure 7 Inverse of a 3x3 Matrix

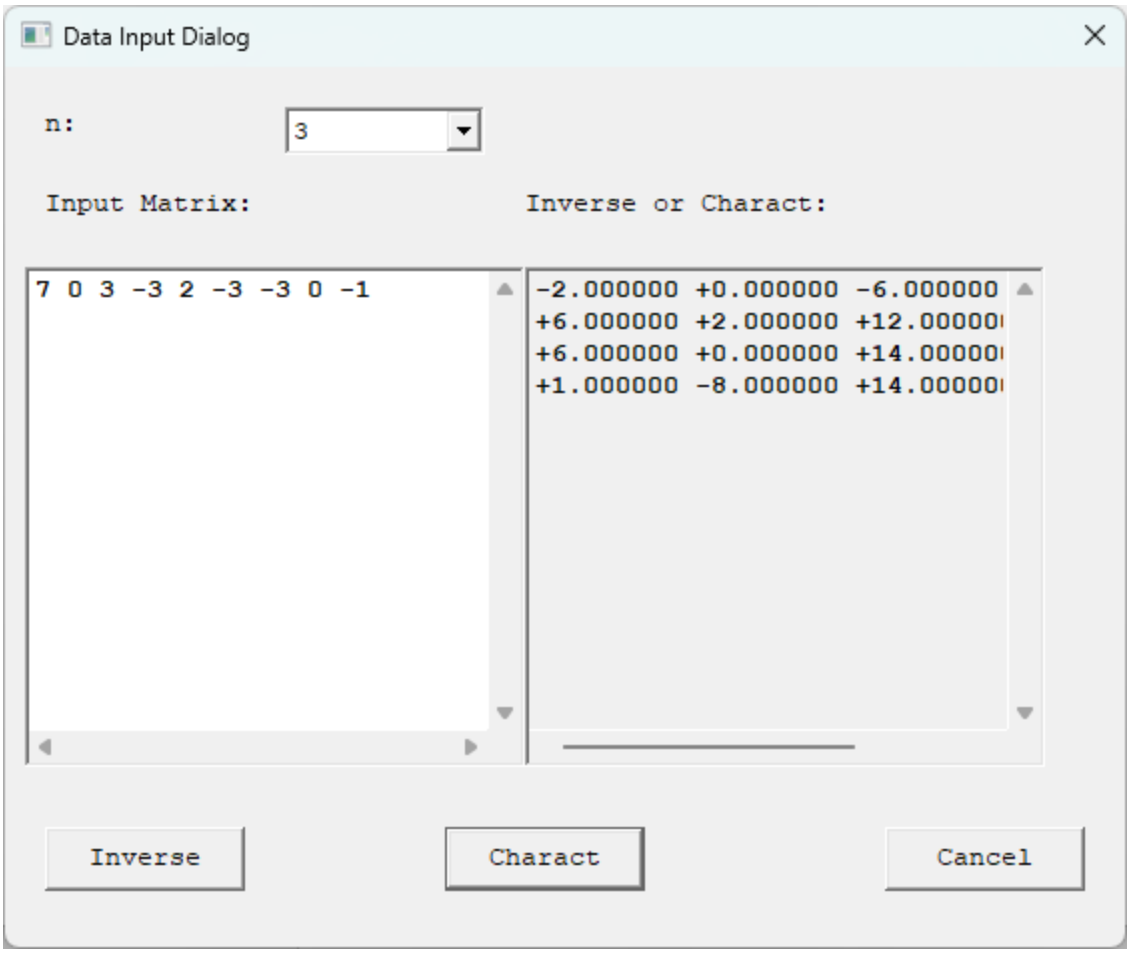

Figure 8 Characteristic Polynomial of a 3x3 Matrix

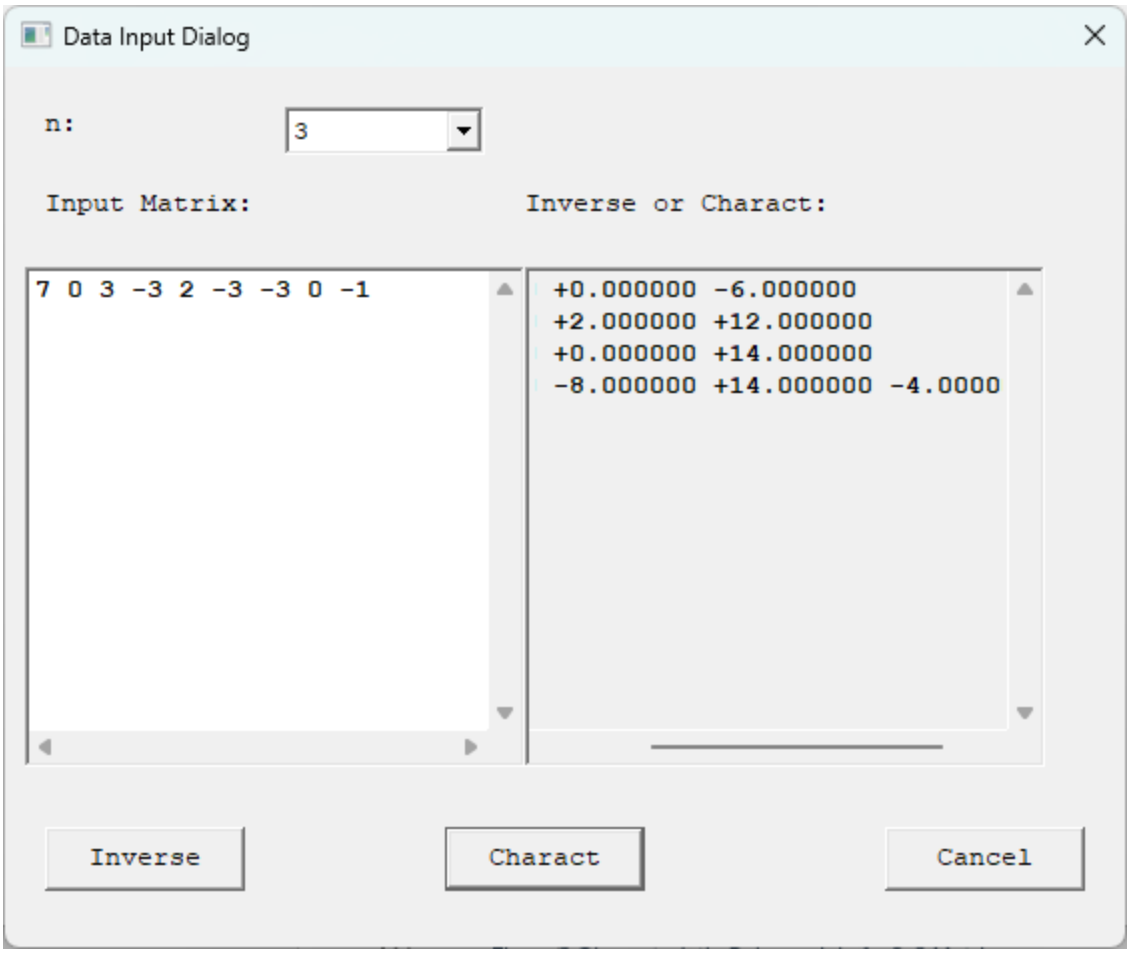

Figure 9 Characteristic Polynomial of a 3x3 Matrix Continued

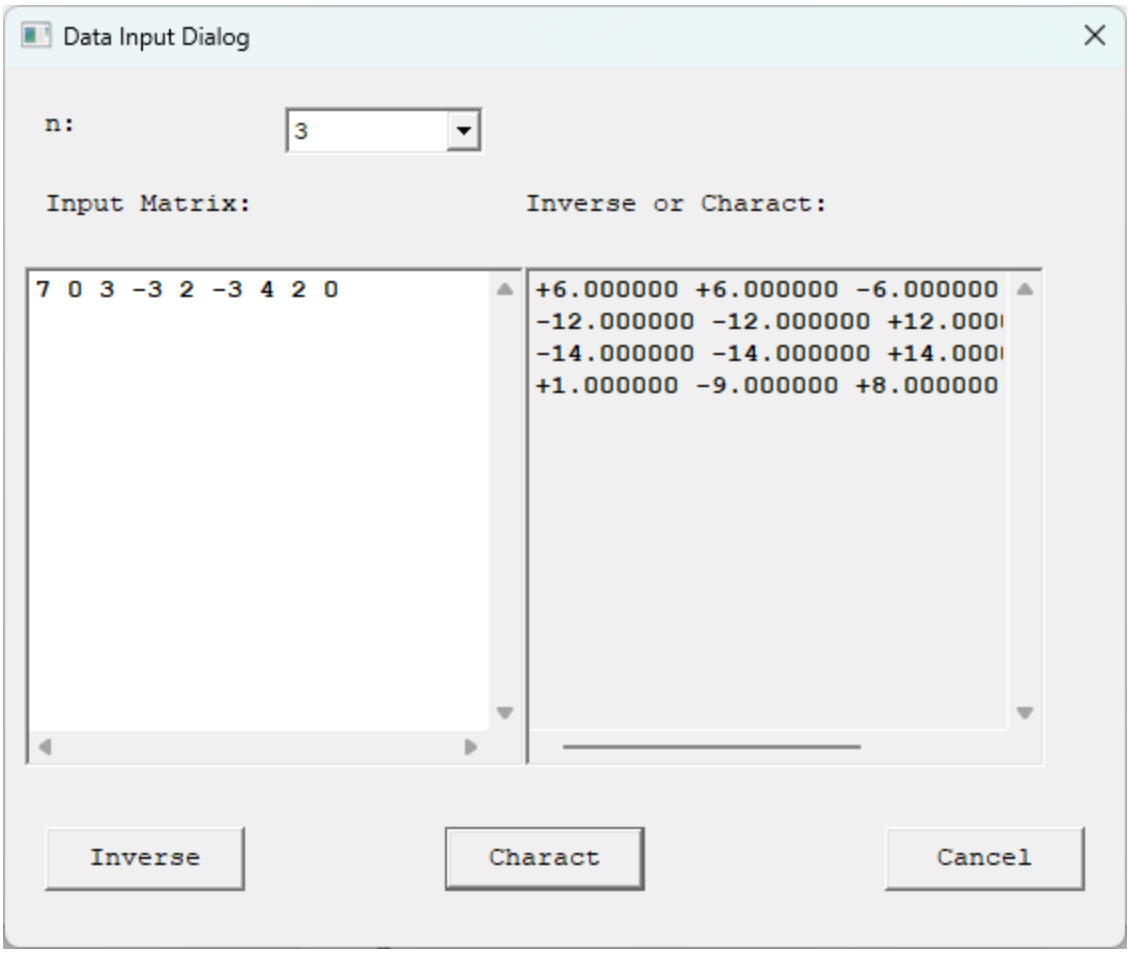

Figure 9 Characteristic Polynomial of a 3x3 Matrix