

Blog Entry © Thursday, April 2, 2026, by James Pate Williams, Jr., Matrix Inverses that Exist and Their Characteristic Polynomials

Reference: [5.2: The Characteristic Polynomial - Mathematics LibreTexts](#)

Inverse of the matrix:

$$A = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}, A^{-1} = \frac{1}{5-4} \begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix}$$

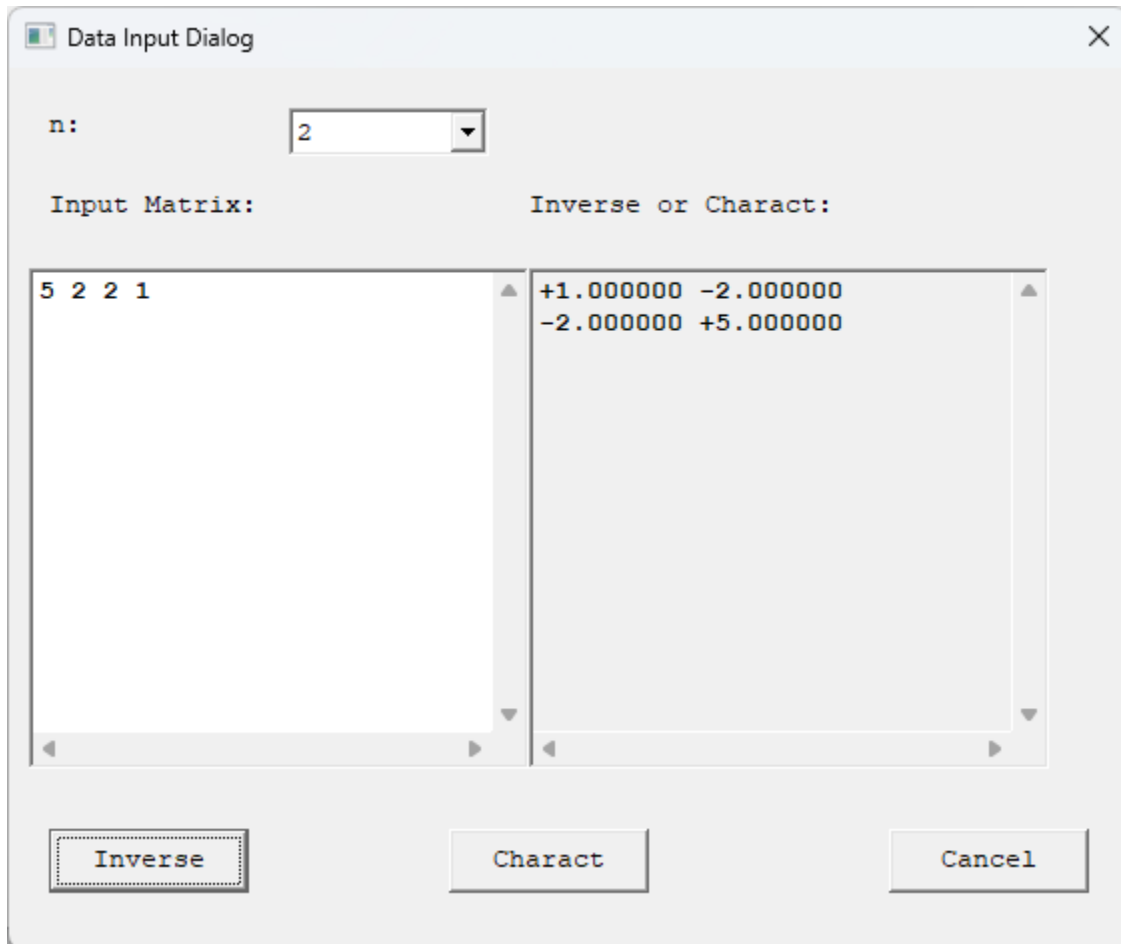


Figure 1 Inverse of a 2x2 Matrix

The characteristic polynomial is computed from the secular determinant:

$$\begin{vmatrix} 5-\lambda & 2 \\ 2 & 1-\lambda \end{vmatrix} = (5-\lambda)(1-\lambda) - 4 = 5 - 4 - \lambda - 5\lambda + \lambda^2 = 1 - 6\lambda + \lambda^2 = 0$$

The eigenvalues are:

$$\lambda = \frac{6 \pm \sqrt{36-4}}{2} = 3 \pm \frac{\sqrt{32}}{2} = 3 \pm \frac{\sqrt{16 \cdot 2}}{2} = 3 \pm \frac{4\sqrt{2}}{2} = 3 \pm 2\sqrt{2}$$

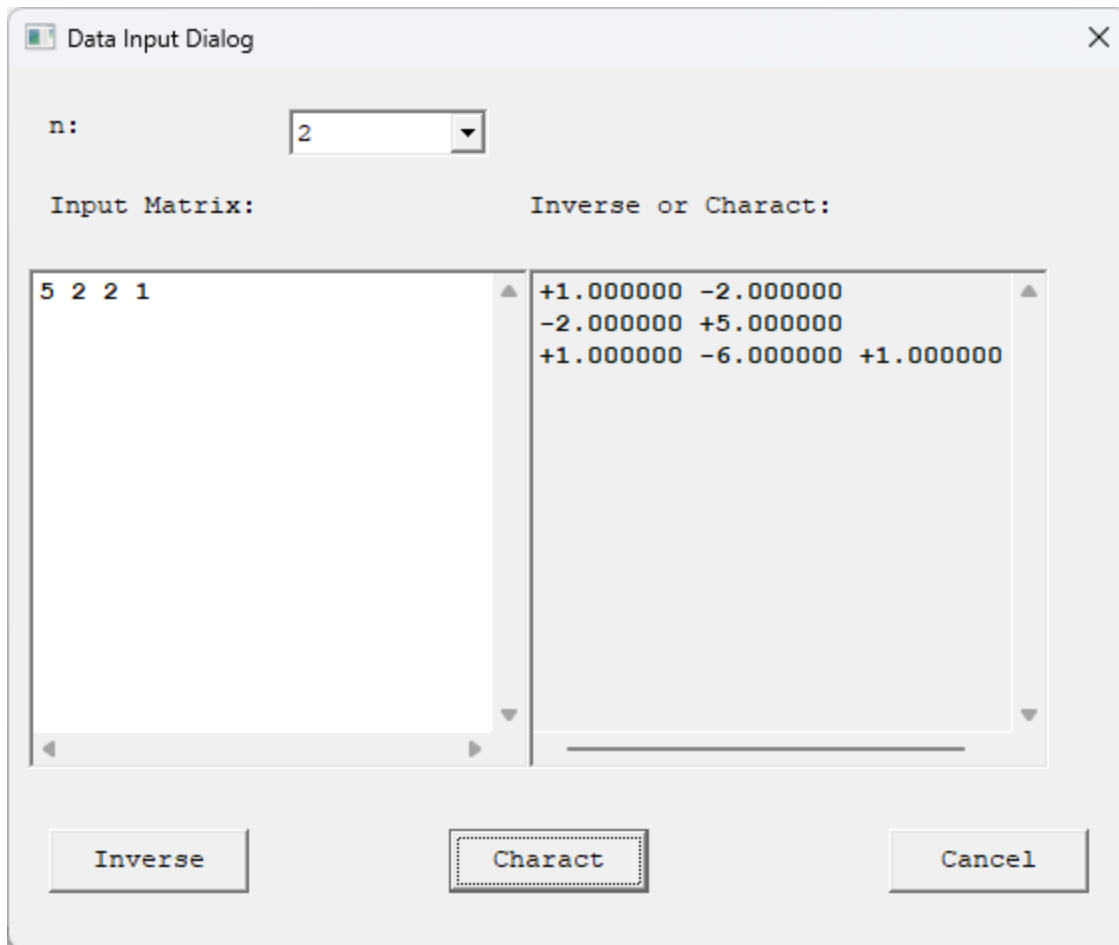


Figure 2 Characteristic Polynomial of a 2x2 Matrix

A 3x3 matrix:

$$A = \begin{bmatrix} 0 & 6 & 8 \\ 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \end{bmatrix}$$

The inverse of the 3x3 matrix is numerically calculated:

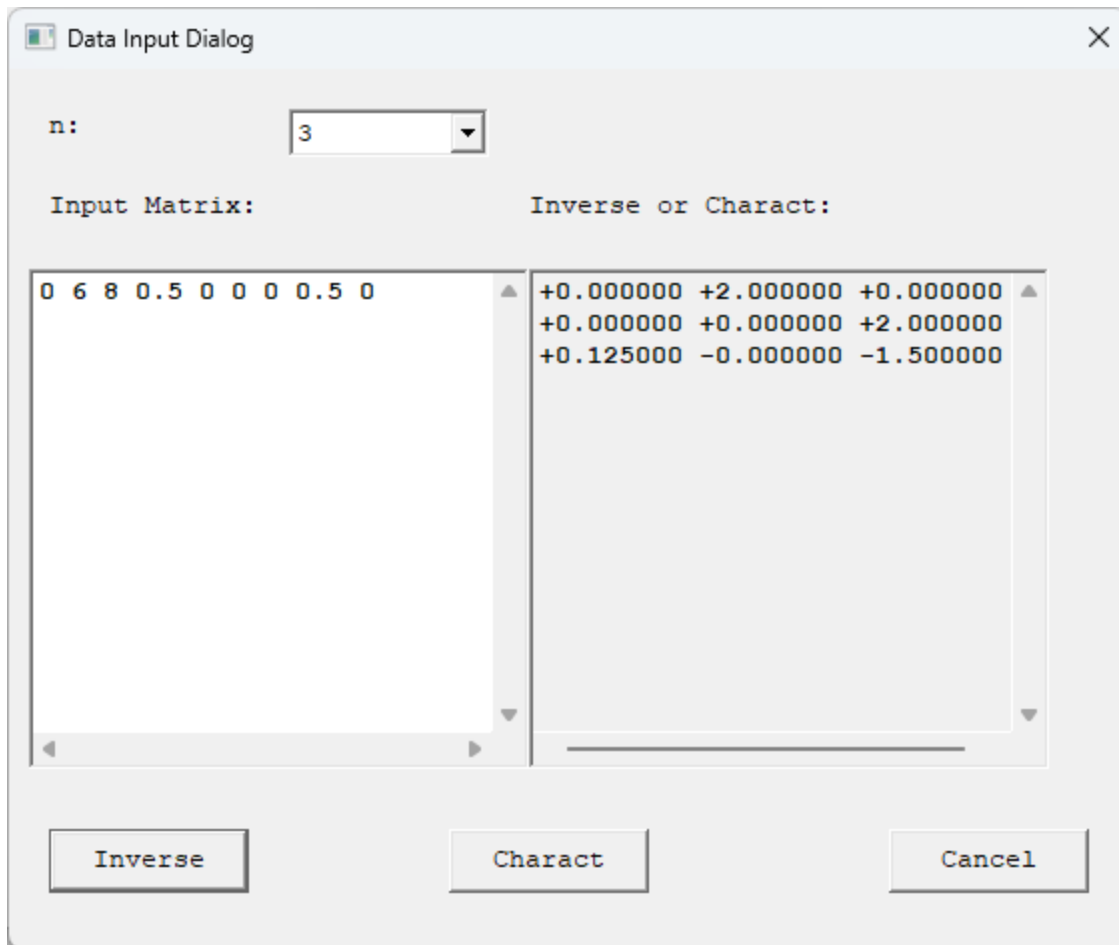


Figure 3 Inverse of a 3x3 Matrix

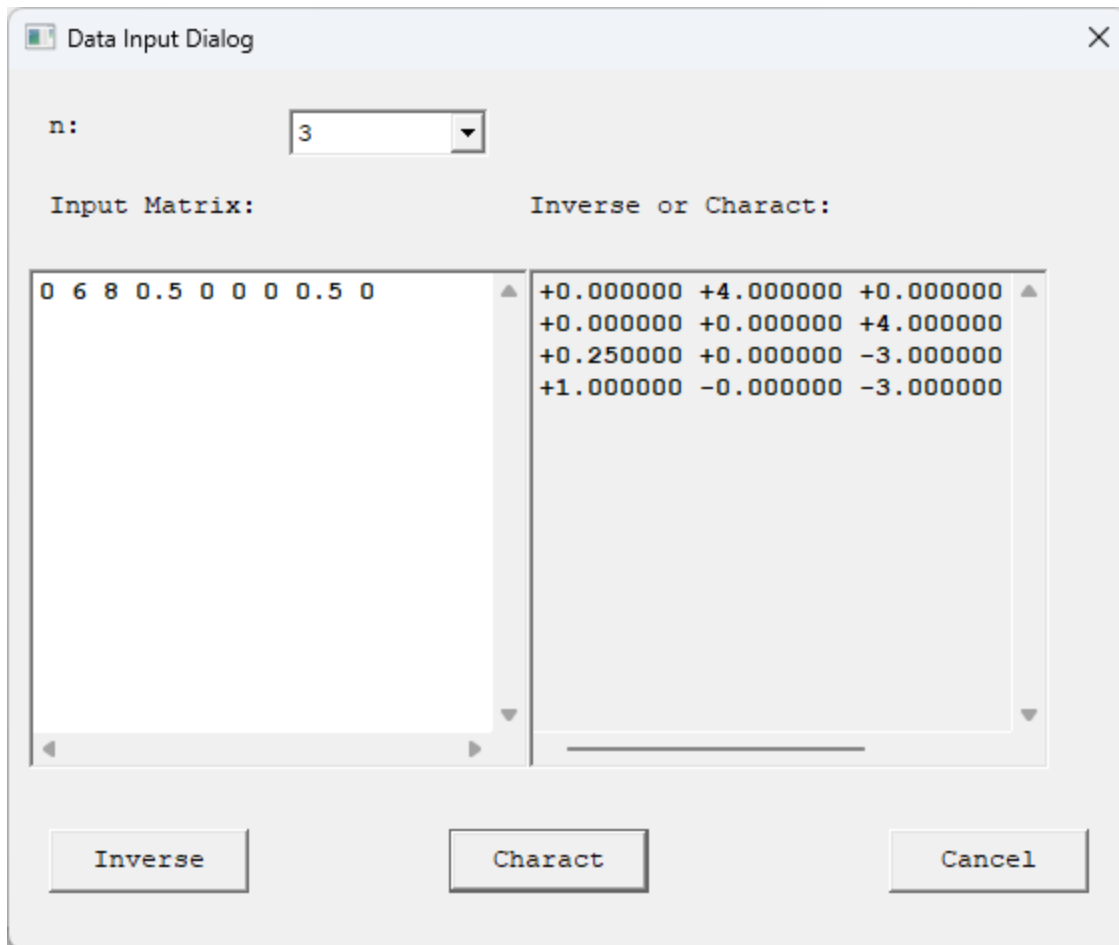


Figure 4 Characteristic Polynomial of a 3x3 Matrix

Data Input Dialog

n:

Input Matrix: Inverse or Charact:

141592653589793 101 0 0 0 0	-0.000000 -0.000000 -0.000000
	-0.000000 -0.000000 -0.000000
	-0.000000 -0.000000 -0.000000
	-0.000000 -0.000000 -0.000000
	+1.000000 -5.141593 +7.283185

Figure 5 Characteristic Polynomial of a 4x4 Matrix

Data Input Dialog

n: 4

Input Matrix: Inverse or Charact:

141592653589793 101 0 0 0 0	0.000000 -1481.380544
	0.000000 +268.442480
	0.000000 -101.000000
	0.000000 +3.141593
	7.283185 -3.141593 -0.000000

Inverse Charact Cancel

Figure 6 Characteristic Polynomial of a 4x4 Matrix Continued

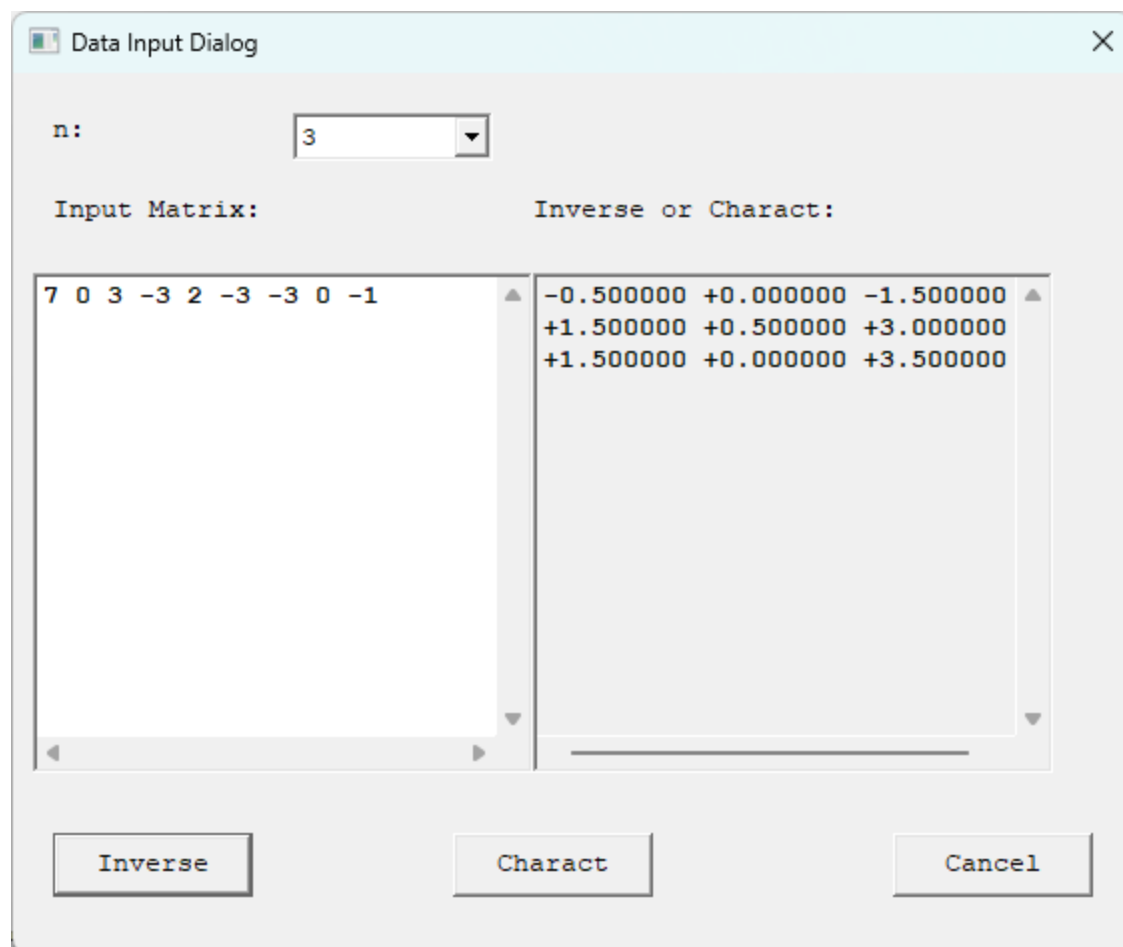


Figure 7 Inverse of a 3x3 Matrix

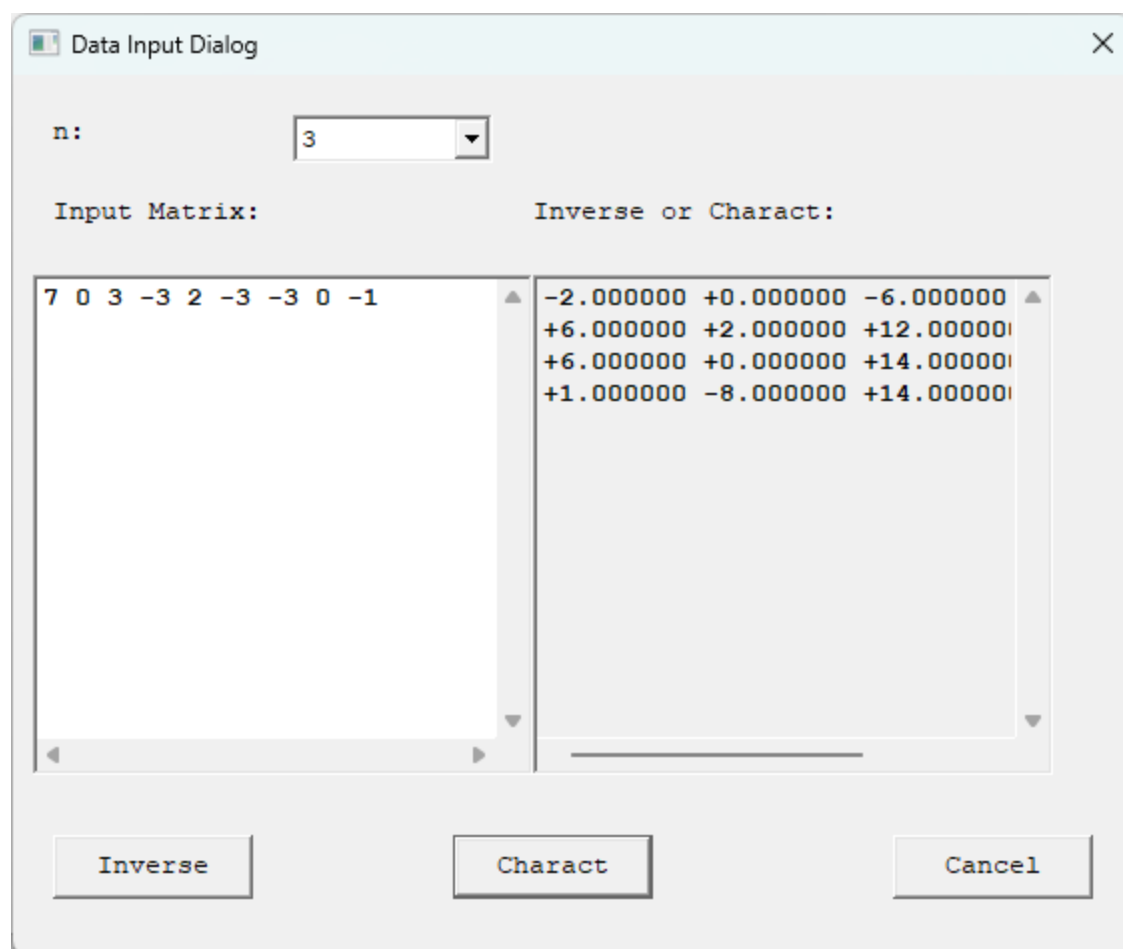


Figure 8 Characteristic Polynomial of a 3x3 Matrix

Data Input Dialog

n:

Input Matrix: Inverse or Charact:

7 0 3 -3 2 -3 -3 0 -1	+0.000000 -6.000000
	+2.000000 +12.000000
	+0.000000 +14.000000
	-8.000000 +14.000000 -4.000000

Figure 9 Characteristic Polynomial of a 3x3 Matrix Continued

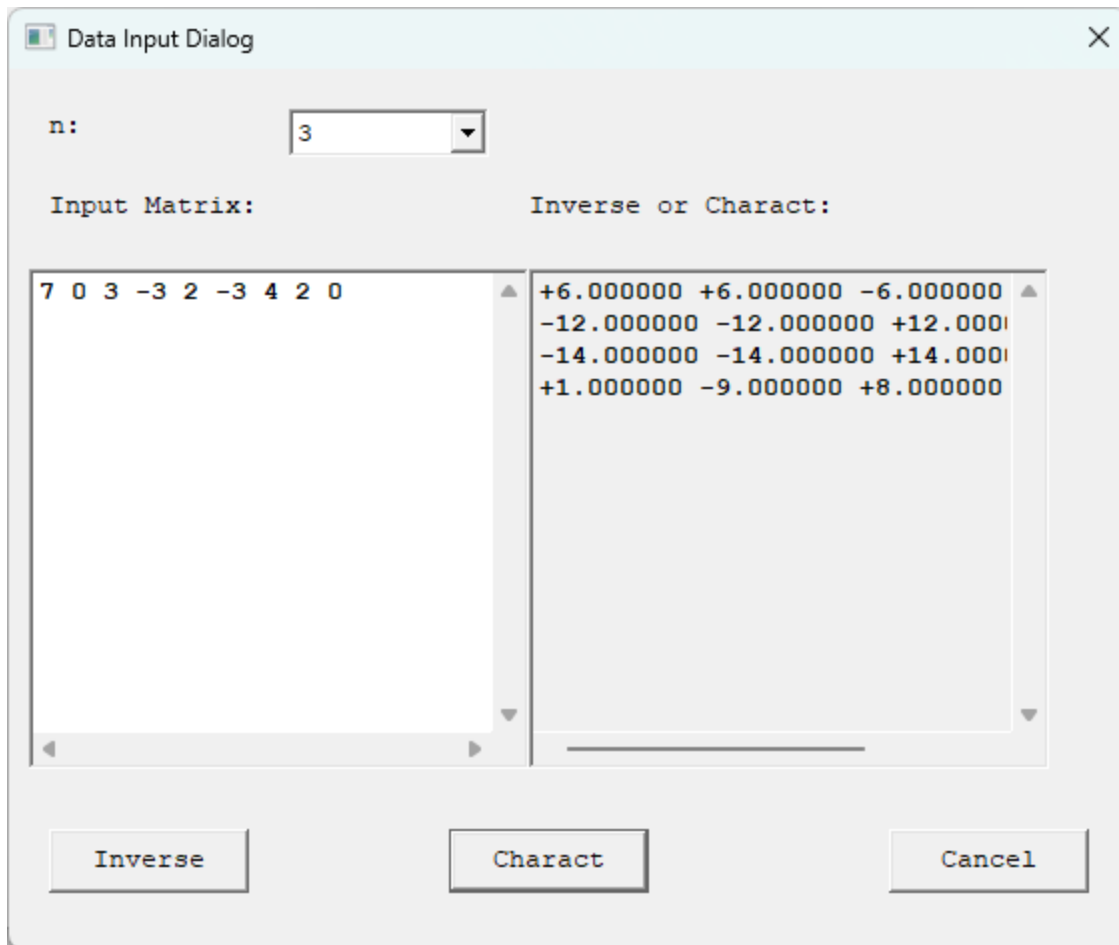


Figure 9 Characteristic Polynomial of a 3x3 Matrix