Blog Entry © April 1, 2026, by James Pate Williams, Jr. More Romberg Integration Results

```
Romberg integration of f(x) = exp(- x * x)
number of trapezoidal intervals = 1024
number of rows in table (<= 8) = 8
integral =  0.74682413280       evals = 131073
number of trapezoidal intervals = 131072
integral =  0.74682413281       evals = 131073
number of Simpson's   intervals = 256
integral =  0.74682413281       evals = 257
integral =  0.74682413279
Romberg integration of g(x) = exp(-x) * J0(x)
number of trapezoidal intervals = 1024
number of rows in table (<= 8) = 8
integral =  0.70710679233       evals = 131073
number of trapezoidal intervals = 131072
integral =  0.70710678555       evals = 131073
number of Simpson's   intervals = 2048
integral =  0.70710678106       evals = 2049
integral =  0.70710678119

D:\Romberg\x64\Debug\Romberg.exe (process 32356) exited with
code 0 (0x0).
Press any key to close this window . . .

Romberg integration of f(x) = exp(- x * x)
number of trapezoidal intervals = 2048
number of rows in table (<= 8) = 8
integral =  0.74682413281       evals = 262145
number of trapezoidal intervals = 262144
integral =  0.74682413281       evals = 262145
number of Simpson's   intervals = 1024
integral =  0.74682413281       evals = 1025
integral =  0.74682413279
Romberg integration of g(x) = exp(-x) * J0(x)
number of trapezoidal intervals = 2048
number of rows in table (<= 8) = 8
integral =  0.70710678397       evals = 262145
number of trapezoidal intervals = 262144
integral =  0.70710678228       evals = 262145
number of Simpson's   intervals = 2048
integral =  0.70710678106       evals = 2049
integral =  0.70710678119

D:\Romberg\x64\Debug\Romberg.exe (process 17972) exited with
code 0 (0x0).
Press any key to close this window . . .
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Romberg integration of f(x) = exp(- x * x)
number of trapezoidal intervals = 4096
number of rows in table (<= 8) = 8
integral =  0.74682413281       evals = 524289
number of trapezoidal intervals = 524288
integral =  0.74682413281       evals = 524289
number of Simpson's   intervals = 256
integral =  0.74682413281       evals = 257
integral =  0.74682413279
Romberg integration of g(x) = exp(-x) * J0(x)
number of trapezoidal intervals = 4096
number of rows in table (<= 8) = 8
integral =  0.70710678188       evals = 524289
number of trapezoidal intervals = 524288
integral =  0.70710678146       evals = 524289
number of Simpson's   intervals = 4096
integral =  0.70710678118       evals = 4097
integral =  0.70710678119

D:\Romberg\x64\Debug\Romberg.exe (process 17440) exited with
code 0 (0x0).
Press any key to close this window . . .

Romberg integration of f(x) = exp(- x * x)
number of trapezoidal intervals = 4096
number of rows in table (<= 8) = 8
integral =  0.74682413281       evals = 524289
number of trapezoidal intervals = 524288
integral =  0.74682413281       evals = 524289
number of Simpson's   intervals = 256
integral =  0.74682413281       evals = 257
integral =  0.74682413279
Romberg integration of g(x) = exp(-x) * J0(x)
number of trapezoidal intervals = 4096
number of rows in table (<= 8) = 8
integral =  0.70710678188       evals = 524289
number of trapezoidal intervals = 524288
integral =  0.70710678146       evals = 524289
number of Simpson's   intervals = 8192
integral =  0.70710678119       evals = 8193
integral =  0.70710678119

D:\Romberg\x64\Release\Romberg.exe (process 14708) exited with
code 0 (0x0).
Press any key to close this window . . .
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The last integral evaluates to:

$$g(x) = \int_0^{\infty} e^{-x} J_0(x) dx = \sin(45°)\,, J_0(x)\ is\ a\ Bessel\ Function\ of\ the\ First\ Kind$$

We use limits 0 to 30. Also, we have:

$$g(x) = \int_0^{\infty} e^{-x} J_0(x) dx = \frac{1}{\sqrt{2}}$$