Vector Analysis by James Pate Williams, Jr. Exercises and Supplementary Problems from ***Introduction to Vector Analysis Fourth Edition*** © 1979 by Harry F. Davis and Arthur David Snider Selected Exercises from Chapter 1 Page 48

1. Simplify

$$\left(\vec{A} \times \vec{B}\right) \cdot \vec{C} = \varepsilon_{ijk} A_i B_j C_k = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \begin{matrix} \hat{\imath} & \hat{\jmath} \\ A_x & A_y \\ B_x & B_y \end{matrix} \cdot \vec{C}$$

$$= \left[\left(A_y B_z - A_z B_y\right)\hat{\imath} + \left(A_z B_x - A_x B_z\right)\hat{\jmath} + \left(A_x B_y - A_y B_x\right)\hat{k}\right] \cdot \vec{C}$$

$$= \left(A_y B_z - A_z B_y\right) C_x + \left(A_z B_x - A_x B_z\right) C_y + \left(A_x B_y - A_y B_x\right) C_z$$

2. Simplify

$$\left(\vec{A} \times \vec{B}\right) \cdot \left(\vec{C} \times \vec{D}\right) = \varepsilon_{ijk} A_i B_j \hat{e}_k \varepsilon_{lmn} C_l D_m \hat{e}_n = \varepsilon_{ijk} A_i B_j \varepsilon_{lmk} C_l D_m$$

$$= \varepsilon_{ijk} \varepsilon_{lmk} A_i B_j C_l D_m = \left(\delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}\right) A_i B_j C_l D_m$$

$$= A_i B_j C_i D_j - A_i B_j C_j D_i = A_i B_j \left(C_i D_j - C_j D_i\right)$$

3. Simplify

$$\left(\vec{A} \times \vec{B}\right) \cdot \left(\vec{B} \times \vec{C}\right) \times \left(\vec{C} \times \vec{A}\right)$$

First, we need to compute the second and third factors:

$$\left(\vec{B} \times \vec{C}\right) \times \left(\vec{C} \times \vec{A}\right) = \varepsilon_{ijk} B_i C_j \hat{e}_k \varepsilon_{lmn} C_l A_m \hat{e}_n$$

$$\vec{B} \times \vec{C} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix} \begin{matrix} \hat{\imath} & \hat{\jmath} \\ B_x & B_y \\ C_x & C_y \end{matrix}$$

$$= \left(B_y C_z - B_z C_y\right)\hat{\imath} + \left(B_z C_x - B_x C_z\right)\hat{\jmath} + \left(B_x C_y - B_y C_x\right)\hat{k}$$

$$\vec{C} \times \vec{A} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ C_x & C_y & C_z \\ A_x & A_y & A_z \end{vmatrix} \begin{matrix} \hat{\imath} & \hat{\jmath} \\ C_x & C_y \\ A_x & A_y \end{matrix}$$

$$= \left(C_y A_z - C_z A_y\right)\hat{\imath} + \left(C_z A_x - C_x A_z\right)\hat{\jmath} + \left(C_x A_y - C_y A_x\right)\hat{k}$$

$$\left(\vec{B} \times \vec{C}\right) \times \left(\vec{C} \times \vec{A}\right)$$

$$= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ B_y C_z - B_z C_y & B_z C_x - B_x C_z & B_x C_y - B_y C_x \\ C_y A_z - C_z A_y & C_z A_x - C_x A_z & C_x A_y - C_y A_x \end{vmatrix} \begin{matrix} \hat{\imath} & \hat{\jmath} \\ B_y C_z - B_z C_y & B_z C_x - B_x C_z \\ C_y A_z - C_z A_y & C_z A_x - C_x A_z \end{matrix}$$

Like any good computer scientist and software engineer, I will write Win32 C/C++ console application to verify my formulas.

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Exercise 1 (a)  30
Exercise 1 (b)  -13
Exercise 1 (c)  5
Exercise 1 (d)  1
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Exercise 2      5
Exercise 3      0
Exercise 4      0.666667
Exercise 5      1
Exercise 10     0.648886        0.648886

D:\VectorAnalysis\x64\Debug\VectorAnalysis.exe (process 12500)
exited with code 0 (0x0).
Press any key to close this window . . .
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