

Blog Entry © Thursday, May 14, 2026, by James Pate Williams, Jr. More Numerical Integration Results, Etc:

```
romberg t-table
1.29612
0.73846 0.98461
1.08473 1.20016 1.21453
1.25229 1.30814 1.31534 1.31694
1.33468 1.36214 1.36574 1.36654 1.36674
1.37553 1.38914 1.39094 1.39134 1.39144 1.39147
1.39587 1.40265 1.40355 1.40375 1.40379 1.40381 1.40381
table of ratios
2.13259
2.06664 1.99613
2.03365 1.99953 2.00001
2.01694 1.99994 2.00000 2.00000
2.00850 1.99999 2.00000 2.00000 2.00000
analytic integral of sine = 1.41614683655
simpson's rule integral   = 1.41614683655
monte carlo integral      = 1.41680565415
infinite series integral  = 1.41614683655
romberg integral          = 1.41614683395
actual spherical volume   = 4.18879020479
approx spherical volume   = 4.19731498304
```

```
C:\Users\James\source\repos\NumericalIntegrals\x64\Debug\Numeric
alIntegrals.exe (process 14048) exited with code 0 (0x0).
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Press any key to close this window . . .
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$$P(x) = x^5 + 0 + x^3 + x^2 + 0 + 1 \text{ mod } 2$$

$$Q(x) = x^3 + x^2 + x + 1 \text{ mod } 2$$

$$\begin{aligned} R(x) &= P(x)Q(x) = (x^5 + 0 + x^3 + x^2 + 0 + 1)(x^3 + x^2 + x + 1) \text{ mod } 2 \\ &= (x^5 + 0 + x^3 + x^2 + 0 + 1) + (x^6 + 0 + x^4 + x^3 + 0 + x) \\ &\quad + (x^7 + 0 + x^5 + x^4 + 0 + x^2) + (x^8 + 0 + x^6 + x^5 + 0 + x^3) \text{ mod } 2 \\ &= x^8 + x^7 + 0 + x^5 + 0 + x^3 + 0 + x + 1 \text{ mod } 2 \end{aligned}$$

$$P(x) = x^7 + x^6 + 0 + x^4 + x^3 + 0 + x + 1 \text{ mod } 2$$

$$Q(x) = x^5 + 0 + x^3 + x^2 + x + 1 \text{ mod } 2$$

$$\begin{aligned}
R(x) &= P(x)Q(x) \\
&= (x^7 + x^6 + 0 + x^4 + x^3 + 0 + x + 1) \\
&\quad + (x^8 + x^7 + 0 + x^5 + x^4 + 0 + x^2 + x) \\
&\quad + (x^9 + x^8 + 0 + x^5 + x^4 + 0 + x^3 + x^2) \\
&\quad + (x^{10} + x^9 + 0 + x^7 + x^6 + 0 + x^4 + x^3) \\
&\quad + (x^{12} + x^{11} + 0 + x^9 + x^8 + 0 + x^6 + x^5) \text{ mod } 2 \\
&= x^{12} + x^{11} + x^{10} + x^9 + x^8 + x^7 + x^6 + x^5 + 0 + x^3 + 0 + 0 + 1 \text{ mod } 2
\end{aligned}$$