

Blog Entry © Monday June 29, 2026, by James Pate Williams, Jr. and Microsoft Copilot Tests of Simpson's Rule and the Monte Carlo Method for Two-Dimensional Functions

We created a Win32 C/C++ testbed for the two integration algorithms. Below is the opening dialog developed by Microsoft Copilot for my usage:

Quadrature 2d

Integration Bounds (b, d)

b: 0

d: 0

Steps (m, n)

m: 0

n: 0

Method

Simpson's Rule

Monte Carlo

Enable Multitasking

Verbose Output

Run Clear

The first run is for the electron-electron interaction of Helium's Slater type 1s and 2s orbitals:

Quadrature 2d

Integration Bounds (b, d)

b:

d:

Steps (m, n)

m:

n:

Method

Simpson's Rule

Monte Carlo

Two-Dimensional Quadrature  
Electron-Electron Integrals  
Lower Bound x = +0.0050000  
Lower Bound y = +0.0050000  
Upper Bound x = +5.0000000  
Upper Bound y = +5.0000000  
Number Steps x = 1000  
Number Steps y = 1000  
Slater Zeta1 = +2.0000000  
Slater Zeta2 = +2.0000000  
Multithreading  
Simpson's Rule  
1s-1s = +1.2571408130  
1s-2s = -2.3551564430  
2s-2s = +7.6063080238

Enable Multitasking

Verbose Output

Quadrature 2d

Integration Bounds (b, d)

b:

d:

Steps (m, n)

m:

n:

Method

Simpson's Rule

Monte Carlo

Two-Dimensional Quadrature  
Electron-Electron Integrals

Lower Bound x = +0.0050000  
Lower Bound y = +0.0050000  
Upper Bound x = +5.0000000  
Upper Bound y = +5.0000000  
Number Steps x = 1000  
Number Steps y = 1000  
Slater Zeta1 = +2.0000000  
Slater Zeta2 = +2.0000000  
Sequential  
Simpson's Rule  
1s-1s = +1.2571408130  
1s-2s = -2.3551564430  
2s-2s = +7.6063080238

Enable Multitasking

Verbose Output

A simple first-order perturbation value of the 1s-1s electron-electron interaction is 1.25. An elementary variational calculation has a value of around 1.05 using the Slater zetas values of 1.69.

Quadrature 2d

Integration Bounds (b, d)

b:

d:

Steps (m, n)

m:

n:

Method

Simpson's Rule

Monte Carlo

Two-Dimensional Quadrature  
Electron-Electron Integrals  
Lower Bound x = +0.0050000  
Lower Bound y = +0.0050000  
Upper Bound x = +5.0000000  
Upper Bound y = +5.0000000  
Number Steps x = 1000  
Number Steps y = 1000  
Slater Zeta1 = +2.0000000  
Slater Zeta2 = +2.0000000  
Multithreading  
Monte Carlo  
1s-1s = +1.2523084820  
1s-2s = -2.3537814375  
2s-2s = +7.6274315650

Enable Multitasking

Verbose Output

Quadrature 2d

Integration Bounds (b, d)

b:

d:

Steps (m, n)

m:

n:

Method

Simpson's Rule

Monte Carlo

Two-Dimensional Quadrature

Electron-Electron Integrals

Lower Bound x = +0.0050000

Lower Bound y = +0.0050000

Upper Bound x = +5.0000000

Upper Bound y = +5.0000000

Number Steps x = 1000

Number Steps y = 1000

Slater Zeta1 = +2.0000000

Slater Zeta2 = +2.0000000

Sequential

Monte Carlo

1s-1s = +1.2574972344

1s-2s = -2.3578585891

2s-2s = +7.6157347416

Enable Multitasking

Verbose Output

Run

Clear

Next, we experiment with the two-dimensional function:

$$f(x, y) = e^{-x^2 - y^2}$$

The exact value over the domain:

$$(0, 0), (\infty, \infty)$$

Is:

$$\frac{\pi}{4} \sim 0.78539816339744830961566084581988$$

Quadrature 2d

Integration Bounds (b, d)

b:

d:

Steps (m, n)

m:

n:

Method

Simpson's Rule

Monte Carlo

Two-Dimensional Quadrature  
Exp(-x \* x - y \* y)  
Lower Bound x = +0.000000  
Lower Bound y = +0.000000  
Upper Bound x = +6.000000  
Upper Bound y = +6.000000  
Number Steps x = 10000  
Number Steps y = 10000  
Multithreading  
Simpson's Rule  
expr = +0.7862844901 23

Enable Multitasking

Verbose Output

Quadrature 2d

Integration Bounds (b, d)

b:

d:

Steps (m, n)

m:

n:

Method

Simpson's Rule

Monte Carlo

Two-Dimensional Quadrature  
Exp(-x \* x - y \* y)  
Lower Bound x = +0.000000  
Lower Bound y = +0.000000  
Upper Bound x = +6.000000  
Upper Bound y = +6.000000  
Number Steps x = 10000  
Number Steps y = 10000  
Sequential  
Simpson's Rule  
expr = +0.7862844901 32

Enable Multitasking

Verbose Output

Quadrature 2d

Integration Bounds (b, d)

b:

d:

Steps (m, n)

m:

n:

Method

Simpson's Rule

Monte Carlo

Two-Dimensional Quadrature  
Exp(-x \* x - y \* y)  
Lower Bound x = +0.000000  
Lower Bound y = +0.000000  
Upper Bound x = +6.000000  
Upper Bound y = +6.000000  
Number Steps x = 10000  
Number Steps y = 10000  
Multithreading  
Monte Carlo  
expr = +0.7856446378 49

Enable Multitasking

Verbose Output

Quadrature 2d

Integration Bounds (b, d)

b:

d:

Steps (m, n)

m:

n:

Method

Simpson's Rule

Monte Carlo

Enable Multitasking

Verbose Output

```

Two-Dimensional Quadrature
Exp(-x * x - y * y)
Lower Bound x = +0.00000000
Lower Bound y = +0.00000000
Upper Bound x = +6.00000000
Upper Bound y = +6.00000000
Number Steps x = 10000
Number Steps y = 10000
Sequential
Monte Carlo
expr = +0.7856446378 30

```

```

Two-Dimensional Quadrature
Electron-Electron Integrals
Lower Bound x = +0.0050000000
Lower Bound y = +0.0050000000
Upper Bound x = +5.0000000000
Upper Bound y = +5.0000000000
Number Steps x = 1000
Number Steps y = 1000
Slater Zeta1 = +2.0000000000
Slater Zeta2 = +2.0000000000
Multithreading
Simpson's Rule

```

1s-1s = +1.2571408130            79 milliseconds  
 1s-2s = -2.3551564430            48 milliseconds  
 2s-2s = +7.6063080238            50 milliseconds

## Two-Dimensional Quadrature

## Electron-Electron Integrals

Lower Bound x = +0.0050000000  
 Lower Bound y = +0.0050000000  
 Upper Bound x = +5.0000000000  
 Upper Bound y = +5.0000000000  
 Number Steps x = 1000  
 Number Steps y = 1000  
 Slater Zeta1 = +2.0000000000  
 Slater Zeta2 = +2.0000000000

## Sequential

## Simpson's Rule

1s-1s = +1.2571408130            86 milliseconds  
 1s-2s = -2.3551564430            91 milliseconds  
 2s-2s = +7.6063080238            84 milliseconds

## Two-Dimensional Quadrature

## Electron-Electron Integrals

Lower Bound x = +0.0050000000  
 Lower Bound y = +0.0050000000  
 Upper Bound x = +5.0000000000  
 Upper Bound y = +5.0000000000  
 Number Steps x = 1000  
 Number Steps y = 1000  
 Slater Zeta1 = +2.0000000000  
 Slater Zeta2 = +2.0000000000

## Multithreading

## Monte Carlo

1s-1s = +1.2523084820            90 milliseconds  
 1s-2s = -2.3537814375            69 milliseconds  
 2s-2s = +7.6274315650            61 milliseconds

## Two-Dimensional Quadrature

## Electron-Electron Integrals

Lower Bound x = +0.0050000000  
 Lower Bound y = +0.0050000000  
 Upper Bound x = +5.0000000000  
 Upper Bound y = +5.0000000000  
 Number Steps x = 1000  
 Number Steps y = 1000  
 Slater Zeta1 = +2.0000000000  
 Slater Zeta2 = +2.0000000000

## Sequential

## Monte Carlo

1s-1s = +1.2548623378            69 milliseconds

1s-2s = -2.3521996553      48 milliseconds  
 2s-2s = +7.6112852979      48 milliseconds

Two-Dimensional Quadrature

Exp(-x \* x - y \* y)

Lower Bound x = +0.0000000000

Lower Bound y = +0.0000000000

Upper Bound x = +6.0000000000

Upper Bound y = +6.0000000000

Number Steps x = 10000

Number Steps y = 10000

Multithreading

Simpson's Rule

expr = +0.7862844901      1917 milliseconds

Two-Dimensional Quadrature

Exp(-x \* x - y \* y)

Lower Bound x = +0.0000000000

Lower Bound y = +0.0000000000

Upper Bound x = +6.0000000000

Upper Bound y = +6.0000000000

Number Steps x = 10000

Number Steps y = 10000

Sequential

Simpson's Rule

expr = +0.7862844901      3292 milliseconds

Two-Dimensional Quadrature

Exp(-x \* x - y \* y)

Lower Bound x = +0.0000000000

Lower Bound y = +0.0000000000

Upper Bound x = +6.0000000000

Upper Bound y = +6.0000000000

Number Steps x = 10000

Number Steps y = 10000

Multithreading

Monte Carlo

expr = +0.7856446378      4074 milliseconds

Two-Dimensional Quadrature

Exp(-x \* x - y \* y)

Lower Bound x = +0.0000000000

Lower Bound y = +0.0000000000

Upper Bound x = +6.0000000000

Upper Bound y = +6.0000000000

Number Steps x = 10000

Number Steps y = 10000

Sequential

Monte Carlo

**expr = +0.7854913461      3143 milliseconds**