Blog Entry © Thursday – Saturday, June 11 - 13, 2026, by James Pate Williams, Jr. C/C++ Translation of S. D. Conte and Carl de Boor's Cooley-Tukey Fast Fourier Transform Algorithm

References: ***Elementary Numerical Analysis: An Algorithmic Approach Third Edition*** © 1980 by S. D. Conte and Carl de Boor Section 6.5 pages 268 to 277 and Section 6.6 pages 277 to 283. Especially Equation (6.57) page 275 and FORTRAN source code on pages 281 to 282.

"Introduction to Algorithms" by Thomas H. Cormen, Charles E. Leiserson, and Ronald L. Rivest, pages 794 – 795 and page 788.

http://astronomy.swin.edu.au/~pbourke/analysis/dft/

Conte and de Boor use two triply indexed FORTRAN complex value array. We use a standard template library three vector complex value array. We are attempting to approximate the following function using a trigonometric Fourier Series and Fast Fourier Transforms:

$$f(x) = x^2 \sin x, \forall x \epsilon [0, 2\pi)$$

$$f(x) = \frac{a_0}{2} + \frac{1}{\pi} \sum_{n=1}^{\infty} (a_n \cos \mathrm{nx} + b_n \sin \mathrm{nx})$$

We computed the following table consisting of x, f(x), Fourier series interpolating function f(x), and the Cooley-Tukey FFT interpolating function. We use n1 = 8, n2 = 2, n3 = 1, thus n0 = 16.

```
== Menu ==
1 Cooley-Tukey
2 FFT
3 Iterative FFT
4 Recursive FFT
5 DFT
6 Vandermonde DFT
7 Exit
Option 1 - 7 = 1
n1 = 8
n2 = 2
n3 = 1
Forward Transform
    1    0.3926990817   0.05901459513   (0.05901459513,0)        (-99.17668936,0)
    2    0.7853981634    0.4361790125   (0.4361790125,0)         (55.82574788,89.51850363)
    3     1.178097245     1.282264523   (1.282264523,0) (64.21004246,-85.67628876)
    4     1.570796327       2.4674011   (2.4674011,0)    (-82.91577632,-29.8717747)
    5     1.963495408     3.561845898   (3.561845898,0) (4.490691329,55.95499614)
    6      2.35619449     3.925611112   (3.925611112,0) (19.08286558,-23.56049327)
    7     2.748893572     2.891715161   (2.891715161,0) (-18.51808266,7.487674959)
    8     3.141592654               0   (1.208677944e-15,0)      (12.43038413,4.324696831)
    9     3.534291735    -4.780182205   (-4.780182205,0)         (-1.888055706,-9.294608434)
   10     3.926990817    -10.90447531   (-10.90447531,0)         (-9.79823693,8.176247996)
   11     4.319689899    -17.23933415   (-17.23933415,0)         (18.94041156,-0.3623096274)
   12      4.71238898     -22.2066099   (-22.2066099,0) (-21.88538543,-10.79696347)
   13     5.105088062    -24.07807827   (-24.07807827,0)         (17.31025702,20.46847469)
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   14     5.497787144    -21.37277161   (-21.37277161,0)        (-8.030599569,-23.1824763)
   15     5.890486225     -13.2782839   (-13.2782839,0) (0.096858918,16.83848543)
   16     6.283185307               0   (-9.669423551e-15,0)    (-0.1564991358,-4.204438574)
== Menu ==
1 Cooley-Tukey
2 FFT
3 Iterative FFT
4 Recursive FFT
5 DFT
6 Vandermonde DFT
7 Exit
Option 1 - 7 =
```

Next, we use n = 16 for the forward and backward online reference FFT calculations which yields the table illustrated below, the first column is x, the second f(x), third Fourier series results, the real component of the transforms and the imaginary component of the transforms:

```
== Menu ==
1 Cooley-Tukey
2 FFT
3 Iterative FFT
4 Recursive FFT
5 DFT
6 Vandermonde DFT
7 Exit
Option 1 - 7 = 2
n0 = 16
Forward Transform
    0               0               0               0    -6.202231497               0
    1    0.3926990817   0.05901459513   0.05882118227      -1.4892083    -6.329329993
    2    0.7853981634    0.4361790125    0.4357768557     2.177934072    -0.888045166
    3     1.178097245     1.282264523     1.281617803     0.872358025   -0.1861510375
    4     1.570796327       2.4674011     2.466439137    0.5110154998   -0.0691398989
    5     1.963495408     3.561845898     3.560425206    0.3613425252   -0.03193846874
    6      2.35619449     3.925611112     3.923399068    0.2894670281   -0.01568714102
    7     2.748893572     2.891715161      2.88773905    0.2555077499   -0.006614673456
    8     3.141592654               0               0    0.2453982971               0
Backward Transform
    0               0               0               0               0               0
    1    0.3926990817   0.05901459513   0.05882118227   0.05901459513               0
    2    0.7853981634    0.4361790125    0.4357768557    0.4361790125               0
    3     1.178097245     1.282264523     1.281617803     1.282264523               0
    4     1.570796327       2.4674011     2.466439137       2.4674011               0
    5     1.963495408     3.561845898     3.560425206     3.561845898               0
    6      2.35619449     3.925611112     3.923399068     3.925611112               0
    7     2.748893572     2.891715161      2.88773905     2.891715161               0
    8     3.141592654               0               0               0               0
== Menu ==
1 Cooley-Tukey
2 FFT
3 Iterative FFT
4 Recursive FFT
5 DFT
6 Vandermonde DFT
7 Exit
Option 1 - 7 =
```

The iterative FFT algorithm of the second reference has the forward transformation:

```
== Menu ==
1 Cooley-Tukey
2 FFT
3 Iterative FFT
4 Recursive FFT
5 DFT
6 Vandermonde DFT
```

```
7 Exit
Option 1 - 7 = 3
n0 = 16
Forward Transform
    0              0              0   (-20.35701138,-0.2064430595)
    1    0.3926990817   0.05901459513   (4.819270675,16.27772632)
    2    0.7853981634    0.4361790125   (15.62182249,52.25777893)
    3     1.178097245     1.282264523   (57.20284364,-34.40373367)
    4     1.570796327       2.4674011   (11.53187054,-47.4301466)
    5     1.963495408     3.561845898   (-60.29328685,36.76552812)
    6      2.35619449     3.925611112   (-29.51742424,-21.00538234)
    7     2.748893572     2.891715161   (12.99244233,-17.48895703)
    8     3.141592654              0   (42.96128056,42.95198628)
    9     3.534291735    -4.780182205   (-6.486947257,5.531224302)
   10     3.926990817    -10.90447531   (-38.22609167,-9.51223571)
   11     4.319689899    -17.23933415   (29.95591938,12.59478305)
   12      4.71238898     -22.2066099   (54.69029989,4.684603379)
   13     5.105088062    -24.07807827   (-26.86547618,-58.57447875)
   14     5.497787144    -21.37277161   (-36.70474619,-21.74016088)
   15     5.890486225     -13.2782839   (-11.32476574,39.29790765)
== Menu ==
1 Cooley-Tukey
2 FFT
3 Iterative FFT
4 Recursive FFT
5 DFT
6 Vandermonde DFT
7 Exit
Option 1 - 7 =
```

Next, we investigate a discrete DFT and its inverse:

```
== Menu ==
1 Cooley-Tukey
2 FFT
3 Iterative FFT
4 Recursive FFT
5 DFT
6 Vandermonde DFT
7 Exit
Option 1 - 7 = 5
n0 = 16
Forward Transform
    0              0              0              0   (-6.202231497,0)
    1    0.3926990817   0.05901459513    0.3926990817   (-1.4892083,-6.329329993)
    2    0.7853981634    0.4361790125    0.7853981634   (2.177934072,-0.888045166)
    3     1.178097245     1.282264523     1.178097245   (0.872358025,-0.1861510375)
    4     1.570796327       2.4674011     1.570796327   (0.5110154998,-0.0691398989)
    5     1.963495408     3.561845898     1.963495408   (0.3613425252,-0.03193846874)
    6      2.35619449     3.925611112      2.35619449   (0.2894670281,-0.01568714102)
    7     2.748893572     2.891715161     2.748893572   (0.2555077499,-0.006614673456)
    8     3.141592654              0     3.141592654   (0.2453982971,1.90777814e-15)
    9     3.534291735    -4.780182205     3.534291735   (0.2555077499,0.006614673456)
   10     3.926990817    -10.90447531     3.926990817   (0.2894670281,0.01568714102)
   11     4.319689899    -17.23933415     4.319689899   (0.3613425252,0.03193846874)
   12      4.71238898     -22.2066099      4.71238898   (0.5110154998,0.0691398989)
   13     5.105088062    -24.07807827     5.105088062   (0.872358025,0.1861510375)
   14     5.497787144    -21.37277161     5.497787144   (2.177934072,0.888045166)
   15     5.890486225     -13.2782839     5.890486225   (-1.4892083,6.329329993)
Inverse Transform
    0              0              0              0              0
    1    0.3926990817   0.05901459513    0.3926990817   0.05901459513
    2    0.7853981634    0.4361790125    0.7853981634    0.4361790125
    3     1.178097245     1.282264523     1.178097245     1.282264523
    4     1.570796327       2.4674011     1.570796327       2.4674011
    5     1.963495408     3.561845898     1.963495408     3.561845898
    6      2.35619449     3.925611112      2.35619449     3.925611112
    7     2.748893572     2.891715161     2.748893572     2.891715161
    8     3.141592654              0     3.141592654              0
    9     3.534291735    -4.780182205     3.534291735    -4.780182205
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   10     3.926990817    -10.90447531     3.926990817    -10.90447531
   11     4.319689899    -17.23933415     4.319689899    -17.23933415
   12      4.71238898     -22.2066099      4.71238898     -22.2066099
   13     5.105088062    -24.07807827     5.105088062    -24.07807827
   14     5.497787144    -21.37277161     5.497787144    -21.37277161
   15     5.890486225     -13.2782839     5.890486225     -13.2782839
== Menu ==
1 Cooley-Tukey
2 FFT
3 Iterative FFT
4 Recursive FFT
5 DFT
6 Vandermonde DFT
7 Exit
Option 1 - 7 =
```

Finally, we test our Vandermonde DFT algorithms:

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== Menu ==
1 Cooley-Tukey
2 FFT
3 Iterative FFT
4 Recursive FFT
5 DFT
6 Vandermonde DFT
7 Exit
Option 1 - 7 = 6
n0 = 16
Vandermonde DFT
    0              0              0              0   (-99.23570395,0)
    1    0.3926990817   0.05901459513   0.05882118227   (-23.8273328,101.2692799)
    2    0.7853981634    0.4361790125    0.4357768557   (34.84694516,14.20872266)
    3     1.178097245     1.282264523     1.281617803   (13.9577284,2.9784166)
    4     1.570796327       2.4674011     2.466439137   (8.176247996,1.106238382)
    5     1.963495408     3.561845898     3.560425206   (5.781480403,0.5110154998)
    6      2.35619449     3.925611112     3.923399068   (4.631472449,0.2509942564)
    7     2.748893572     2.891715161      2.88773905   (4.088123998,0.1058347753)
    8     3.141592654               0              0   (3.926372753,-1.452792721e-13)
    9     3.534291735    -4.780182205    -2.88773905   (4.088123998,-0.1058347753)
   10     3.926990817    -10.90447531    -3.923399068   (4.631472449,-0.2509942564)
   11     4.319689899    -17.23933415    -3.560425206   (5.781480403,-0.5110154998)
   12      4.71238898     -22.2066099    -2.466439137   (8.176247996,-1.106238382)
   13     5.105088062    -24.07807827    -1.281617803   (13.9577284,-2.9784166)
   14     5.497787144    -21.37277161   -0.4357768557   (34.84694516,-14.20872266)
   15     5.890486225     -13.2782839   -0.05882118227  (-23.8273328,-101.2692799)
Inverse Vandermonde DFT
    0              0              0              0   (6.949996134e-14,2.131628207e-14)
    1    0.3926990817   0.05901459513   0.05882118227   (0.05901459513,5.506706202e-14)
    2    0.7853981634    0.4361790125    0.4357768557   (0.4361790125,2.131628207e-14)
    3     1.178097245     1.282264523     1.281617803   (1.282264523,3.15303339e-14)
    4     1.570796327       2.4674011     2.466439137   (2.4674011,2.309263891e-14)
    5     1.963495408     3.561845898     3.560425206   (3.561845898,7.238654121e-14)
    6      2.35619449     3.925611112     3.923399068   (3.925611112,1.421085472e-14)
    7     2.748893572     2.891715161      2.88773905   (2.891715161,-5.329070518e-15)
    8     3.141592654               0              0   (-1.398881011e-14,-2.309263891e-14)
    9     3.534291735    -4.780182205    -2.88773905   (-4.780182205,2.131628207e-14)
   10     3.926990817    -10.90447531    -3.923399068   (-10.90447531,9.769962617e-15)
   11     4.319689899    -17.23933415    -3.560425206   (-17.23933415,8.881784197e-15)
   12      4.71238898     -22.2066099    -2.466439137   (-22.2066099,9.103828802e-15)
   13     5.105088062    -24.07807827    -1.281617803   (-24.07807827,-1.110223025e-15)
   14     5.497787144    -21.37277161   -0.4357768557   (-21.37277161,9.325873407e-15)
   15     5.890486225     -13.2782839   -0.05882118227  (-13.2782839,-4.707345624e-14)
== Menu ==
1 Cooley-Tukey
2 FFT
3 Iterative FFT
4 Recursive FFT
5 DFT
6 Vandermonde DF7 ExitOption 1 - 7 =
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