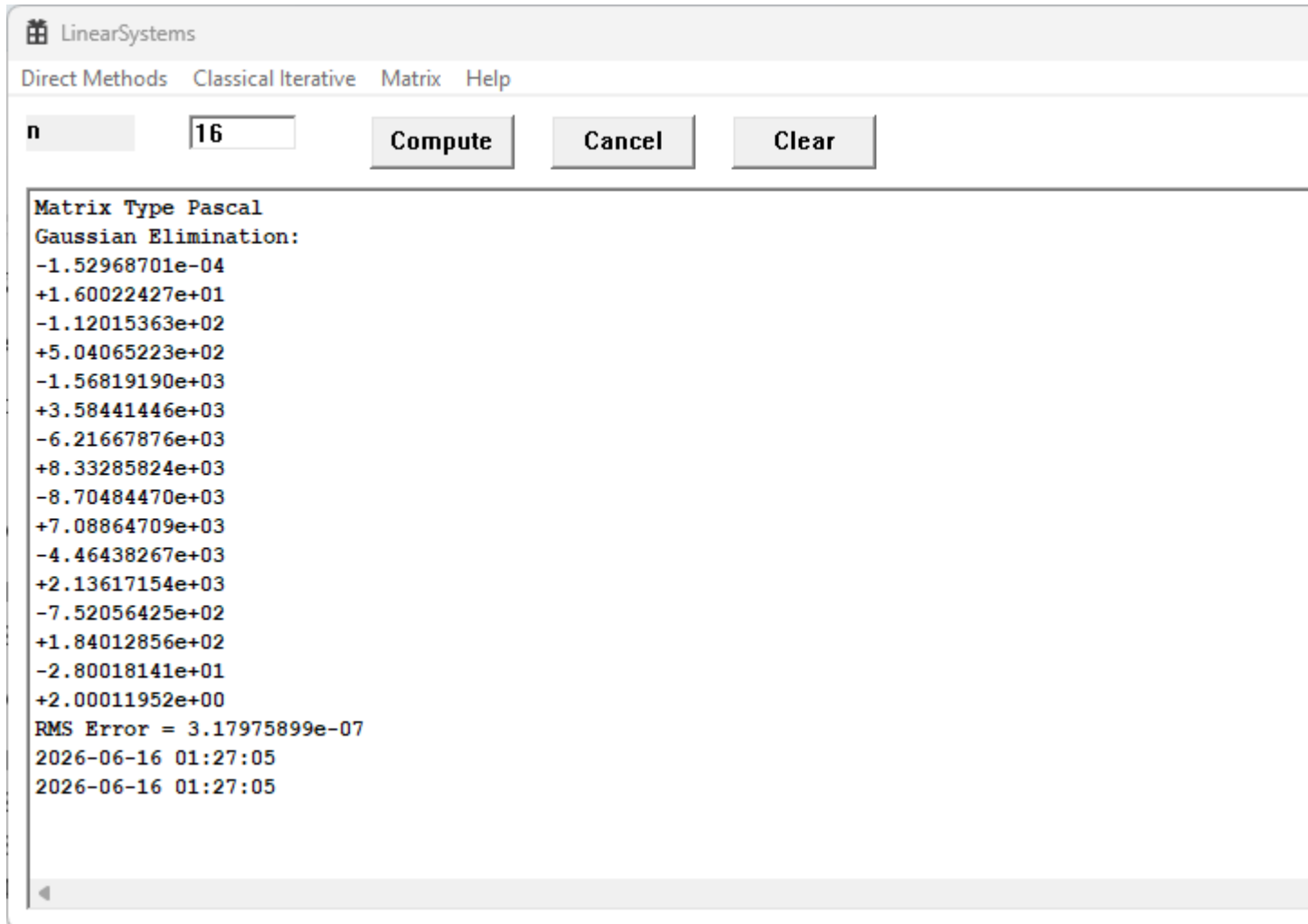


Blog Entry Tuesday, June 16, 2026, by James Pate Williams, Jr., Linear Systems of Equations Solutions

Reference: ***Elementary Numerical Analysis: An Algorithmic Approach Third Edition*** © 1980 by S. D. Conte and Carl de Boor

We created a C/C++ Windows Desktop application to solve linear systems of equations. The input dialog is illustrated below:



The direct methods are:

1. Gaussian Elimination
2. Lower-Upper Decomposition

The classical iterative methods are:

1. Jacobi
2. Gauss-Seidel

3. Successive Over Relaxation (SOR) with $\omega = 1.5$
4. Gradient Method

The methods were translated from the FORTRAN 77 source code in the above reference.

The matrices are:

1. Pascal
2. Other
3. Cauchy
4. Lehmer

Matrix Type Pascal

Gaussian Elimination:

```

-1.52968701e-04
+1.60022427e+01
-1.12015363e+02
+5.04065223e+02
-1.56819190e+03
+3.58441446e+03
-6.21667876e+03
+8.33285824e+03
-8.70484470e+03
+7.08864709e+03
-4.46438267e+03
+2.13617154e+03
-7.52056425e+02
+1.84012856e+02
-2.80018141e+01
+2.00011952e+00
RMS Error = 3.17975899e-07
2026-06-16 01:01:39
2026-06-16 01:01:39

```

Matrix Type Pascal

LU Decomposition:

```

+2.55075539e+00
-1.72191290e+00
-3.05204772e-01
+1.48025779e+00
+3.01287626e-02
+2.07697713e-02
-4.53912856e-02
-7.06066386e-04
-1.03512758e-02
-1.99401065e-03
-2.56569847e-03

```

+3.81628430e-03
+2.85462394e-03
-7.54402484e-04
+5.56817781e-06
+2.92220067e-04
RMS Error = 4.38798962e+03
2026-06-16 01:13:46
2026-06-16 01:13:46

Matrix Type Pascal
Jacobi Method:
+2.00000000e+00
+1.00000000e+00
+5.00000000e-01
+2.50000000e-01
+1.25000000e-01
+6.25000000e-02
+3.12500000e-02
+1.56250000e-02
+7.81250000e-03
+3.90625000e-03
+1.95312500e-03
+9.76562500e-04
+4.88281250e-04
+2.44140625e-04
+1.22070313e-04
+6.10351563e-05
RMS Error = 2.01777203e+03
2026-06-16 01:14:40
2026-06-16 01:14:40

Matrix Type Pascal
Gauss-Seidel Method:
+1.71747506e+00
-4.30295905e+00
+1.35523519e+01
-1.85776140e+01
+1.19156469e+01
+1.15980112e+00
-5.55060262e+00
+1.13031344e+00
+1.61244721e+00
-2.93715387e-01
-5.14007623e-01
+1.06175413e-02
+1.64895746e-01
+2.12147170e-02

-6.14791098e-02
+1.56148701e-02
RMS Error = 4.32366456e-01
2026-06-16 01:15:22
2026-06-16 01:15:23

Matrix Type Pascal
Successive Over Relaxation:
+1.67746153e+00
-3.69063090e+00
+1.03856434e+01
-1.01421640e+01
-1.29670215e+00
+1.36355156e+01
-1.20883761e+01
+2.28725540e+00
+2.02170886e+00
-3.14937078e-01
-6.54021225e-01
+1.35424947e-02
+1.92784798e-01
+2.50966175e-02
-6.94695767e-02
+1.72942415e-02
RMS Error = 3.70751996e-01
2026-06-16 01:15:49
2026-06-16 01:15:50

Matrix Type Pascal
Gradient Method:
+1.43765188e-02
+2.32123411e-02
+3.32797463e-02
+4.25235834e-02
+4.88886706e-02
+5.11703697e-02
+4.88920078e-02
+4.09418767e-02
+2.45931663e-02
-1.61299468e-03
-2.91688312e-02
-3.25222577e-02
+1.21558159e-02
+5.78321019e-02
-4.77749898e-02
+1.10898183e-02
RMS Error = 5.08611680e+00

2026-06-16 01:16:12
2026-06-16 01:16:15

Matrix Type Other
Gaussian Elimination:

+1.09411765e+01
+2.13823529e+01
+3.03235294e+01
+3.77647059e+01
+4.37058824e+01
+4.81470588e+01
+5.10882353e+01
+5.25294118e+01
+5.24705882e+01
+5.09117647e+01
+4.78529412e+01
+4.32941176e+01
+3.72352941e+01
+2.96764706e+01
+2.06176471e+01
+1.00588235e+01
RMS Error = 5.12148661e-15
2026-06-16 01:16:57
2026-06-16 01:16:57

Matrix Type Other
LU Decomposition:

+1.09411765e+01
+2.13823529e+01
+3.03235294e+01
+3.77647059e+01
+4.37058824e+01
+4.81470588e+01
+5.10882353e+01
+5.25294118e+01
+5.24705882e+01
+5.09117647e+01
+4.78529412e+01
+4.32941176e+01
+3.72352941e+01
+2.96764706e+01
+2.06176471e+01
+1.00588235e+01
RMS Error = 5.12148661e-15
2026-06-16 01:17:21
2026-06-16 01:17:21

Matrix Type Other

Jacobi Method:

+2.50000000e-01

+8.75000000e-01

+1.18750000e+00

+1.34375000e+00

+1.42187500e+00

+1.46093750e+00

+1.48046875e+00

+1.49023438e+00

+1.49511719e+00

+1.49755859e+00

+1.49877930e+00

+1.49938965e+00

+1.49969482e+00

+1.49984741e+00

+1.49992371e+00

+4.99961853e-01

RMS Error = 1.33512696e+00

2026-06-16 01:17:42

2026-06-16 01:17:42

Matrix Type Other

Gauss-Seidel Method:

+1.09411764e+01

+2.13823529e+01

+3.03235294e+01

+3.77647058e+01

+4.37058823e+01

+4.81470587e+01

+5.10882352e+01

+5.25294117e+01

+5.24705881e+01

+5.09117646e+01

+4.78529411e+01

+4.32941176e+01

+3.72352941e+01

+2.96764705e+01

+2.06176470e+01

+1.00588235e+01

RMS Error = 2.51931554e-09

2026-06-16 01:18:11

2026-06-16 01:18:11

Matrix Type Other

Successive Over Relaxation:

+1.09411765e+01

+2.13823529e+01
+3.03235294e+01
+3.77647059e+01
+4.37058823e+01
+4.81470588e+01
+5.10882353e+01
+5.25294117e+01
+5.24705882e+01
+5.09117647e+01
+4.78529412e+01
+4.32941176e+01
+3.72352941e+01
+2.96764706e+01
+2.06176471e+01
+1.00588235e+01
RMS Error = 9.72176378e-10
2026-06-16 01:18:31
2026-06-16 01:18:31

Matrix Type Other
Gradient Method:
+1.09411764e+01
+2.13823529e+01
+3.03235293e+01
+3.77647058e+01
+4.37058822e+01
+4.81470587e+01
+5.10882352e+01
+5.25294116e+01
+5.24705881e+01
+5.09117646e+01
+4.78529410e+01
+4.32941175e+01
+3.72352940e+01
+2.96764705e+01
+2.06176470e+01
+1.00588235e+01
RMS Error = 5.86330373e-09
2026-06-16 01:19:00
2026-06-16 01:19:00

Matrix Type Cauchy
Gaussian Elimination:
-2.79365376e+02
+1.07638693e+05
-6.49210907e+06
+1.46635650e+08

-1.63708715e+09
+1.02164551e+10
-3.72105125e+10
+7.55834356e+10
-5.85204289e+10
-8.10543320e+10
+2.31128139e+11
-1.61104169e+11
-9.21446432e+10
+2.21007670e+11
-1.36447327e+11
+3.00425878e+10
RMS Error = 1.43948611e-06
2026-06-16 01:19:28
2026-06-16 01:19:28

Matrix Type Cauchy

LU Decomposition:

-9.68137450e-01
+6.71033559e+00
+7.99957755e+00
+2.29990429e+01
-1.55961428e+02
-6.78219132e+01
-1.20241483e+02
+4.40072721e+02
+1.24556555e+02
+1.93955370e+02
-9.56114476e+02
-2.46047158e+02
-3.61428765e+02
+1.37815728e+03
+3.32961295e+02
-3.44828514e+02
RMS Error = 6.91171484e+00
2026-06-16 01:19:55
2026-06-16 01:19:55

Matrix Type Cauchy

Jacobi Method:

+1.00000000e+00
+8.33333333e+00
+1.44857143e+01
+2.09863396e+01
+2.75049479e+01
+3.40248632e+01
+4.05443759e+01

+4.70634143e+01
+5.35820966e+01
+6.01005300e+01
+6.66187909e+01
+7.31369308e+01
+7.96549846e+01
+8.61729762e+01
+9.26909218e+01
+9.92088332e+01
RMS Error = 2.19160629e+01
2026-06-16 01:20:42
2026-06-16 01:20:42

Matrix Type Cauchy
Gauss-Seidel Method:
-6.23603338e+01
+8.07343371e+03
-1.10314924e+05
+4.12173277e+05
-3.61830068e+05
-3.23219816e+05
+6.41361864e+04
+3.36527170e+05
+3.55200454e+05
+1.73240395e+05
-9.05158051e+04
-3.17340011e+05
-4.13838582e+05
-3.16825277e+05
+1.01796346e+04
+5.82922979e+05
RMS Error = 2.56812596e-02
2026-06-16 01:21:00
2026-06-16 01:21:02

Matrix Type Cauchy
Successive Over Relaxation:
-1.99527868e+02
+1.71862183e+04
-2.09115064e+05
+7.90039609e+05
-9.49430961e+05
-9.22078393e+03
+2.22656802e+04
+4.57284989e+05
+3.16405571e+05
+1.66447831e+05

-1.35766795e+05
-3.39303483e+05
-4.31061767e+05
-3.14818947e+05
+2.25623612e+04
+6.05538429e+05
RMS Error = 2.14963994e-02
2026-06-16 01:21:24
2026-06-16 01:21:26

Matrix Type Cauchy
Gradient Method:
+8.04322195e+01
-4.55833945e+03
+3.27709294e+04
-4.94797038e+04
-3.16589340e+04
+2.14724105e+04
+5.55962254e+04
+5.50817303e+04
+2.65447725e+04
-1.52098721e+04
-5.46192899e+04
-7.84598564e+04
-7.67179668e+04
-4.24686548e+04
+2.86107293e+04
+1.38801107e+05
RMS Error = 6.39716222e-02
2026-06-16 01:21:53
2026-06-16 01:21:56

Matrix Type Lehmer
Gaussian Elimination:
+5.64868196e-01
+2.43289375e-01
+1.36096435e-01
+8.24999647e-02
+5.03420826e-02
+2.89034945e-02
+1.35902173e-02
+2.10525944e-03
-6.82748559e-03
-1.39736816e-02
-1.98205693e-02
-2.46929756e-02
-2.88157810e-02

-3.23496142e-02
-3.54122697e-02
-3.80920932e-02
RMS Error = 2.26339869e-15
2026-06-16 01:22:25
2026-06-16 01:22:25

Matrix Type Lehmer
LU Decomposition:
+5.37902788e-01
+3.90335278e-02
+2.88778857e-02
-2.07085444e-02
-7.81163327e-03
-6.34700418e-03
+5.44125575e-03
+1.31309255e-03
+1.46852070e-03
-1.98940516e-03
-6.55527876e-04
-8.28666354e-04
+9.51786867e-04
+1.76946496e-04
-2.06962908e-03
-3.45911261e-02
RMS Error = 3.53506077e+00
2026-06-16 01:22:50
2026-06-16 01:22:50

Matrix Type Lehmer
Jacobi Method:
0.00000000e+00
+5.00000000e-01
-1.66666667e-01
+2.91666667e-01
-1.58333333e-01
+2.20833333e-01
-1.44642857e-01
+1.83705357e-01
-1.32911706e-01
+1.60255456e-01
-1.23314619e-01
+1.43822432e-01
-1.15406403e-01
+1.31519685e-01
-1.08786862e-01
+1.21878903e-01

RMS Error = 3.56087258e+00
2026-06-16 01:23:11
2026-06-16 01:23:11

Matrix Type Lehmer
Gauss-Seidel Method:

- nan
- nan
- nan
- nan
- nan
- nan
- nan
- nan
- nan
- nan
- nan
- nan
- nan
- nan
- nan
- nan

RMS Error = -nan(ind)
2026-06-16 01:23:42
2026-06-16 01:23:44

Matrix Type Lehmer
Successive Over Relaxation:

- nan
- nan
- nan
- nan
- nan
- nan
- nan
- nan
- nan
- nan
- nan
- nan
- nan
- nan
- nan
- nan

RMS Error = -nan(ind)
2026-06-16 01:24:05
2026-06-16 01:24:07

Matrix Type Lehmer

Gradient Method:

-7.93392102e+150

-7.54997085e+150

-7.12561126e+150

-6.65541680e+150

-6.13292877e+150

-5.55039740e+150

-4.89844248e+150

-4.16560130e+150

-3.33771707e+150

-2.39709858e+150

-1.32134421e+150

-8.16630121e+148

+1.35957748e+150

+3.05252669e+150

+5.06552616e+150

+7.49418047e+150

RMS Error = 4.17807849e+152

2026-06-16 01:24:26

2026-06-16 01:24:26