Solutions of a Linear Second Order Ordinary Differential Equation Initial Value Problem by James Pate Williams, Jr.

References: 17.2: Nonhomogeneous Linear Equations - Mathematics LibreTexts and ***A Numerical Library in C for Scientists and Engineers*** © 1994 by H. T. Lau, PhD See rk2 pages 456-459.

$$y'' - 9y = -6\cos(3x), y(0) = 1, y'(0) = 1$$

According to the first reference the general solution is:

$$y(x) = c_1 e^{3x} + c_2 e^{-3x} + \frac{1}{3}\cos(3x)$$

$$y'(x) = 3c_1 e^{3x} - 3c_2 e^{-3x} - \sin(3x)$$

$$c_1 + c_2 + \frac{1}{3} = 1$$

$$3c_1 - 3c_2 = 1$$

$$c_1 + c_2 = \frac{2}{3}$$

$$c_1 - c_2 = \frac{1}{3}$$

$$2c_1 = 1, c_1 = \frac{1}{2}$$

$$c_2 = \frac{2}{3} - \frac{1}{2} = \frac{4}{6} - \frac{3}{6} = \frac{1}{6}$$

$$y(x) = \frac{1}{2}e^{3x} + \frac{1}{6}e^{-3x} + \frac{1}{3}\cos(3x)$$

More generally we have:

$$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$M^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Now we develop an infinite power series solution using Frobenius Method:

$$y(x) = \sum_{n=0}^{\infty} a_n x^n$$

$$y'(x) = \sum_{n=0}^{\infty} n a_n x^{n-1}$$

$$y''(x) = \sum_{n=0}^{\infty} n(n-1) a_n x^{n-2}$$

$$n = m + 2, n - 2 = m$$

$$f(x) = \cos(3x)$$

$$f'(x) = -3\sin(3x)$$

$$f''(x) = -9\cos(3x)$$

$$f'''(x) = 27\sin(3x)$$

$$f^{iv}(x) = 81\cos(3x)$$

$$f^{v}(x) = -243\sin(3x)$$

$$f^{vi}(x) = -729\cos(3x)$$

$$f^{n}(x) = (-1)^n 3^{2n}\cos(3x)$$

$$f(x) = -6\cos(3x) = -6\sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n}}{(2n)!} x^{2n}$$

$$(n+2)(n+1)a_{n+2} - 9a_n = 0$$

$$a_{n+2} = \frac{9a_n}{(n+2)(n+1)}$$

$$a_0 = a_1 = 1$$

$$a_2 = \frac{9}{2}$$

$$a_3 = \frac{9}{6}$$

$$a_4 = \frac{9}{4 \cdot 3} \cdot \frac{9}{2} = \frac{27}{8}$$

$$a_5 = \frac{9}{5 \cdot 4} \cdot \frac{9}{6} = \frac{81}{120}$$

$$\sum_{n=0}^{\infty}(n+2)(n+1)a_{n+2}x^n - 9a_nx^n = -6\cos(3x) = -6\sum_{n=0}^{\infty}\frac{(-1)^n 3^{2n}}{(2n)!}x^{2n}$$

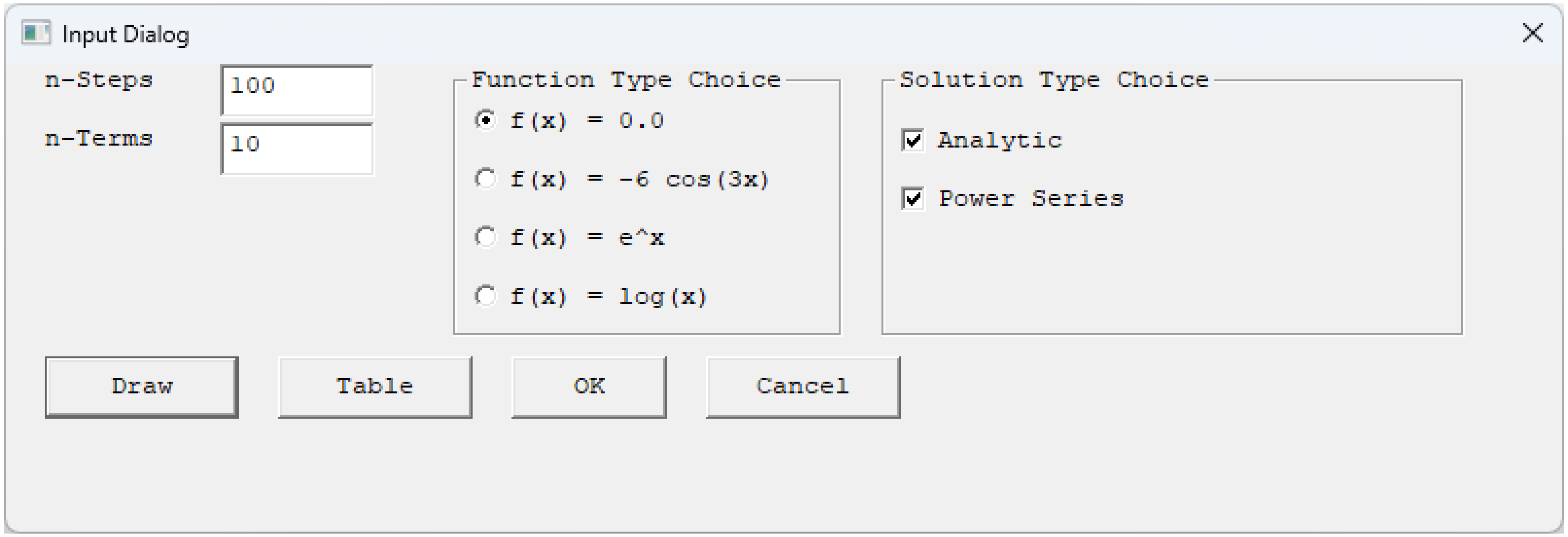
$$(n+2)(n+1)a_{n+2} - 9a_n = -\frac{6(-1)^n 3^{2n}}{(2n)!}$$

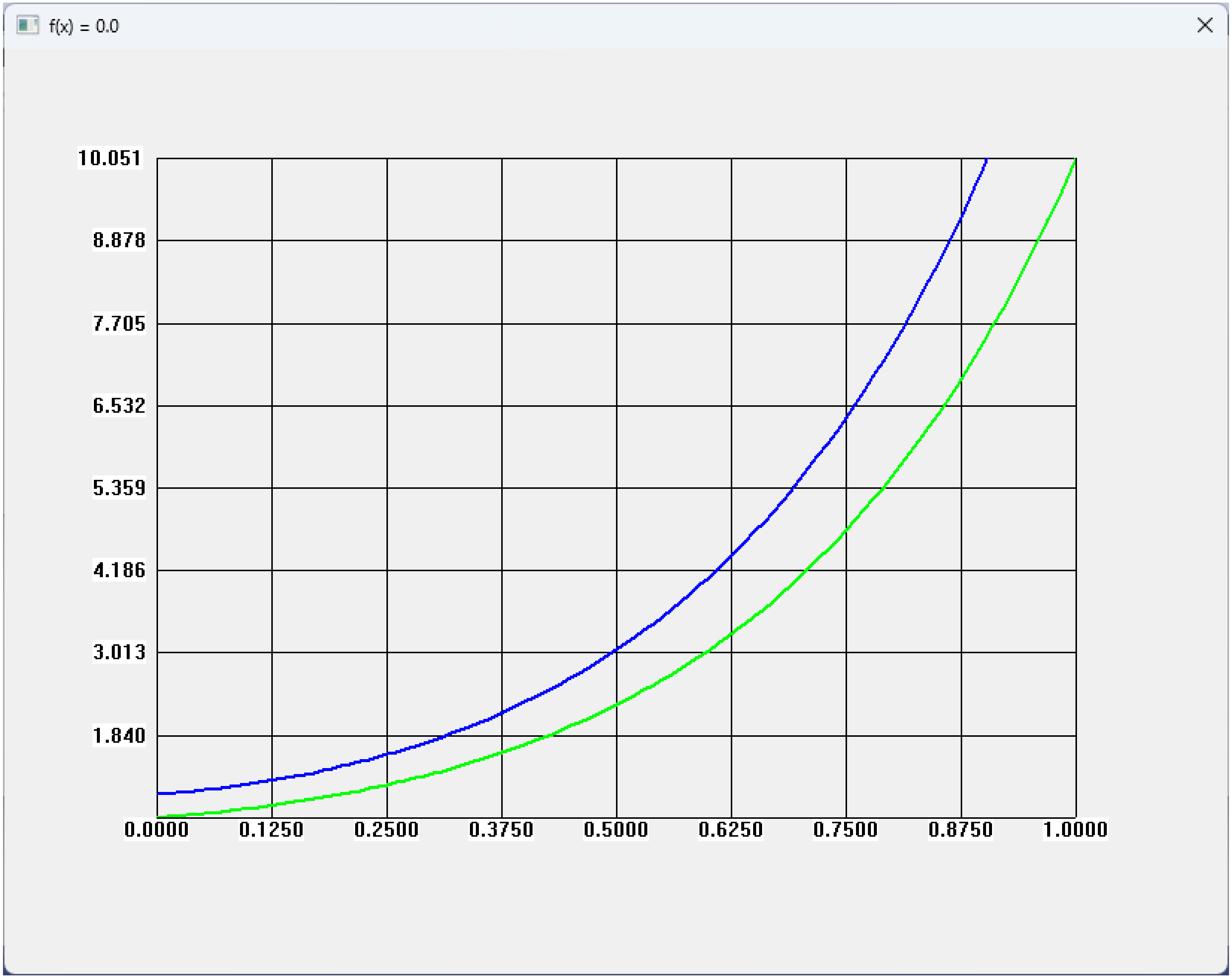
$$a_{n+2} = \frac{9a_n - 6(-1)^n 3^{2n}/(2n)!}{(n+2)(n+1)}$$

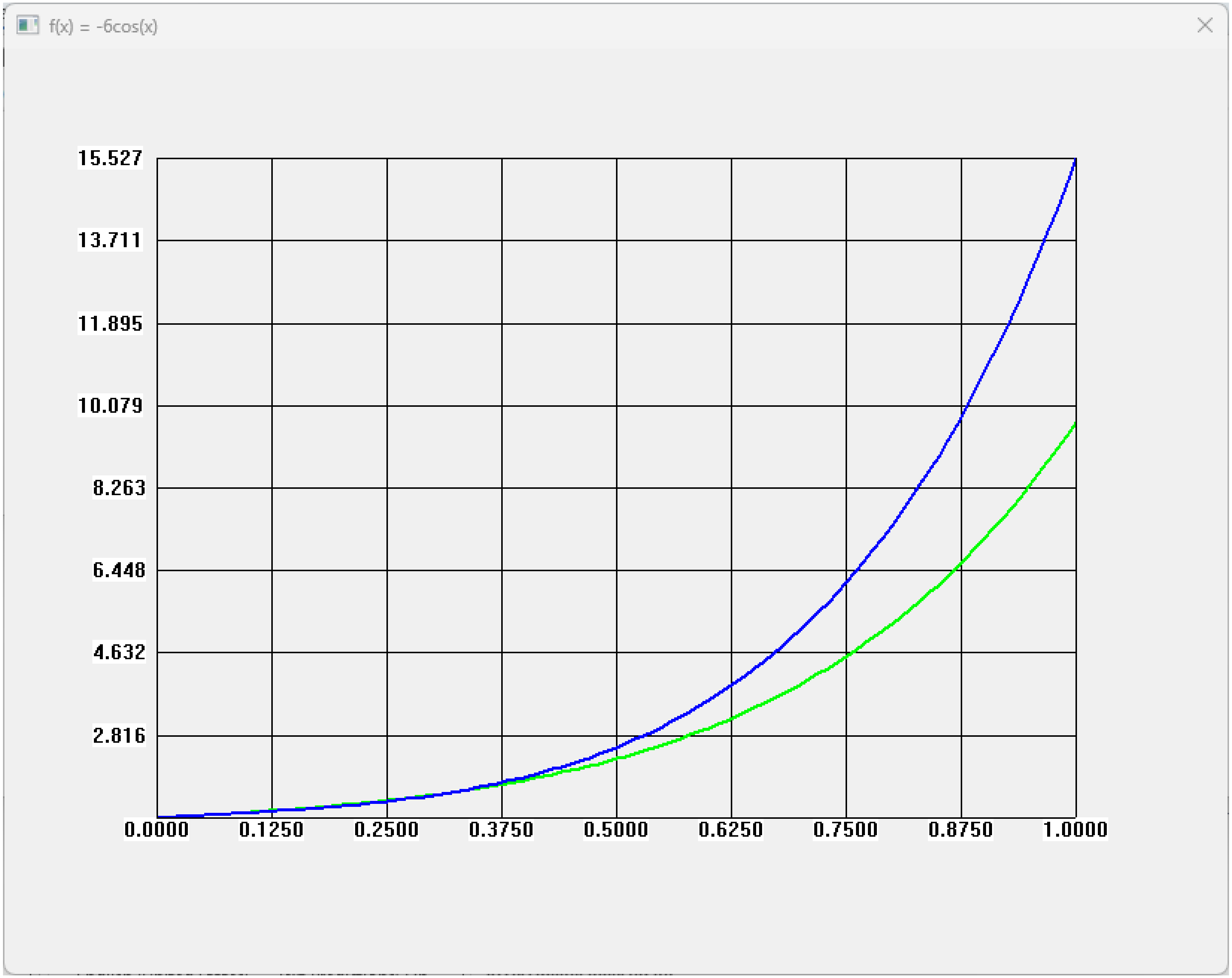
$$y(x) = \frac{1}{2}e^{3x} + \frac{1}{6}e^{-3x} + 9e^x$$

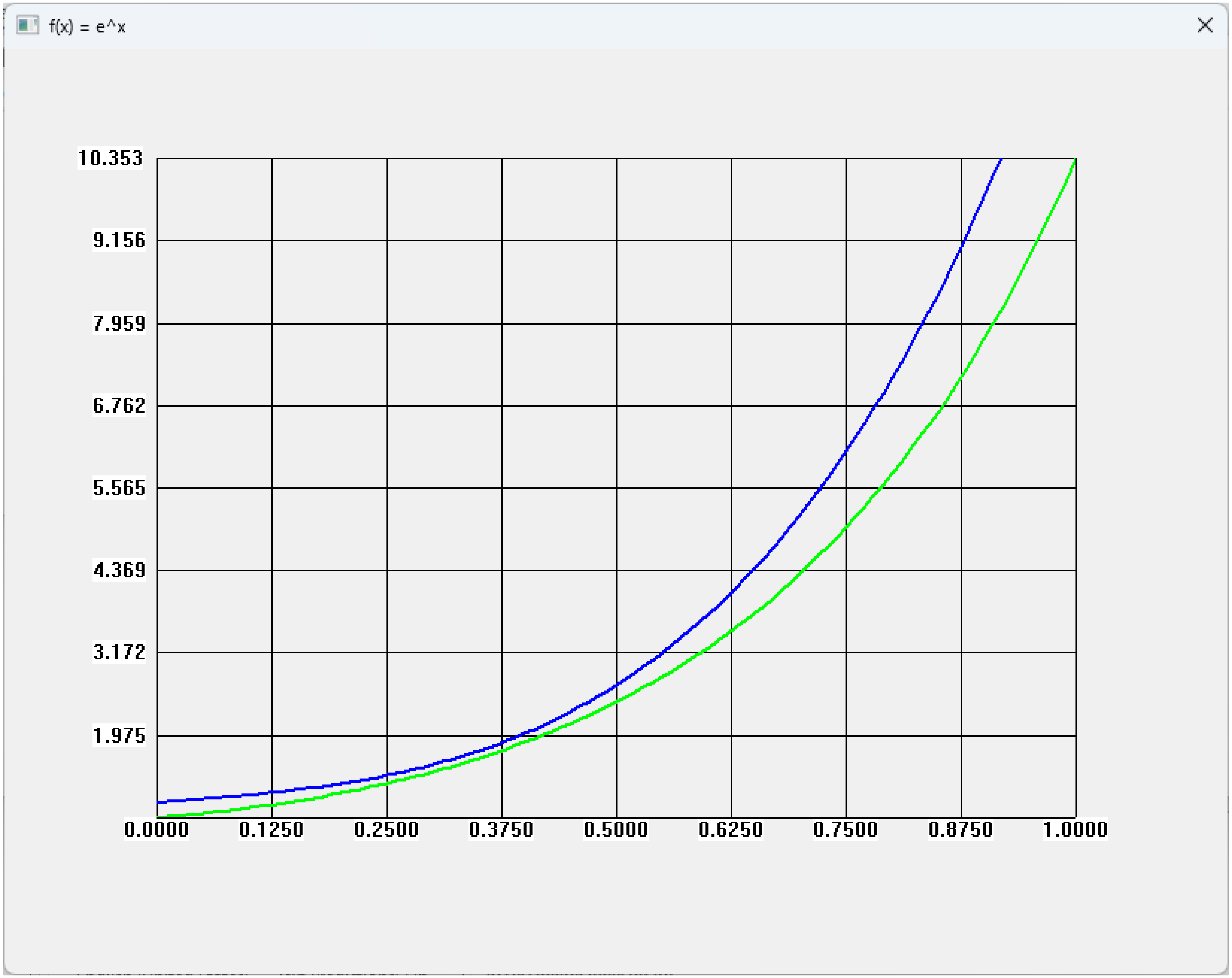
$$y''(x) = 0 + 9e^x = 9e^x$$

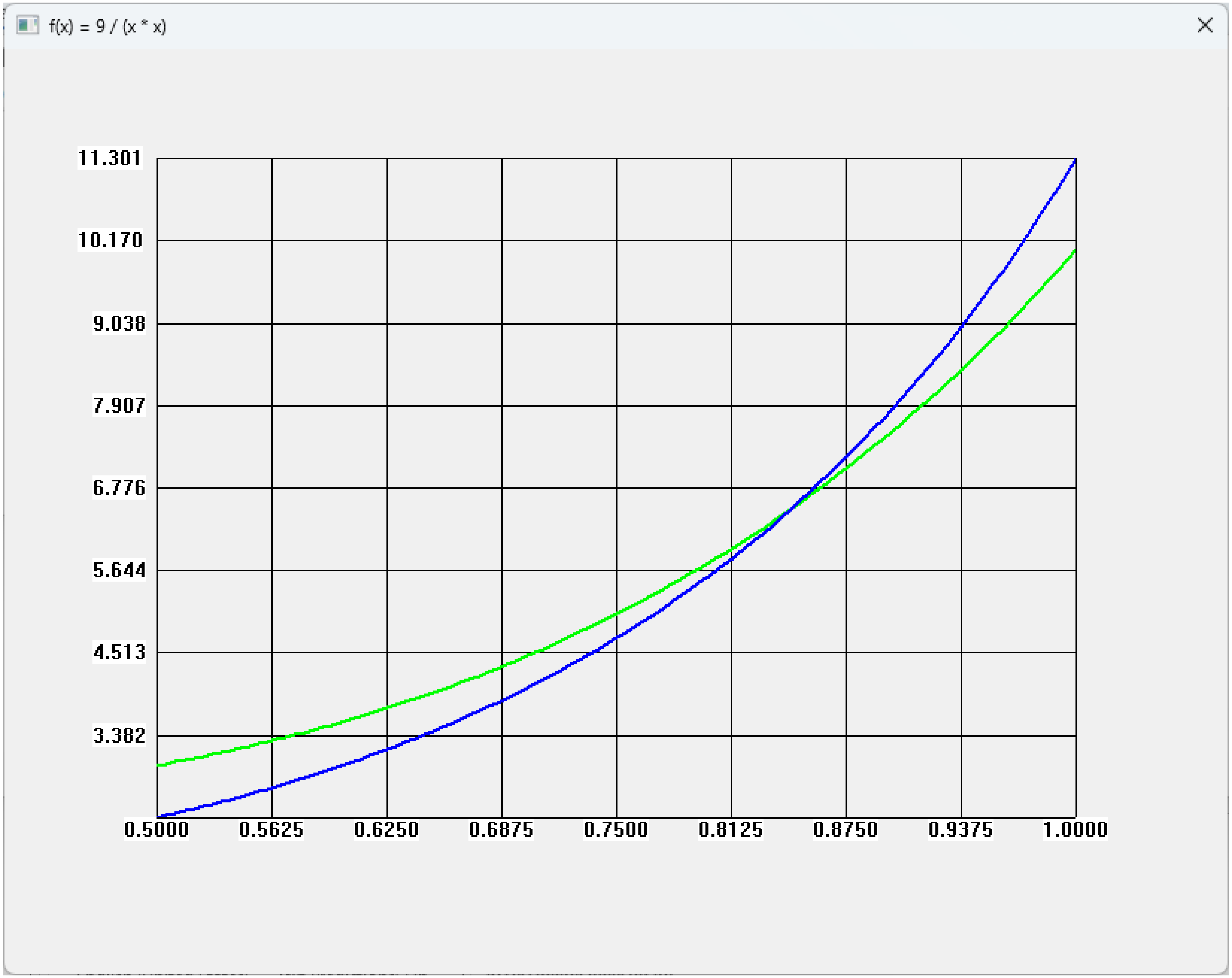
$$a_{n+2} = \frac{a_n}{(n+2)(n+1)} + \frac{9}{n!}$$


Input Dialog
n-Steps
100
n-Terms
10
Function Type Choice
f(x) = 0.0
f(x) = -6 cos(3x)
f(x) = e^x
f(x) = log(x)
Solution Type Choice
Analytic
Power Series
Draw
Table
OK
Cancel


f(x) = 0.0
10.051
8.878
7.705
6.532
5.359
4.186
3.013
1.840
0.0000
0.1250
0.2500
0.3750
0.5000
0.6250
0.7500
0.8750
1.0000


f(x) = -6cos(x)
15.527
13.711
11.895
10.079
8.263
6.448
4.632
2.816
0.0000
0.1250
0.2500
0.3750
0.5000
0.6250
0.7500
0.8750
1.0000


f(x) = e^x
10.353
9.156
7.959
6.762
5.565
4.369
3.172
1.975
0.0000
0.1250
0.2500
0.3750
0.5000
0.6250
0.7500
0.8750
1.0000



f(x) = 9 / (x * x)
11.301
10.170
9.038
7.907
6.776
5.644
4.513
3.382
0.5000
0.5625
0.6250
0.6875
0.7500
0.8125
0.8750
0.9375
1.0000