

Black Body Radiation (Thermal Radiation) by James Pate Williams, Jr.

References: [Planck's law - Wikipedia https://en.wikipedia.org/wiki/Black-body_radiation](https://en.wikipedia.org/wiki/Black-body_radiation)

The black body spectral radiance as a function of the frequency and temperature:

$$B_\nu(\nu, T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{\frac{h\nu}{k_B T}} - 1}$$

Also, in terms of the wavelength and temperature:

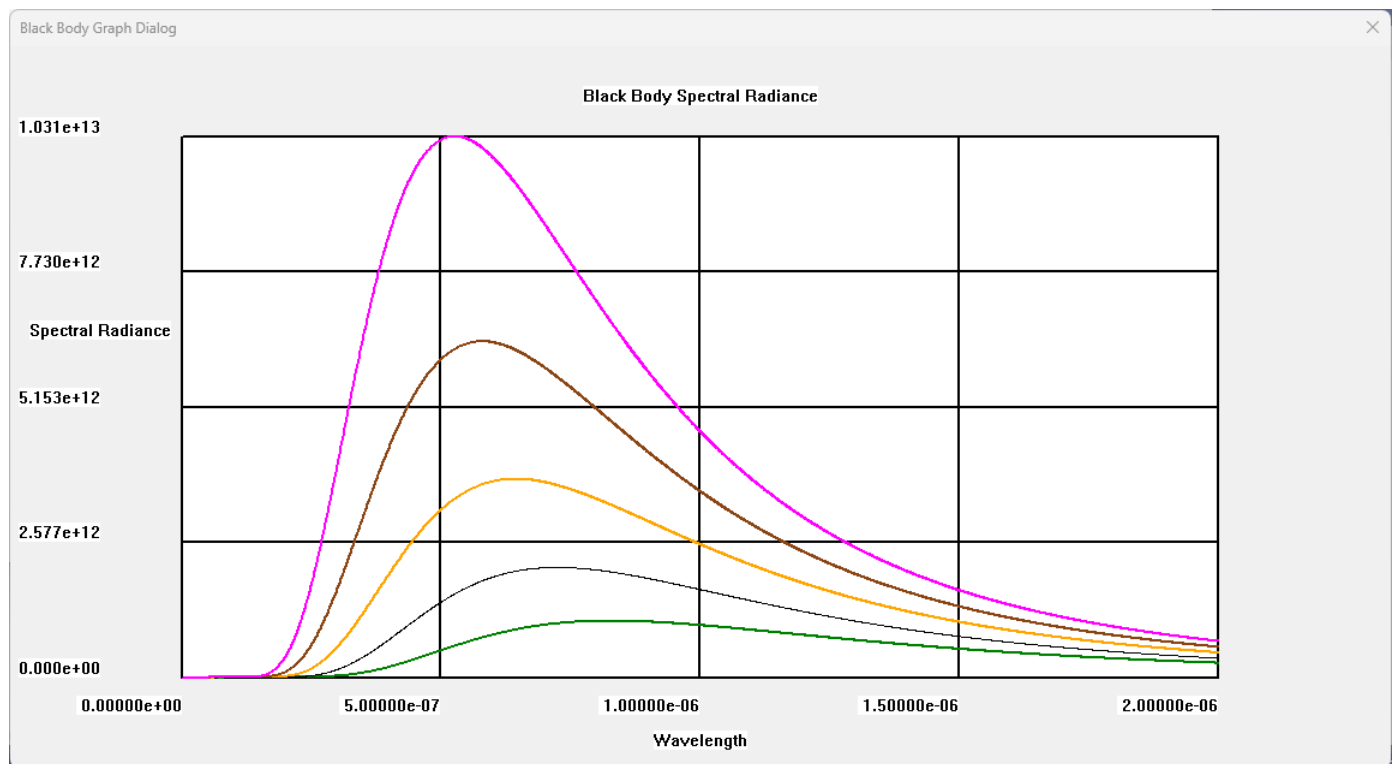
$$B_\lambda(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}$$

Where we have partially used the well-known formula:

$$c = \lambda\nu$$

$$\nu = \frac{c}{\lambda}$$

Graphs for the Black Body Spectral Radiance at the Temperatures 3500, 4000, 4500, 5000, and 5500 Degrees Kelvin



Black Body Table Dialog

https://en.wikipedia.org/wiki/Black-body_radiation

Wave	3500	4000	4500	5000	5500
1	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00
2	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00
3	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00
4	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	5.50264e-259
5	0.000000e+00	0.000000e+00	3.69544e-253	2.18225e-225	1.15034e-202
6	2.16352e-273	3.37896e-236	2.86521e-207	3.97974e-184	3.42602e-165
7	3.21680e-231	2.44159e-199	1.52553e-174	1.04717e-154	1.77813e-138
8	1.25231e-199	9.83851e-172	4.88888e-150	1.11233e-132	1.76780e-118
9	4.34208e-175	2.71298e-150	5.23635e-131	1.40441e-115	5.89908e-103
10	1.75999e-155	3.64488e-133	8.29290e-116	6.37273e-102	1.46314e-90
11	1.85563e-139	3.59684e-119	2.16288e-103	9.08497e-91	1.93406e-80
12	4.02255e-126	1.58969e-107	4.62912e-93	1.72523e-81	5.06192e-72
13	7.49691e-115	1.09943e-97	2.47017e-84	1.18569e-73	6.50388e-65
14	3.33630e-105	2.90663e-89	7.26547e-77	6.01954e-67	7.84396e-59
15	7.49634e-97	5.65325e-82	2.10692e-70	3.80826e-61	1.42791e-53
16	1.49084e-89	1.32141e-75	9.31490e-65	4.44314e-56	5.60133e-49
17	4.03056e-83	5.40165e-70	8.76074e-60	1.28989e-51	6.21541e-45
18	2.06796e-77	5.16911e-65	2.27095e-55	1.17609e-47	2.41037e-41
19	2.61995e-72	1.45763e-60	1.99032e-51	4.04710e-44	3.86009e-38
20	1.01170e-67	1.45593e-56	6.94468e-48	6.08782e-41	2.91704e-35
21	1.41191e-63	5.97800e-53	1.10105e-44	4.50822e-38	1.15874e-32
22	8.18582e-60	1.13967e-49	8.83759e-42	1.81125e-35	2.64272e-30
23	2.21200e-56	1.11550e-46	3.92646e-39	4.27786e-33	3.72199e-28
24	3.06617e-53	6.09539e-44	1.04015e-36	6.34994e-31	3.43956e-26

Compute Cancel

C/C++ function to compute the black body spectral radiance:

```
static double SpectralRadiance(double lambda, double T)
{
    double c = 299792458;
    double h = 6.62607015e-34;
    double k = 1.380649e-23;
    double B = h * c * c / pow(lambda, 5.0) * 1.0 /
        (exp(h * c / k / lambda / T) - 1.0);

    return B;
}
```

Expanding the following frequency-based function as a Taylor's Series and integrating term-by-term, we have Stefan's Law (reference - ***Stellar Atmospheres Second Edition*** by Dimitri Mihalas Exercise 1-3 page 7). Stefan's law governs the total energy density in thermodynamic equilibrium in the interior of a star and in the deepest layers of stellar atmospheres:

$$E_R^* = (8\pi h/c^3) \int_0^{\infty} (e^{h\nu/kT} - 1)^{-1} \nu^3 d\nu = a_R T^4$$

$$a_R = 8\pi^5 k^4 / (15c^3 h^3)$$

$$f(\nu) = (e^{h\nu/kT} - 1)^{-1}$$

$$x = h\nu/kT$$

$$g(x) = (e^x - 1)^{-1} = e^{-x} (1 - e^{-x})^{-1}$$

$$y = e^{-x}$$

$$g(y) = y(1 - y)^{-1}$$

$$g'(y) = (1 - y)^{-1} + y(1 - y)^{-2}$$

$$g''(y) = 2(1 - y)^{-2} + 2y(1 - y)^{-3}$$

$$g'''(y) = 6(1 - y)^{-3} + 6y(1 - y)^{-4}$$

$$g^{iv}(y) = 24(1 - y)^{-4} + 24y(1 - y)^{-5}$$

$$g^v(y) = 120(1 - y)^{-5} + 120y(1 - y)^{-6}$$

$$g(y) = y + y^2 + y^3 + y^4 + y^5 + \dots$$

$$g(x) = \sum_{n=1}^{\infty} e^{-nx}$$

$$\int_0^{\infty} (e^{h\nu/kT} - 1)^{-1} \nu^3 d\nu = (k^4 T^4 / h^4) \sum_{n=1}^{\infty} \int_0^{\infty} x^3 e^{-nx} dx = 6(k^4 T^4 / h^4) \sum_{n=1}^{\infty} 1/n^4 = 6(k^4 T^4 / h^4) \zeta(4)$$

$$E_R^* = 48\pi k^4 T^4 \zeta(4) / (c^3 h^3) = 48\pi^5 k^4 T^4 / (90c^3 h^3) = 8\pi^5 k^4 T^4 / (15c^3 h^3) = a_R T^4$$