

Three Pi Related Integrals by James Pate Williams, Jr.

Gauss Integral

$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$$
$$\int_{-\infty}^{\infty} e^{-x^2} dx = \left[\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-x^2-y^2} dx dy \right]^{1/2} = \left[\int_0^{\infty} \int_0^{2\pi} e^{-r^2} r dr d\theta \right]^{1/2} = \left[-2\pi \frac{1}{2} (e^{-\infty^2} - e^0) \right]^{1/2}$$
$$= \pi^{1/2} = \sqrt{\pi}$$

Cauchy Integral

$$\int_{-\infty}^{\infty} \frac{dx}{x^2 + 1} = \pi$$

Dirichlet Integral

$$\int_{-\infty}^{\infty} \frac{\sin(x) dx}{x} = \pi$$

Function Evaluation of Pi

$$4 \tan^{-1}(1) = \pi$$

For the Gauss Integral I used Simpson's Rule with 16384 steps and left and right endpoints of 600. The other two integrals were computed using Simpson's Rule with an Elitist Evolutionary Hill Climber with parameters the number of candidate solutions and the number of generations. All the floating-point arithmetic was performed using double precision real numbers.

First run of the Win32 C/C++ Desktop Application:

Function	Value of Pi	Runtime (mu-s)
Pi [4 * atan(1)]	= 3.141592653590	24
Gauss Integral	= 3.141592653590	902
Cauchy Integral	= 3.141340969599	31955716
Dirichlet Integral	= 3.141592216944	6912277

C:\Users\James\OneDrive\ThreeIntegrals\x64\Debug\ThreeIntegrals.exe (process 10188) exited with code 0 (0x0).
Press any key to close this window . . .

The parameters for the first Cauchy Integral evaluation were 100000 generations and a population of 1500 candidate solutions and the first Dirichlet Integral evaluation were 10000 generations and 500 candidate solutions.

Function	Value of Pi	Runtime (mu-s)
Pi [4 * atan(1)]	= 3.141592653590	14
Gauss Integral	= 3.141592653590	732
Cauchy Integral	= 3.141346364986	84658624
Dirichlet Integral	= 3.141589948406	30667264

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exe (process 5704) exited with code 0 (0x0).  
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The parameters for the second Cauchy Integral evaluation were 200000 generations and 3000 candidate solutions and the second Dirichlet Integral evaluation were 50000 generations and 1000 candidate solutions.