Maclaurin and Taylor Series Expansions by James Pate Williams, Jr.

Maclaurin Series expansion of the function:

$$f(x) = \cos(\tan^{-1}(x)) = (1 + x^2)^{-1/2}$$

$$f'(x) = -\frac{1}{2}(2x)(1 + x^2)^{-3/2} = -x(1 + x^2)^{-3/2}$$

$$f''(x) = -(1 + x^2)^{-3/2} - x\left(-\frac{3}{2}\right)(2x)(1 + x^2)^{-5/2} - (1 + x^2)^{-3/2} + 3x^2(1 + x^2)^{-5/2}$$

$$f'''(x) = -\left(-\frac{3}{2}\right)(2x)(1 + x^2)^{-5/2} + 6x(1 + x^2)^{-5/2} + (3x^2)\left(-\frac{5}{2}\right)(2x)(1 + x^2)^{-7/2}$$
$$= 9x(1 + x^2)^{-5/2} - 15x^3(1 + x^2)^{-7/2}$$

$$f^{iv}(x) = 9(1 + x^2)^{-5/2} + (9x)\left(-\frac{5}{2}\right)(2x)(1 + x^2)^{-7/2} - 45x^2(1 + x^2)^{-7/2}$$
$$- (15x^3)\left(-\frac{7}{2}\right)(2x)(1 + x^2)^{-9/2}$$
$$= 9(1 + x^2)^{-5/2} - 90x^2(1 + x^2)^{-7/2} + 105x^4(1 + x^2)^{-9/2}$$

$$f^{v}(x) = 9\left(-\frac{5}{2}\right)(2x)(1 + x^2)^{-7/2} - 180x(1 + x^2)^{-7/2} - 90x^2\left(-\frac{7}{2}\right)(2x)(1 + x^2)^{-9/2}$$
$$+ 420x^3(1 + x^2)^{-9/2} + 105x^4\left(-\frac{9}{2}\right)(2x)(1 + x^2)^{-11/2}$$
$$= -225x(1 + x^2)^{-7/2} + 1050x^3(1 + x^2)^{-9/2} - 945x^5(1 + x^2)^{-11/2}$$

$$f^{vi}(x) = -225(1 + x^2)^{-7/2} - 225x\left(-\frac{7}{2}\right)(2x)(1 + x^2)^{-9/2} + 3150x^2(1 + x^2)^{-9/2}$$
$$+ 1050x^3\left(-\frac{9}{2}\right)(2x)(1 + x^2)^{-11/2} - 4725x^4(1 + x^2)^{-11/2}$$
$$- 945x^5\left(-\frac{9}{2}\right)(2x)(1 + x^2)^{-13/2}$$
$$= -225(1 + x^2)^{-7/2} + 4625x^2(1 + x^2)^{-9/2} - 14175x^4(1 + x^2)^{-11/2}$$
$$+ 8505x^6(1 + x^2)^{-13/2}$$

$$f^{vii}(x) = -225\left(-\frac{7}{2}\right)(2x)(1 + x^2)^{-9/2} + 9250x(1 + x^2)^{-9/2} + \cdots$$
$$= 1575x(1 + x^2)^{-9/2} + 9250x(1 + x^2)^{-9/2} + \cdots = 10825x(1 + x^2)^{-9/2} + \cdots$$

$$f(0) = 1, f''(x) = -1, f^{iv}(x) = 9, f^{vi}(x) = -225, f^{vii}(x) = 10825$$

$$f(x) \cong 1 - x^2 + \frac{9}{4!}x^4 - \frac{225}{6!}x^6 + \frac{10825}{8!}x^8 - \cdots$$

Taylor Series expansion of the following function about x = 1:

$$f(x) = \ln(x)$$

$$f'(x) = \frac{1}{x} = x^{-1}$$

$$f''(x) = -x^{-2}$$

$$f'''(x) = 2x^{-3}$$

$$f^{iv}(x) = -6x^{-4}$$

$$f^{v}(x) = 24x^{-5}$$

$$f^{vi}(x) = -120x^{-6}$$

$$f^{vii}(x) = 720x^{-7}$$

$$f^{viii}(x) = 5{,}040x^{-8}$$

$$f^{ix}(x) = 40{,}320x^{-9}$$

$$f^{x}(x) = 362{,}880x^{-10}$$

$$x = 1$$

$$f(x) = \ln(1), f'(x) = 1, f''(x) = -1, f'''(x) = 2, f^{iv}(x) = -6, f^{v}(x) = 24, f^{vi}(x) = -120, f^{vii}(x) = 720, f^{viii}(x) = 5{,}040, f^{ix}(x) = 40{,}320, f^{x}(x) = 362{,}880$$

$$f(x) \cong \ln(1) + (x-1) - \frac{1}{2!}(x-1)^2 + \frac{2}{3!}(x-1)^3 - \frac{6}{4!}(x-1)^4 + \frac{24}{5!}(x-1)^5 - \frac{120}{6!}(x-1)^6 + \frac{720}{7!}(x-1)^7 - \cdots = \ln(1) + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-1)^n}{n}$$

n	n!
0	1
1	1
2	2
3	6
4	24
5	120
6	720
7	5040
8	40320
9	362880
10	3.6288e+06

cos(arctan(x)) = (1 + x^2)^-1/2
Maclaurin Series Expansion of (1 + x^2)^-1/2
 # is the number + 1 of nonzero terms in the summation
x = 0.0150000000

```
#       Series          (1 + x^2)^-1/2
0       1.0000000000    0.9998875190
1       0.9997750000    0.9998875190
2       0.9997750190    0.9998875190
3       0.9997750190    0.9998875190
4       0.9997750190    0.9998875190
Maclaurin Series Expansion of (1 + x^2)^-1/2
x = 0.1250000000
#       Series          (1 + x^2)^-1/2
0       1.0000000000    0.9922778767
1       0.9843750000    0.9922778767
2       0.9844665527    0.9922778767
3       0.9844653606    0.9922778767
4       0.9844653766    0.9922778767
Maclaurin Series Expansion of (1 + x^2)^-1/2
x = 0.5000000000
#       Series          (1 + x^2)^-1/2
0       1.0000000000    0.8944271910
1       0.7500000000    0.8944271910
2       0.7734375000    0.8944271910
3       0.7685546875    0.8944271910
4       0.7696034265    0.8944271910
Taylor Series Expansion of log(x)
About x = 1
x = 1.2500000000
#       Series          ln(x)
0       0.0000000000    0.2231435513
1       0.2500000000    0.2231435513
2       0.2187500000    0.2231435513
3       0.2239583333    0.2231435513
4       0.2229817708    0.2231435513
5       0.2231770833    0.2231435513
6       0.2231363932    0.2231435513
7       0.2231451125    0.2231435513
8       0.2231432052    0.2231435513
9       0.2231436290    0.2231435513
10      0.2231435337    0.2231435513
11      0.2231435554    0.2231435513
12      0.2231435504    0.2231435513
13      0.2231435515    0.2231435513
14      0.2231435513    0.2231435513
15      0.2231435513    0.2231435513
Taylor Series Expansion of log(x)
About x = 1
x = 1.5000000000
#       Series          ln(x)
0       0.0000000000    0.4054651081
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```
1       0.5000000000    0.4054651081
2       0.3750000000    0.4054651081
3       0.4166666667    0.4054651081
4       0.4010416667    0.4054651081
5       0.4072916667    0.4054651081
6       0.4046875000    0.4054651081
7       0.4058035714    0.4054651081
8       0.4053152902    0.4054651081
9       0.4055323041    0.4054651081
10      0.4054346478    0.4054651081
11      0.4054790370    0.4054651081
12      0.4054586920    0.4054651081
13      0.4054680820    0.4054651081
14      0.4054637223    0.4054651081
15      0.4054657568    0.4054651081
16      0.4054648032    0.4054651081
17      0.4054652520    0.4054651081
18      0.4054650400    0.4054651081
19      0.4054651404    0.4054651081
20      0.4054650927    0.4054651081
21      0.4054651154    0.4054651081
22      0.4054651046    0.4054651081
23      0.4054651098    0.4054651081
24      0.4054651073    0.4054651081
25      0.4054651085    0.4054651081
26      0.4054651079    0.4054651081
27      0.4054651082    0.4054651081
28      0.4054651081    0.4054651081
29      0.4054651081    0.4054651081
30      0.4054651081    0.4054651081

C:\Users\James\OneDrive\MaclaurinSeries\x64\Debug\MaclaurinSerie
s.exe (process 27692) exited with code 0 (0x0).
Press any key to close this window . . .
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