Some Results from an Astrophysical Textbook by James Pate Williams, Jr. and the Microsoft Copilot

Reference: ***Stellar Atmospheres*** (c) 1978 by Dimitri Mihalas Chapter 3 “The Grey Atmosphere” see Equations (3-76), (3-68), (3-79), (3-80) and Table 3-2 Section 3-4 pages 71-73.

Using 25-point Gauss-Legendre quadrature, the computed asymptotic Hopf constant was $q(\infty) = 0.7104460885$, agreeing with the value 0.71044609 cited by Mihalas from Placzek and Seidel (1947) to approximately nine significant digits. Using more than 25 points does not increase the accuracy of the computations.

Pseudocode

1. Generate Gauss-Legendre abscissas and weights.

2. Evaluate the H(mu) function.

3. Evaluate the Hopf-function integrals.

4. Compute q(tau) for selected tau values.

5. Compare q(infinity) with literature value.

6. Measure runtime over 1000 trials.

Computer results:

```
Gauss-Legendre n = 25
tau     q(tau)
0.00    0.5773486142
0.01    0.5882374687
0.03    0.6012409234
0.05    0.6107581842
0.10    0.6279171561
0.20    0.6495491466
0.30    0.6633650117
0.40    0.6730903211
0.50    0.6802927332
0.60    0.6858005919
0.80    0.6935333173
1.00    0.6985388039
1.50    0.7051298301
2.00    0.7079164289
2.50    0.7091929996
3.00    0.7098076803
3.50    0.7101139759
```

```
4.00    0.7102704921
5.00    0.7103951668
q_infinity = 0.7104460885
Statistics are measured in microseconds
Number of trials (observations) = 1000
Mean = 1159.1670000000
Median = 1056.5000000000
Minimum = 828
Maximum = 8938
Standard Deviation = 342.8564680280
Total Runtime = 1159 milliseconds

C:\Users\James\OneDrive\LimbDarkeningLaw\x64\Debug\LimbDarkening
Law.exe (process 1592) exited with code 0 (0x0).
Press any key to close this window . . .
```